Homework 2

1. (Computational work) Consider the potential flow past a cylinder of radius \( a \) when it is translating at a constant velocity of \((U, 0)\) along the \( x \) axis in an \( xy \)– coordinate system. Determine the pathlines of fluid elements initially located at \((x/a, y/a) = (0.01, 0), (0.1, 0), (0.5, 0), (1.0, 0), (1.5, 0), (2, 0)\)

2. (Computational work) For the flow above, plot the streaklines of dye traces released at \((x/a, y/a) = (1.01, 0), (1.1, 0), (1.5, 0), (2, 0)\)

3. Given the velocity field \( \mathbf{u} = (u(x, y), v(x, y)) \), consider a short material line segment that passes through the origin, making an angle \( \theta \) with the \( x \) axis.
   (a) Calculate the angular velocity \( d\theta/dt \) of that line as a function of the partial derivatives \( \partial u/\partial x \) etc evaluated at the origin.
   (b) Calculate the average angular velocity of all such lines \( 0 \leq \theta \leq 2\pi \).
   (c) Show that the answer found in (b) is the same as the average for any two mutually perpendicular lines, say \( \theta \) and \( \theta + \pi/2 \).
   (d) Verify that there are just two material lines that rotate at the average angular speed, and that these are identified by
   \[
   \theta_1 = \frac{1}{2} \tan^{-1} \left( \frac{\partial v/\partial x + \partial u/\partial y}{\partial u/\partial x - \partial v/\partial y} \right) \quad \text{and} \quad \theta_2 = \theta_1 + \frac{\pi}{2}.
   \]
   (e) Verify, by use of a trigonometric identity, that the two lines found in (d) are the axes of the principal rates of deformation.
   *Sherman 3.3.*

4. Write down the deformation and rotation tensors and find the magnitudes and directions of the principal strain rates for the flow field
   \[
   (u_r, u_\theta, u_z) = \left(0, \frac{\Gamma}{2\pi r}, 0\right).
   \]

5. Given the velocity field, in spherical polar coordinates, \((r, \theta, \phi)\),
   \[
   \frac{\mathbf{u}}{U} = \left(\frac{u_r}{U}, \frac{u_\theta}{U}, \frac{u_\phi}{U}\right) = \left\{ \cos \theta \left[1 - \left(\frac{a}{r}\right)^3\right], -\sin \theta \left[1 + \frac{1}{2} \left(\frac{a}{r}\right)^3\right], 0 \right\}
   \]
   (a) Calculate all nonzero components of the viscous stress tensor \( \tau \) as functions of \( r \) and \( \theta \).
   (b) Calculate the dissipation function
   \[
   \Phi = \lambda (\text{div} \mathbf{u})^2 + \frac{1}{2} \mu (\text{def} \mathbf{u}) \cdot (\text{def} \mathbf{u})
   \]
   as a function of \( r \) and \( \theta \).
   (c) Calculate the integral of \( \Phi \) over the volume \( a \leq r \leq \infty \).