1. (25%) Consider the super jumbo aircraft A380 cruising at 13 km altitude at Mach number \( M = 0.85 \). Its wing span is \( b = 80 \) m, wing area \( A = b \times c = 845 \) m\(^2\), and mass \( W = 500,000 \) kg. Determine the lift coefficient and the bound circulation of the wing.

At 13 km, \( T = 216.65 \) K, hence \( \sqrt{\frac{\gamma RT}{\mu}} = \sqrt{1.4 \times 287 \times 216.65} = 295 \) m/s, \( U = aM = 295 \times 0.85 = 250 \) m/s.

\[
C_L = \frac{W}{\frac{1}{2} \rho U^2 A} = \frac{500,000 \times 9.81}{\frac{1}{2} \times 0.265 \times (250)^2 \times 845} = 0.700
\]

\[
\Gamma = \frac{W}{\rho Ub} = \frac{500,000 \times 9.81}{0.265 \times (250) \times 80} = 925 \text{ m}^2/\text{s}
\]

2. (25%) Estimate the total skin friction drag on the wing in question 1, assuming (a) laminar flow, and (b) turbulent flow.

\[
2 \times D = 2 \times 0.664 \rho \nu^{1/2} L^{1/2} U^{3/2} \times b =
\]

\[
= 2 \times 0.664 \times 0.265 \times (1.421 \cdot 10^{-5}/0.265)^{1/2} \times (845/80)^{1/2} \times 250^{3/2} \times 80 = 2.6 \cdot 10^3 \text{ N}
\]

\[
2 \times D = 2 \times c_f \frac{1}{2} \rho U^2 A = 2 \times 0.0035 \times \frac{1}{2} \times 0.265 \times 250^2 \times 845 = 4.9 \cdot 10^4 \text{ N}
\]

3. (25%) Consider a vortex sheet of uniform strength \( \gamma \), on the \( x \)-axis, extending from \( x = -a \) to \( x = +a \). Determine the self induced velocity distribution on the sheet. How do you think the sheet will move or deform?

\[
v_{ind} = \frac{-\gamma}{2\pi} \int_{-a}^{a} \frac{d\xi}{x-\xi} = \frac{-\gamma}{2\pi} \ln \left| \frac{x-a}{x+a} \right|
\]

4. (25%) Consider the flow field whose stream function is given as (in dimensionless form)

\[
\psi(x, y) = \ln \left( \frac{\cosh x - \cos y}{\cosh x + \cos y} \right)
\]

(a) Show that \( \psi = \text{constant} \) on \( y = \pm \pi/2 \), hence, it represents a flow between two parallel walls.

(b) Determine the flow velocity along the \( x \)-axis.

(c) Determine the flow velocity along the \( y \)-axis.

(d) Sketch the flow pattern. Can you guess what it is?

\[
(u, v) = (\psi_y, -\psi_x) = \left( \frac{2 \sin y \cosh x}{\cosh^2 x - \cos^2 y}, \frac{-2 \sin x \cos y}{\cosh^2 x - \cos^2 y} \right)
\]