University of California at Berkeley
College of Engineering
Mechanical Engineering Department

ME219, Spring 2012 **Liwei Lin**

Project #2 (part II & III) – 25% Class Grade Weight

Pre-Optimization Preparation for the Optimal Design of a MEMS Resonator
Due Date: 9:00 a.m. Wednesday 4 April 2012

A simple MEMS resonator is constructed from a single cantilever beam and a single lumped proof mass. The structure has been simplified to allow you to focus on optimization, not on flexural formula derivation. The resonator motion is in the x-direction. The resonator is presumed to move perfectly in single-axis motion. You may consider the proof mass to act, dynamically, as a point mass. The figure at the end of this document shows the design as well as some of the variables. For this project, you will perform pre-optimization preparations for the optimal design of the MEMS resonator. This preparation will comprise two parts, 1) Monotonicity Analysis of the Resonator and 2) Familiarization with Numerical Optimization Tools.

PART I1. Monotonicity Analysis. The design objective for this resonator is to make the natural frequency as low as possible. There are constraints on the die, the stresses and the feature width that the fabrication technology can achieve. This is a concise and precise statement of the problem: Minimize the objective function, *obj*, which is the square of the natural frequency of the resonator:

$$obj = \frac{k}{m}$$

Subject to inequality constraints:

g_1:	$h \geq h_{min}$	Fabrication-limited beam width
g_2:	$y_D \geq y_M + L$	Mass and flexure fit inside die
g_3:	$x_D \geq x_M$	Mass fits inside die
g_4:	$\sigma_{max} \geq \sigma$	Stress limited to working stress
g_5:	$\delta \geq \delta_{min}$	Minimum displacement of mass
g_6:	$\delta \leq \frac{1}{2}(x_D - x_M)$	Maximum displacement of mass

And subject to equality constraints:

h_1:	$k = \frac{3EI}{L^3}$	Spring constant of flexure
h_2:	$I = \frac{bh^3}{12}$	Definition of second moment of area
h_3:	$M = x_M y_M b * \rho$	Definition of mass of resonator
h_4:	$\sigma = \sigma(E, h, L, \delta)$	A <u>student-derived</u> stress formula in <u>precisely</u> these specific variables

In this project, the following parameters are held constant and their values are not subject to the wishes of the designer:

b	Thickness of all structures
ρ	Density of material of all structures
h_{min}	Minimum flexure height due to fabrication limits
E	Elastic modulus of material for all structures
δ_{min}	Minimum lateral deflection imposed on the resonator
σ_{max}	Maximum allowable working stress in material
x_D	Maximum x-dimension of die on which resonator fits
y_D	Maximum y-dimension of die on which resonator fits

The following are a few of the variables in the problem and may be freely adjusted by the designer. (Are there any other variables not mentioned here? Identify, define and put them in. You must justify your choices!)

x_M	X-dimension of proof mass
y_M	Y-dimension of proof mass

For this specific optimization problem, please be sure to:

1. Derive the formula for maximum stress in the flexure.
2. Formulate a statement of all constraints in the standard form, i.e., such as: $g_1 = h_{min} - h \leq 0$
3. Formulate a monotonicity table with initial entries (the "always true" case) as was done in class. Work to <u>avoid any regional monotonicities</u>.
4. Formulate cases and sub-cases, each with their own monotonicity table, as was done in class. Take care to proceed in a way that generates the minimum number of cases.
5. Formulate a logic table that summarizes all the design cases as was done in class.

PART III. Familiarization with Numerical Optimization Tools. For this part, please perform both constrained and unconstrained optimization of a mathematical function. The experience gained from this familiarization exercise will assist you greatly when its time to numerically optimize an actual MEMS design. It is recommended that you 1) use MATLAB™ to perform your calculations, 2) perform your calculations via M-Files and 3) submit all MATLAB™ M-Files you wrote.

The mathematical function that you must minimize is the **<u>modified</u>** Rosenbrock Test Function:

$$f(x_1, x_2) = 300 * (x_2 - x_1^2)^2 + (1 - x_1)^2$$

On this function there are two constraints:

$$g_1 = x_2 + \frac{15}{7} x_1 \leq 0$$

and

$$g_2 = \frac{21}{31}(x_1 - 1.25)^2 - (x_2 - 2.17) \leq 0$$

Please perform the following tasks. Note that I'm asking YOU to determine "appropriate" scales, and how you choose that scale is part of your grade.

1. On an appropriate scale, with variables shown over an appropriate range, plot the Rosenbrock Test Function using your own M-File.
2. On an appropriate scale, with variables shown over an appropriate range, plot the Rosenbrock Test Function with inequality constraint g_1 shown.
3. On an appropriate scale, with variables shown over an appropriate range, plot the Rosenbrock Test Function with inequality constraint g_2 shown.
4. Perform a numerical, unconstrained optimization of the Rosenbrock Test Function and plot the optimal point on the graph in Task 1. Show your initial guess for the numerical optimization on this graph as well.
5. Perform a numerical, constrained optimization of the Rosenbrock Test Function, subject to inequality constraint g_1 alone, and plot the optimal point on the graph in Task 2. Show your initial guess for the numerical optimization on this graph as well.
6. Perform a numerical, constrained optimization of the Rosenbrock Test Function, subject to inequality constraint g_2 alone, and plot the optimal point on the graph in Task 3. Show your initial guess for the numerical optimization on this graph as well.
7. Make a table containing all three cases of initial guesses and answers.

Tasks 1 through 6 should be reported in the "Results" section with appropriate text and captions. Task 7 should be reported in the "Summary of Results" section. There is no contribution of Part III to the "Theory" section since you are not developing any theory in Part III. (But Part II DOES have a major contribution to the "Theory" section.) However, there must be a "Code Verification" section so you can show why your numerical results can be trusted. If, however, you use some theory that you must derive as part of your code verification efforts, then derive it and report it in the "Code Verification" section.