Supplemental Homework Problems

1. The atmosphere:

   (a) Determine the density distribution if the atmosphere is taken as isothermal; ie constant temperature.

   (b) Compare the pressure and density at 10 km altitude for isothermal, isentropic, \(-6.5K/km\) lapse-rate, and the standard atmospheres.

2. Bernoulli equation: Consider the flight of jetliner at 900 km/hr at 10km. Determine the pressure reading of a nose-mounted Pitot tube if the flow is treated as

   (a) incompressible

   (b) compressible, isentropic

3. Determine the equation of the streamline passing through \((x, y) = (1, 1)\) in the velocity field given by

   (a) \((u, v) = (y, -x)\)

   (b) \((u, v) = (x, y)\)

4. Determine the equation of the streamline passing through \((x, y) = (1, 1, 1)\) in the three-dimensional velocity field given by \((u, v, w) = (x, y, z)\)

5. Evaluate the divergence and vorticity for the flow fields in #3 above. What can you say about each of the flows?

6. Show that in a potential flow field, the iso-potential lines \(\varphi(x, y) = \text{constant}\) are normal to streamlines \(\psi(x, y) = \text{constant}\).

7. Construct the stream function \(\psi(r, \theta)\) and potential function \(\phi(r, \theta)\) for the velocity field given in cylindrical polar coordinates \((r, \theta)\)

   \[
   (u_r, u_\theta) = \left[ \left( 1 - \frac{1}{r^2} \right) \cos \theta, -\left( 1 + \frac{1}{r^2} \right) \sin \theta \right]
   \]

8. Consider the velocity field in cylindrical polar coordinates \((r, \theta)\)

   \[
   \mathbf{u} = [u_r, u_\theta] = \left[ 0, \frac{\Gamma_o}{2\pi r}(1 - e^{-r^2/\sigma^2}) \right]
   \]

   where \(\sigma\) is a parameter.

   (a) Determine the vorticity field of the flow.

   (b) By taking the limits of the flow as \(r \to 0\) and as \(r \to \infty\), comment on the nature of the flow field near the origin and far way from the origin.
9. Consider the flow given in #8 above. Calculate the circulation integral

\[ \Gamma(r) = \oint_{C} \mathbf{u} \cdot d\mathbf{l} \]

where \( C \) is a circle of radius \( r \). Plot \( \Gamma(r) \).

10. Determine the velocity along the centerline of a pair of line vortices that are separated by \( b \) and have circulations \( +\Gamma \) and \( -\Gamma \).

11. Determine the velocity along the centerline of a vortex ring of radius \( R \) and circulation \( \Gamma \).

12. The sail of a sail boat can be considered as half-wing flying right on the water surface. Sketch the vortex diagram shortly after the sailboat started sailing, say, normal to the wind direction. Give both side and top views of your sketch and clearly indicate vortex rotation directions.

13. Evaluate the principal value integral

\[ I = \text{P} \int_{0}^{\infty} \frac{dx}{x^3 - 1} \]

\textit{Hint: consider}

\[ \lim_{\epsilon \to 0} \left( \int_{0}^{1-\epsilon} + \int_{1+\epsilon}^{\infty} \right) \]

14. Verify the following integral

\[ \int_{0}^{\pi} \sin n\theta \sin \theta d\theta \cos \theta - \cos \eta = -\pi \cos n\eta \]

\textit{Hint: See von Mises}

15. Find the center of pressure for the lift distribution

\[ \gamma(x) = 2\alpha U \sqrt{\frac{c-x}{x}} \]

16. Consider a curved thin airfoil whose camber is given as

\[ z = 4z_m \left[ \frac{x}{c} - \left( \frac{x}{c} \right)^2 \right] \]

Determine the location of the aerodynamic center of the airfoil.

17. Assuming that the flow over a thin airfoil may be approximated as that over a flat plate at zero angle of attack for purposes of drag calculations, estimate the drag coefficient at low and high Reynolds numbers.

18. Consider a vortex panel of linearly varying strength \( \gamma(x) = \gamma_0 + \gamma_1(x/l) \) whose end points are at (0,0) and (1,0). Determine the induced velocity at an arbitrary point \((x, y)\).
19. Consider a span loading on a planar wing of span $b$

$$\Gamma(y) = \Gamma_0 \left[ 1 - \left( \frac{y}{b/2} \right)^2 \right]^{3/2}$$

where $\Gamma_0$ is the root circulation. Determine the lift and induced drag coefficients of the wing.

20. Consider a symmetric supersonic airfoil whose surfaces are sinus curves $y_{u,l} = \epsilon \sin(\pi x/c)$. The airfoil is flying supersonically at an angle of attack $\alpha$. Determine its sectional lift and drag coefficients.

21. The pressure coefficient at a point on a two-dimensional airfoil is -0.4 at $M = 0.1$. Estimate the pressure coefficient at that point at $M = 0.75$.

22. A two-dimensional airfoil has a minimum pressure coefficient of -0.782 at $M=0.3$. Determine the critical Mach number of the airfoil.

23. A symmetric diamond-shaped airfoil of chord $c$ has a maximum thickness $t$ at a distance $ac$ from its leading edge. What is its wave drag coefficient due to its thickness at supersonic speeds. Find the value of $a$ for minimum drag.