

## ME 290-R

### Homework # 5

Due at start of class, Monday April 22

Solid modeling CAD systems support geometric queries about the models, such as their volumes. In this homework, we will consider how such queries are supported.

1) Recall that for two vectors, the magnitude of the cross product gives the area of a symmetric quadrilateral defined by the two vectors when placed tail to tail. Thus for any three points P, Q, and R in  $\mathfrak{R}^3$ , the formula  $\frac{1}{2}\|(Q-P) \times (R-P)\|$  gives the area of the triangle they define.

If P, Q, and R all lie in the  $xy$  plane, the formula

$$\frac{1}{2} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot ((Q-P) \times (R-P))$$

gives the *signed area* of the triangle they define (e.g. positive for a counter-clockwise triangle viewed from a point on the positive  $z$  axis in a right handed coordinate system).

a) Confirm that the signed area of a triangle with vertices  $P = (0,0)$ ,  $Q = (x_i, y_i)$ , and  $R = (x_{i+1}, y_{i+1})$  is

$$\frac{1}{2}(x_i y_{i+1} - x_{i+1} y_i).$$

b) For a planar polygon with  $n - 1$  vertices  $v_1, \dots, v_{n-1}, v_n$ ,  $v_n = v_1$  and  $v_i = (x_i, y_i)$  for each  $i$ , prove that the signed area of the polygon is

$$\frac{1}{2} \sum_{i=1}^{n-1} (x_i y_{i+1} - x_{i+1} y_i).$$

2) Write a matlab program that uses this formula to calculate the signed area of an arbitrary planar input polygon. You may assume it is a simple polygon (one loop, no holes) with a non-self-intersecting boundary. Your program should read its input from a file that contains the  $x$  and  $y$  coordinates of the vertices in counter-clockwise order, one vertex per line, with a space between the two coordinates. A sample input file will be provided on the website. Turn in a printout of your code and email a soft-copy to me for testing (in a zip file with problem 4).

3) Drawing upon your understanding of the formula above, develop a similar formula based on tetrahedrons instead of triangles to calculate the volume of a 3D polyhedron defined by triangular faces. Give the formula and explain its derivation.

4) Write a matlab program that uses the formula you developed in 3) to calculate the signed volume of an arbitrary 3D input polyhedron defined by oriented triangular faces provided in .STL format. You may assume the input has a watertight, non-self-intersecting boundary. A link to a matlab .STL reader will be provided on the class website.

Again, turn in a printout of your code and email a soft-copy to me for testing (in a zip file with problem 2).

Two STL files generated in SolidWorks, named with their volume as calculated by SolidWorks, will also be available from the class website under the homework link. To generate your own STL files in SolidWorks, use the full version available in 2105/2107 Etchevery to save parts as ASCII STL files (click on “options” after you select .stl as the “save as” format to select “ASCII” instead of the default binary .stl format). To see the part volume SolidWorks calculates, look under the Tools menu for mass properties. (I have not verified that the volumes SolidWorks calculated for these parts are correct, or in the same units as the STL files, etc. — I have seen SolidWorks output STL files that didn’t match the part modeled — so no guarantees on this data! If your program gets different results, make a simpler part

whose volume you can calculate by hand to test whether the bug is yours or SolidWorks.)

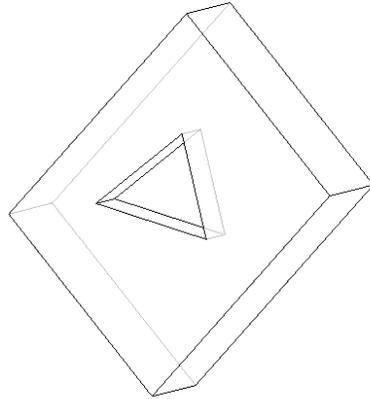


Figure 1: *4332.stl is for this rectangular part with a pocket*

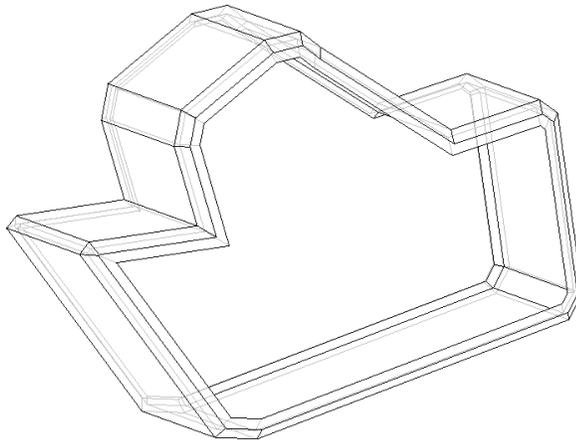


Figure 2: *56278.stl is this shelled part*