A New Efficient Computational Method for Thermal Radiation in Participating Media

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- Background
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  - Radiative Transfer Equation
  - Finite Volume Radiation Method
- Q_L Method
  - Formulation
  - Solution Procedure
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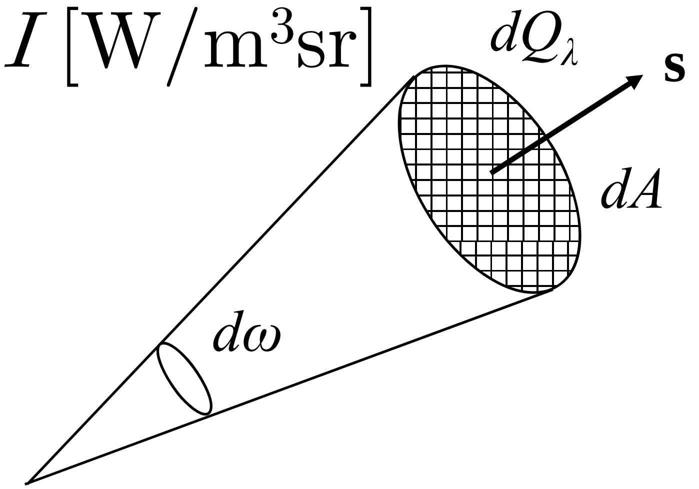
- Summary

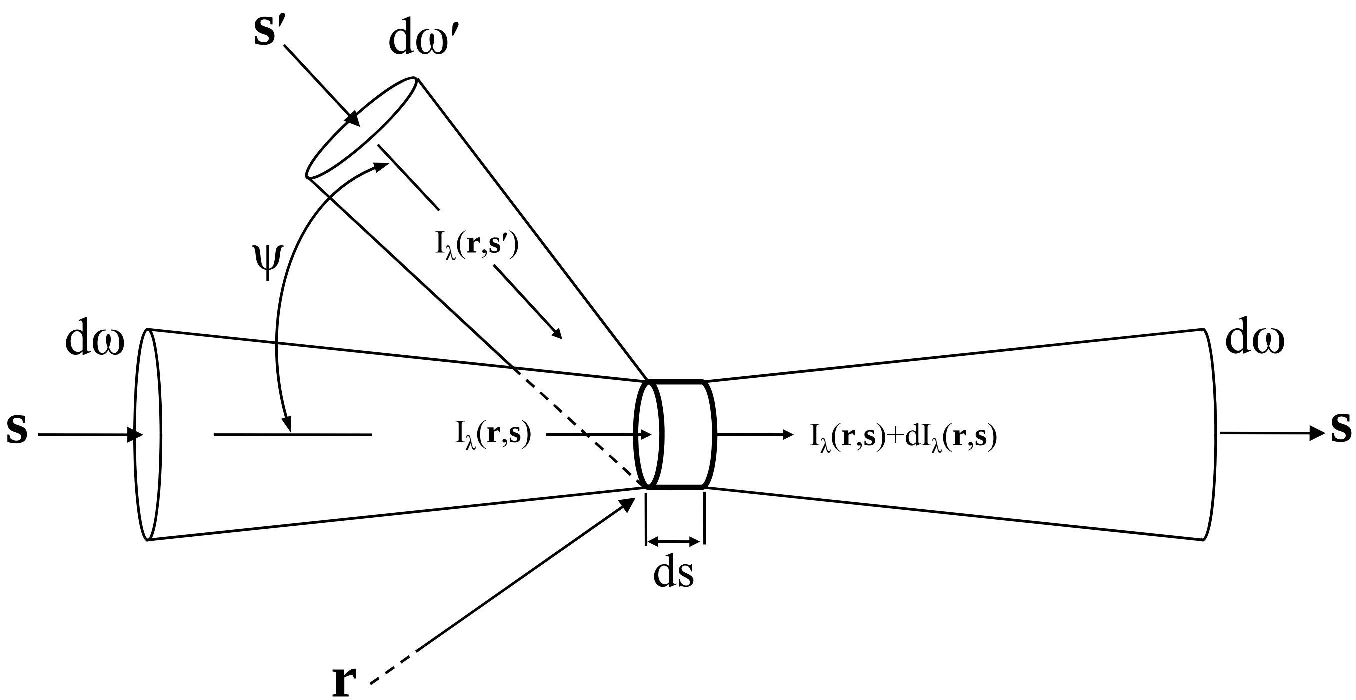
Thermal Radiation

- Mode of heat transfer cased by photon exchange or electromagnetic waves
- Very important in high-temperature and large-scale systems: furnaces & engines
- Working variable: Intensity I [W/m^{3}sr]

$$I_\lambda(\mathbf{r}, \mathbf{s}) = \lim_{dA, d\omega, d\lambda \to 0} \frac{dQ_\lambda}{dA\, d\omega\, d\lambda}$$



Radiative Transfer Equation (RTE)



$$I_b = \frac{\sigma T^4}{\pi}$$

$$\frac{dI_\lambda}{ds} = -K_\lambda I_\lambda - \sigma_\lambda^s I_\lambda + K_\lambda I_{b\lambda} + \frac{\sigma_\lambda^s}{4\pi}\int_{4\pi} I_\lambda(\mathbf{r},\mathbf{s}')\Phi(\mathbf{s}',\mathbf{s})d\omega'$$

Radiation is Complex

- Intensity is a 6-dimensional variable:
  - depends on position(3), direction(2), wavelength(1), and local temperature
  - Gray approximation: dependence on wavelength is neglected

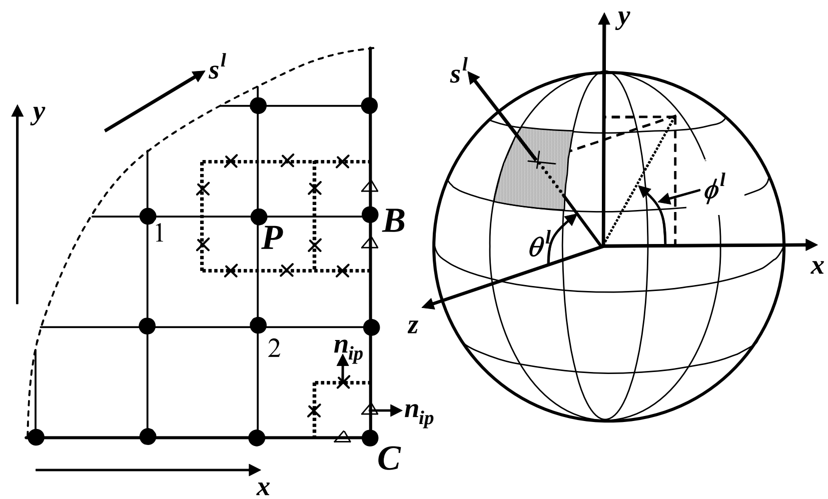
- Integro-differential nature of the RTE

- Interdirectional coupling is mainly caused by scattering and wall reflection

Computational Methods

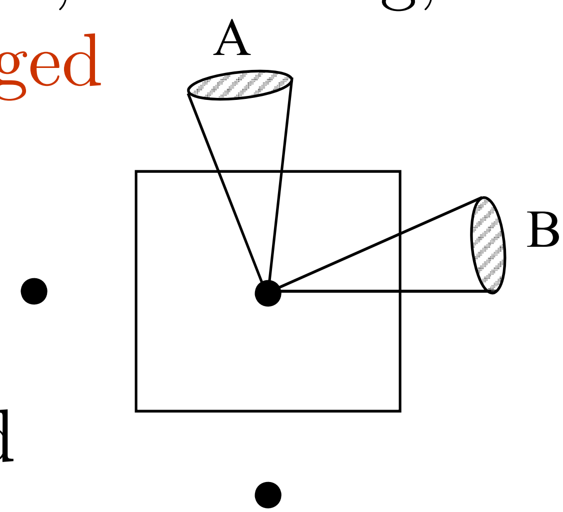
- P_1 Method
  - Assumes isotropic distribution for intensity
  - Forms a single equation for average intensity at each node
  - Is inexpensive but inaccurate

- Finite Volume Method (FVM)
  - Discretizes direction to L finite solid angles
  - Forms L equations for intensity at each node
  - Is very accurate but expensive



Solution Procedure for FVM

- In the conventional FVM (explicit update):
  - Previously calculated temperature, scattering, and wall reflection terms are lagged
  - New intensity is calculated
  - Temperature, scattering, and wall reflection terms are updated



- Weakly participating: practical and inexpensive
- Strongly participating: impractical and expensive
- The available acceleration schemes are not effective

The Q_L Method

- Originally formulated by G. Raithby in 1996
- Integrating the RTE over V_P and 4π

$$\int_{4\pi}\int_{A_s} I(\mathbf{s}\cdot\mathbf{n})\,dA_s\,d\omega = \int_{4\pi}\int_{V_P}\left[-\left(K+\sigma^s\right)I + KI_b + \frac{\sigma^s}{4\pi}\int_{4\pi} I(\mathbf{r},\mathbf{s}')\Phi(\mathbf{s}',\mathbf{s})\,d\omega'\right]dV\,d\omega$$

- Scattering terms are cancelled in RHS

$$\mathrm{RHS} = 4\pi\int_{V_P} K\left(I_b - I_a\right)dV \qquad I_a = \frac{1}{4\pi}\int_{4\pi} I\,d\omega$$

Radiation Energy Equation

- Defining the *radiant heat flux vector*

$$\mathbf{q}(\mathbf{r}) = \int_{4\pi} I\mathbf{s}d\omega$$

- LHS becomes

$$\text{LHS} = \int_{A_{s,P}} \mathbf{q} \cdot \mathbf{n} dA_s$$

- Radiation Energy Equation (REE)

$$\int_{A_{s,P}} \mathbf{q} \cdot \mathbf{n} dA_s = 4\pi \int_{V_P} K\left(I_b - I_a\right) dV$$

Final discrete equation of the REE

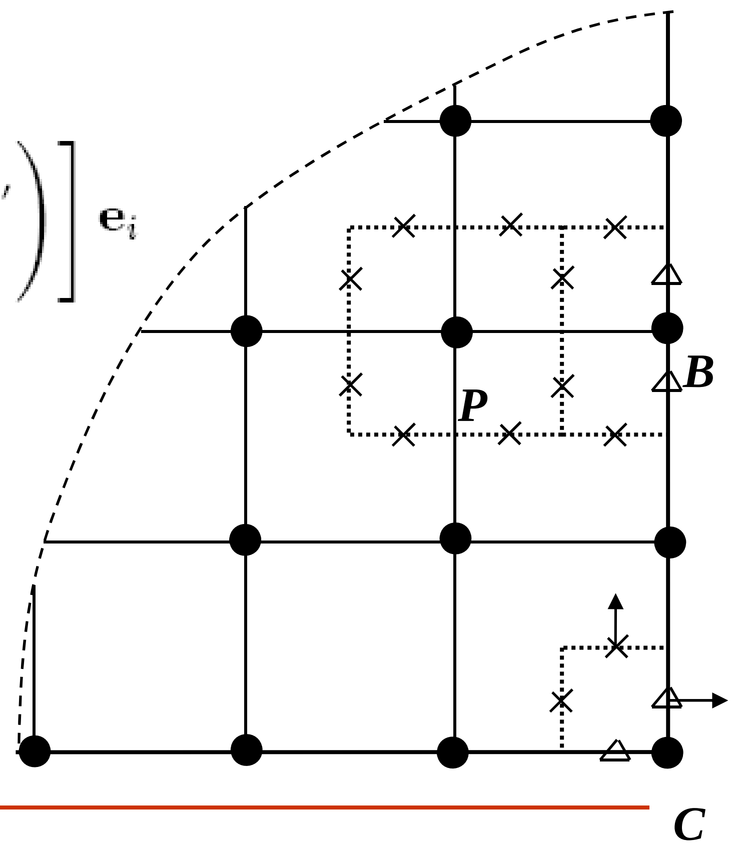
$$\sum_{ip} \mathbf{q}_{ip} \cdot \mathbf{n}_{ip} A_{s,ip} = -4\pi K_P V_P \left(I_{a,P} - I_{b,P} \right)$$

- For interior integration points

$$\mathbf{q}(\mathbf{r}) = \left[-\frac{1}{\kappa} \frac{\partial}{\partial x_j} \left(I_a \sum_{l=1}^{L} \alpha^l D_{ij}^l \right) + \Omega \left(I_a \sum_{l'=1}^{L} \alpha^{l'} F_i^{l'} \right) \right] \mathbf{e}_i$$

- For boundary integration points

$$q_{ip} = \epsilon_s I_{a,p} \sum_{\mathbf{N}^l \cdot \mathbf{n}_{ip} > 0} \alpha_P^l \mathbf{N}^l \cdot \mathbf{n}_{ip} - \epsilon_s \sigma T_s^4$$



In a multi-process CFD code:

- Shift in paradigm: solving for I_a instead of I^l
- The single equation for I_a can be coupled with equations of other processes
- Since the interdirectional coupling is avoided, the phase weights are easily provided
- It is expected to obtain the accuracy of the FVM with the solution cost of the P_1 method

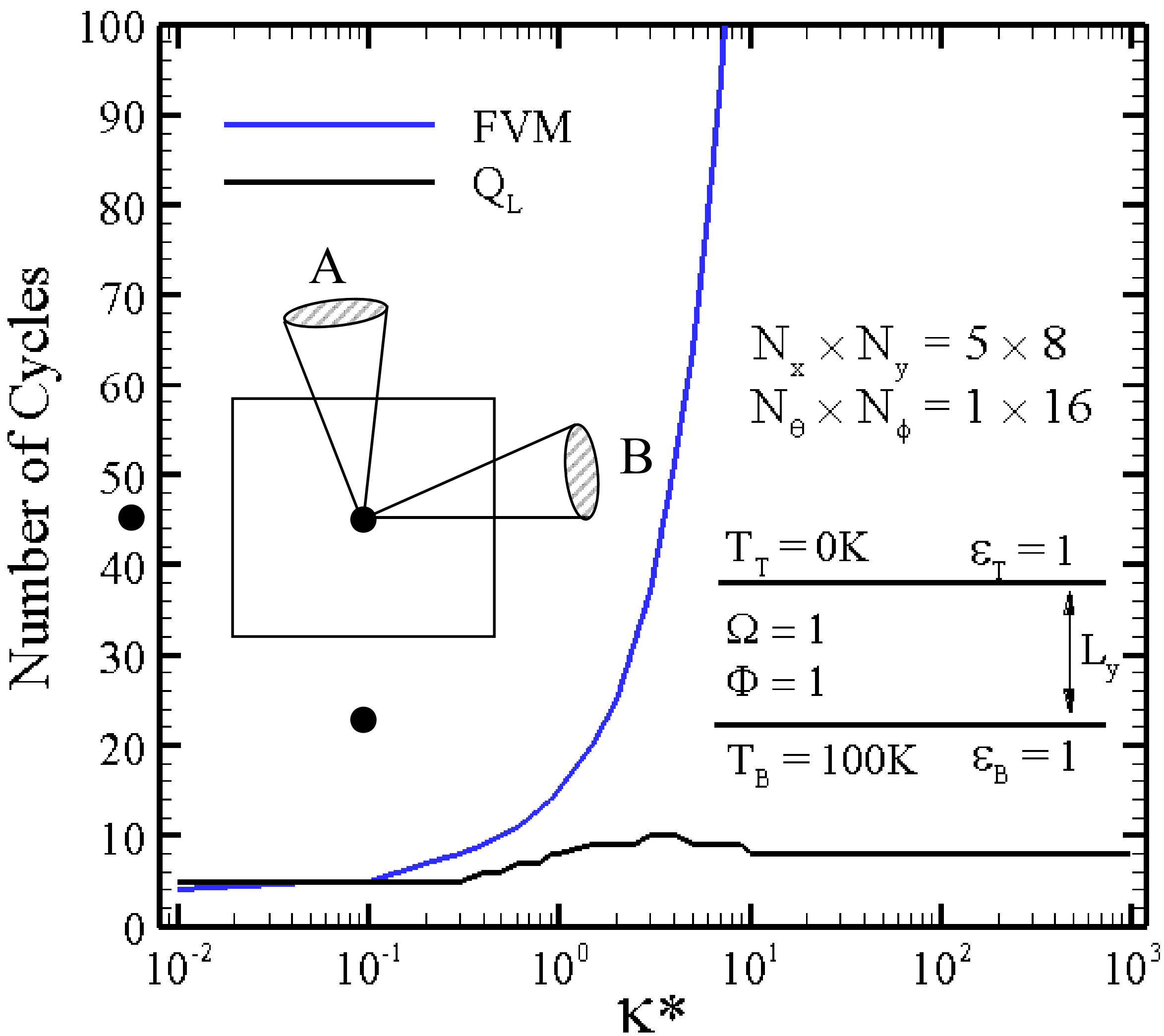
Problem Description

- Several Radiation-only problems are solved with the Q_L Method and FVM (explicit update)
- Radiation-only: temperature field is known or disconnected from the radiation field
- Walls are isothermal and diffuse-gray with constant and known temperature and emissivity
- Enclosed gray medium has known and constant K, σ^s, and $\Phi(\mathbf{s}', \mathbf{s})$

Numerical Solution

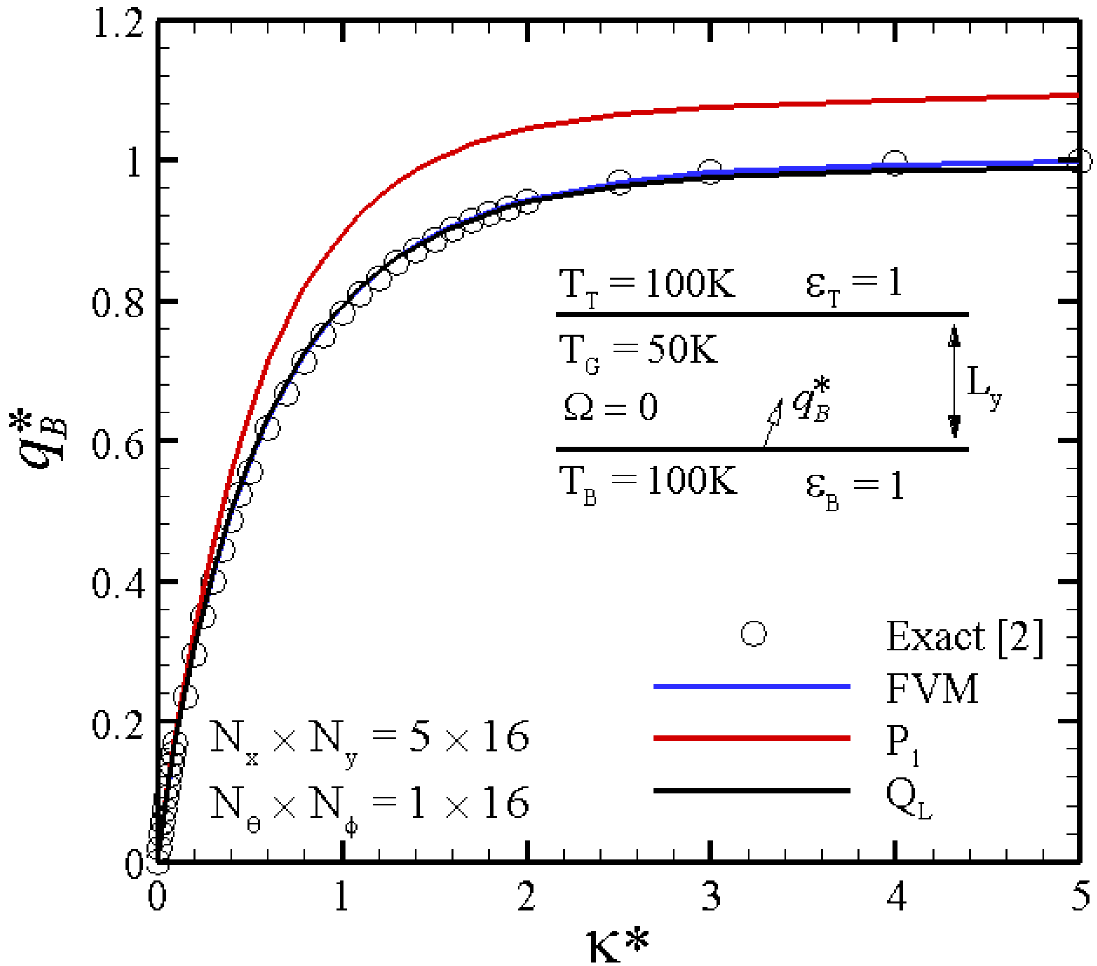
- Spatial grid: Vertex-centered finite volume Cartesian grid with $N_x \times N_y$ control volumes
- To efficiently solve the I_a equations, the Additive Correction Multigrid solver with V-cycle is used.
- Convergence criterion: reducing maximum scaled residual below 10^{-5}

Case 1



Number of Cycles
100
90
80
70
60
50
40
30
20
10
0
FVM
Q_L
A
B
N_x × N_y = 5 × 8
N_θ × N_ϕ = 1 × 16
T_T = 0K ε_T = 1
Ω = 1
Φ = 1
L_y
T_B = 100K ε_B = 1
10^-2
10^-1
10^0
10^1
10^2
10^3
κ*

Case 3: Isothermal Absorbing-Emitting Medium



$$q_B^* = q_B / \sigma \left(T_B^4 - T_G^4 \right)$$

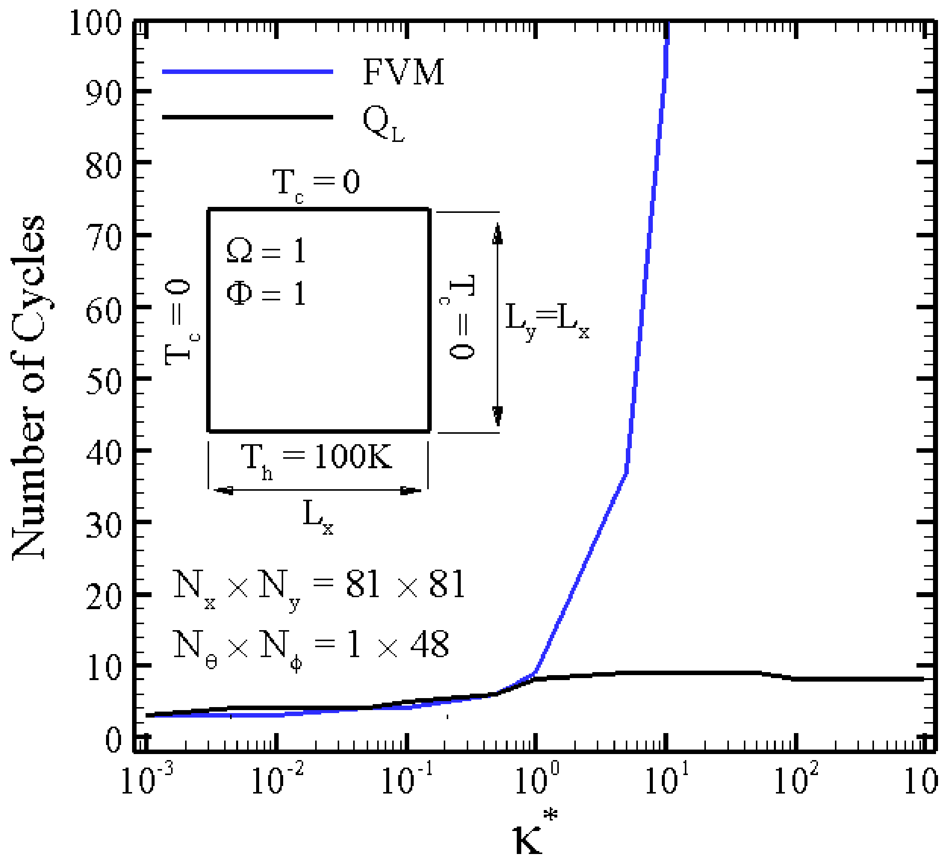
- FVM converges with 2 cycles
- Max. error is 4%
- Q_L method converges with 4 cycles
- Max error is 4%
- Max. error of P_1: 15%

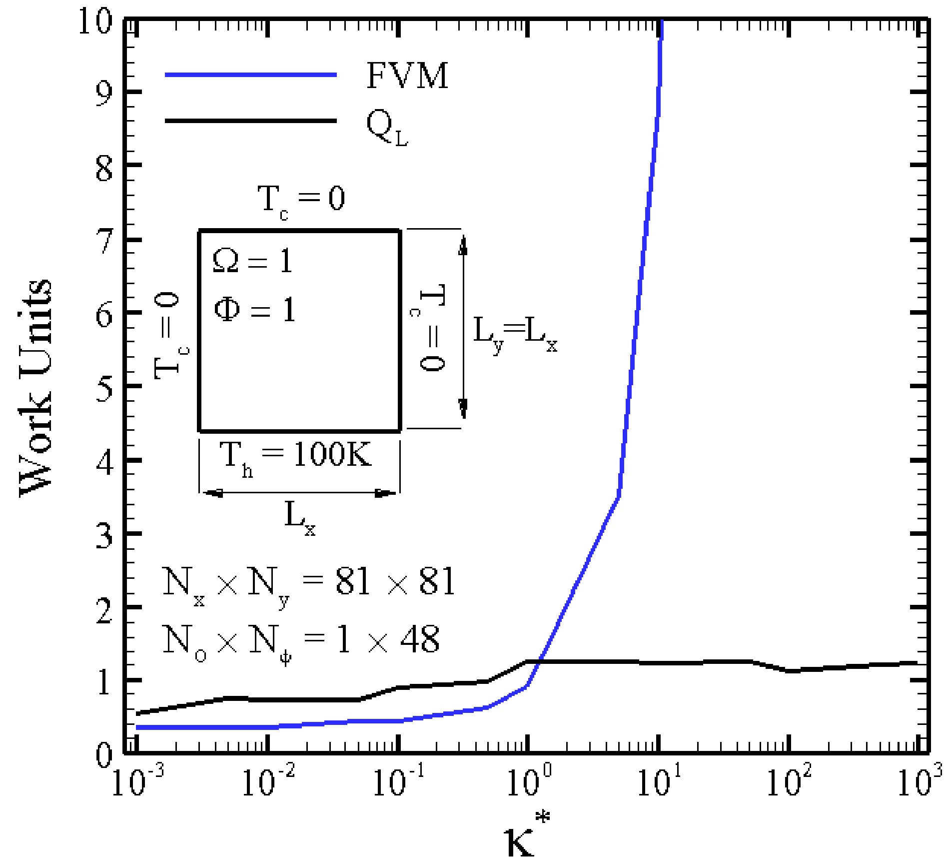
Computational Time

- One WU for a specific set of spatial and angular grids is defined as the computational time required to do 10 explicit updates in the FVM on that grid (i.e. 10 FVM-cycles)

- The number of work units is found by dividing the CPU time for the solution by the CPU time for 10 FVM cycles.

Case 4: Solution Cost





Summary

- The Q_L method was formulated for the general case and applied to several test cases
- This method was found to have a good accuracy on the tested coarse grids
- It was shown that this method has a rapid convergence rate, regardless of grid size and optical thickness
- Its solution cost is comparable to cost of the FVM in the weakly participating media and is much lower in the strongly participating media.

Summary

It was discussed that when applied to a multi-process CFD code:

- The Q_L method incorporates one single equation per node for thermal radiation

- It is expected to have an accuracy comparable to the FVM and solution cost similar to the P1 method.