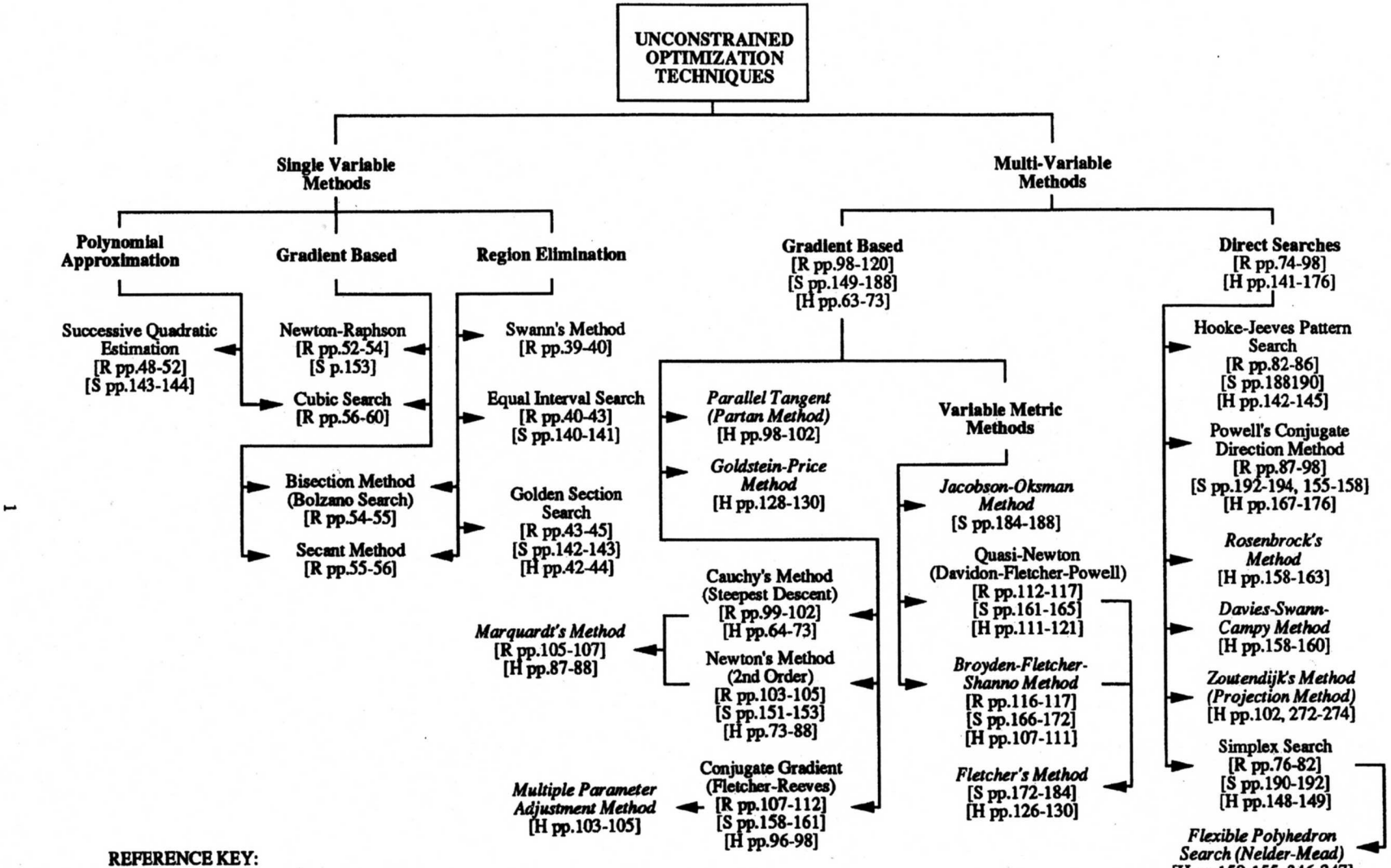
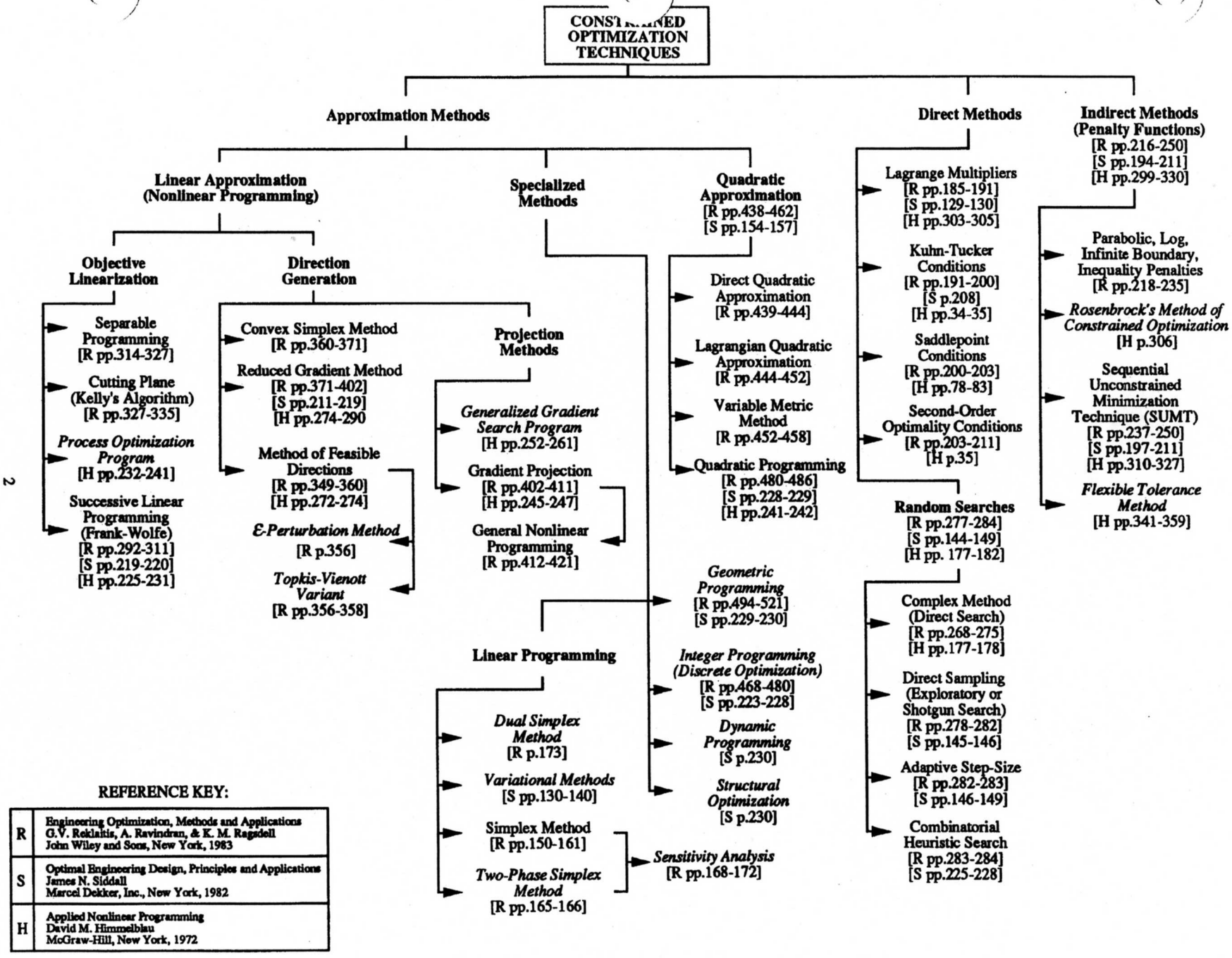


REFERENCE KEY:

R	Engineering Optimization, Methods and Applications G.V. Reklaitis, A. Ravindran, & K. M. Ragsdell John Wiley and Sons, New York, 1983
S	Optimal Engineering Design, Principles and Applications James N. Siddall Marcel Dekker, Inc., New York, 1982
H	Applied Nonlinear Programming David M. Himmelblau McGraw-Hill, New York, 1972



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DESCRIPTION OF UNCONSTRAINED METHODS

Single Variable Methods:

Bisection Method (Bolzano Search) - the minima of a unimodal objective is determined on an interval by region elimination utilizing the derivative of the objective.

Cubic Search - the function value and derivative are used to approximate the objective with a cubic. The minima of the approximation is used for the subsequent step.

Equal Interval Search - reduces the interval containing the optimum of a unimodal objective function by 0.5 with two function evaluations each step.

Golden Section Search - reduces the interval containing the optimum of a unimodal objective by 0.38297 with one function evaluation each step.

Newton-Raphson - the root of a linear approximation of the derivative of the objective function determines the next step. The second derivative is used in the approximation.

Secant Method - combines Newton's method with region elimination. The root of a secant approximation of the derivative determines the next step.

Successive Quadratic Estimation - approximates the objective function as a quadratic on an interval. The minima of the approximation is used for the subsequent step.

Swann's Method - an heuristic expanding pattern search to bracket the optimum.

Multi-variable Methods:

Broyden-Fletcher-Shanno Method - extension of Davidon-Fletcher-Powell method which eliminates the one-dimension search at each step but requires more complicated matrix calculations.

Cauchy's Method - classic method of local steepest descent. First derivative information of the objective is used to determine search direction for the next step. Effectively uses successive linear approximation of the objective.

Conjugate Gradient Method (Fletcher-Reeves) - quadratically convergent using only first order information. Quadratic approximation and conjugate directions are used. Easily implemented.

DSC Method (Davies-Swann-Campy) - extension of Fletcher's method which locates the minimum of the objective in each of the orthonormal directions like the Davidon-Fletcher-Powell method.

Fletcher's Method - combines Broyden-Fletcher-Shanno and Davidon-Fletcher-Powell. Eliminates one dimensional search at each stage and uses factorizing of the inverse hessian matrix.

Flexible Polyhedron Search (Nelder-Mead) - same as simplex search but allows irregular polyhedron vertex spacing to improve performance.

Goldstein-Price Method - approximates the hessian using a half factorial design and performs a matrix inversion. Similar performance to Davidon-Fletcher-Powell Method.

Hooke-Jeeves Pattern Search - combines exploratory and pattern moves with heuristic rules.

Jacobson-Oksman Method - Similar to Fletcher's method. Models objective as homogeneous function yielding an optimum in N+2 steps for an N dimensional homogeneous objective.

Marquardt's Method - combines Newton's and Cauchy's methods, requiring second-order information. Effectively uses Cauchy far from the solution and Newton close to the solution.

Multiple Parameter Adjustment Method - modification of Conjugate Gradient method which provides for more optimal search directions for non-quadratic objective functions.

Newton's Method - first and second derivative information is used to implement successive quadratic approximation of the objective. Minimum of approximation determines next step.

Parallel Tangent Method - a parallel to the tangent to the objective at the last search step is used as the search direction for the current search.

Powell's Conjugate Direction Method - approximates the objective function as a quadratic. A set of transformation vectors (conjugate directions) are calculated which will determine the optimum of a truly quadratic objective in exactly N searches for an N dimensional objective.

Quasi-Newton Method (Davidon-Fletcher-Powell) - uses a changing metric matrix at each step to determine the search direction. A robust technique but requires storage of a large matrix.

Rosenbrock's Method - similar to Hooke-Jeeves pattern search but the orthonormal search directions are aligned with the quadratic approximation to the objective at each step.

Simplex Search - utilizes a set of N+1 search points arranged on a polyhedron equidistantly (a regular simplex) for an N dimensional objective. The function is evaluated at each vertex. The worst is reflected through the centroid to determine the new search direction.

Zoutendijk's Projection Method - utilizes a projection matrix to calculate conjugate directions.

DESCRIPTION OF CONSTRAINED METHODS

Approximation Methods:

Convex Simplex Method - generalizes the linear simplex method to linearly constrained nonlinear objective functions.

Cutting Plane - uses successive linear programming on decreasing subinterval which is reduced via cutting planes.

Direct Quadratic Approximation - similar to successive linear programming but uses quadratic approximation of each nonlinear function and solves resulting series of approximating subproblems.

Dual Simplex Method - modified simplex method incorporating Duality Theory.

Dynamic Programming - suited for allocation problems and sequential decision problems.

ε-Perturbation Method - extension of method of Feasible Directions which uses an iterative adjustment on the positive tolerance parameter, ε, which is used in forming the active constraint set.

General Nonlinear Programming - extension of Gradient Projection to nonlinear constraints.

Generalized Gradient Search Program - uses steepest descent search interior to a feasible region, forward difference approximation of objective partial derivatives, and history of successful steps.

Geometric Programming - constrained optimization of problems involving a special class of polynomial functions, *posynomials*, in the objective and constraints.

Gradient Projection - directly calculates the projection of a linearly constrained objective gradient onto the constraint surfaces avoiding some unsatisfactory features of the Reduced Gradient method.

Integer Programming - linear programming with some discrete objective variables.

Lagrangian Quadratic Approximation - utilizes the Lagrangian function to extend Newton's method to accommodate constraints.

Method of Feasible Directions - utilizes a metric matrix which leads to a feasible direction that gives the greatest improvement in the objective without violating any constraint.

Process Optimization Program - similar to successive linear programming. Linearization of the problem is repeated at each iteration.

Quadratic Programming - an optimization problem involving a quadratic objective function with linear constraints.

Reduced Gradient Method - the gradient of the objective function after transformation into a reduced space is used to determine the search direction.

Sensitivity Analysis - study of how changes in input data (uncertain coefficients) affects the simplex (linear programming) solution.

Separable Programming - certain types of nonlinear constrained optimization problems can be solve by reformulation into equivalent problems involving only linear functions.

Simplex Method - method of sequential steps for iterative solution of linear programming problems.

Structural Optimization - structural design methods using knowledge of modes of failure.

Successive Linear Programming - objective and constraint functions are linearly approximated at each iteration. Descent directions are determined from the approximation.

Topkis-Vienott Variant Method - extension of method of Feasible Directions which eliminates active constraint formulation and substitutes an alternative direction finding subproblem avoiding jamming.

Two-Phase Simplex Method - extension of simplex method to handle introduction of artificial variables needed to put a problem in the proper form for simplex method solution.

Variable Metric Method - approximates second derivative matrices using Quasi-Newton update formulas requiring only differences of gradients.

Variational Methods - use of methods similar to Variational Calculus which give the from of a continuous solution directly. Variational problems arise from the nature of the formulation.

Indirect Methods:

Flexible Tolerance Method - utilizes information from near-feasible as well as feasible points. The restriction on feasible points is gradually tightened. Many unconstrained methods can then be applied.

Parabolic, Log, Infinite Boundary, Inequality Penalties - forms of various penalty functions. The penalty terms enforce constraints by penalizing the objective when constraints are violated.

Rosenbrock's Method of Constrained Optimization - objective is modified so that it vanishes outside the feasible region and assumed negative in the interior. Then Rosenbrock's method for unconstrained minimization is applied.

Sequential Unconstrained Minimization Technique (SUMT) - classic penalty function algorithm. An unconstrained minimization is done in each variable direction of the augmented (penalized) objective function.

Direct Methods:

Adaptive Step-Size - random trial points are used to generate a search direction while step length in that direction is based on previous history of successes and failures.

Combinatorial Heuristic Search - utilizes a random search over a discretized finite grid with a heuristic termination algorithm.

Complex Method - modification of the simplex method which utilizes a set of randomly generated vertices. Infeasible points are retracted towards the centroid of previously generated points until feasible.

Direct Sampling - a series of random trial points are generated. The objective value of each feasible point is compared to the current best objective value.

Kuhn-Tucker Conditions - optimality conditions for the general nonlinear objective problem with (or without) equality and inequality constraints.

Lagrange Multipliers - Lagrange multipliers are parameters used to combine equality constraints into the objective function. Unconstrained optimization can then proceed.

Saddlepoint Conditions - optimality conditions for non-differentiable functions.

Second-Order Optimality Conditions - optimality conditions for twice differentiable functions.

COMPARISON OF BOOKS

Relkaitis, Ravindran, & Ragsdell - most complete with clear detailed derivations and discussions.

Siddall - less clear derivations, fewer methods covered, more application oriented.

Himmelblau - older methods and terminology covered.