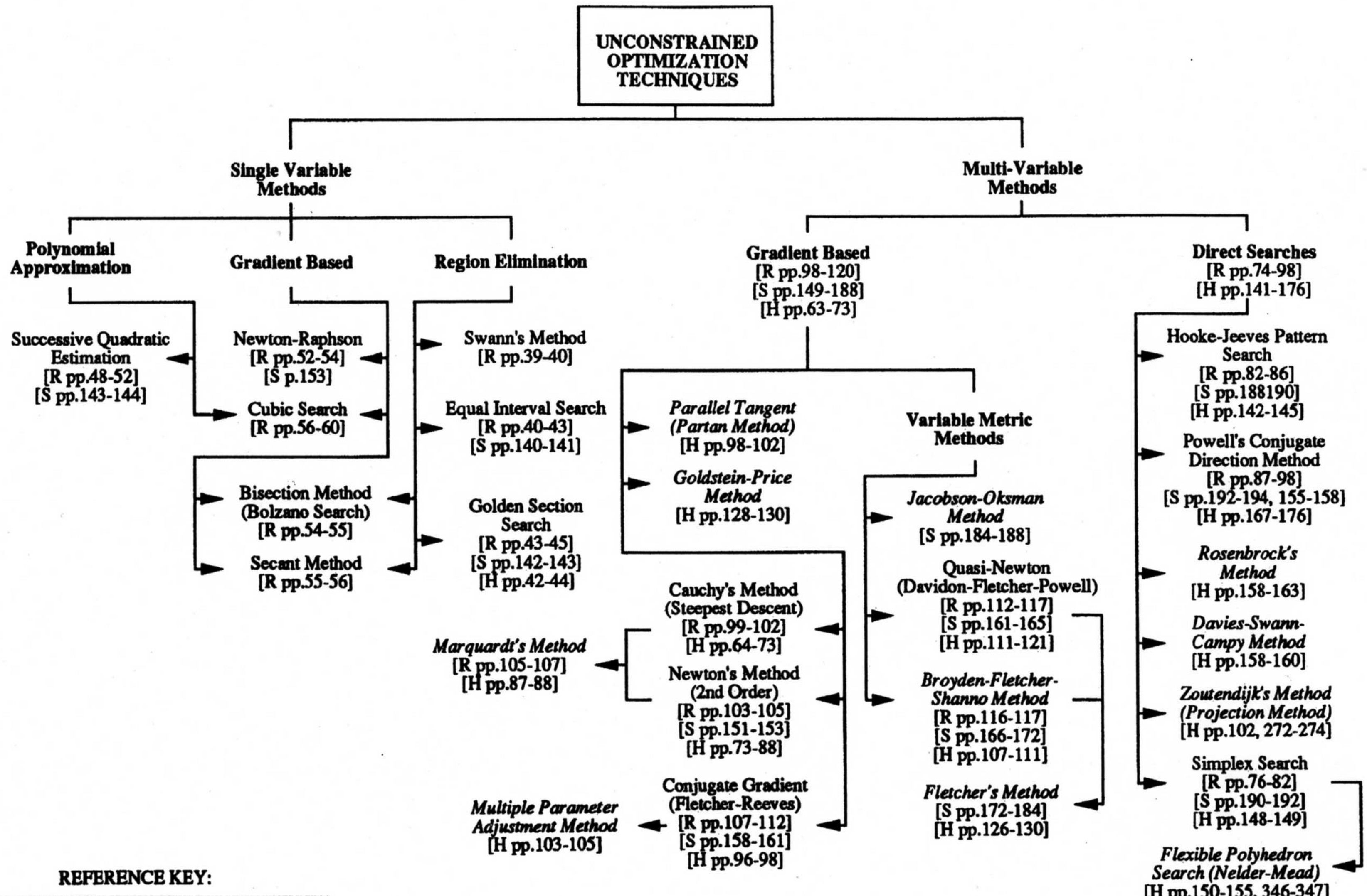
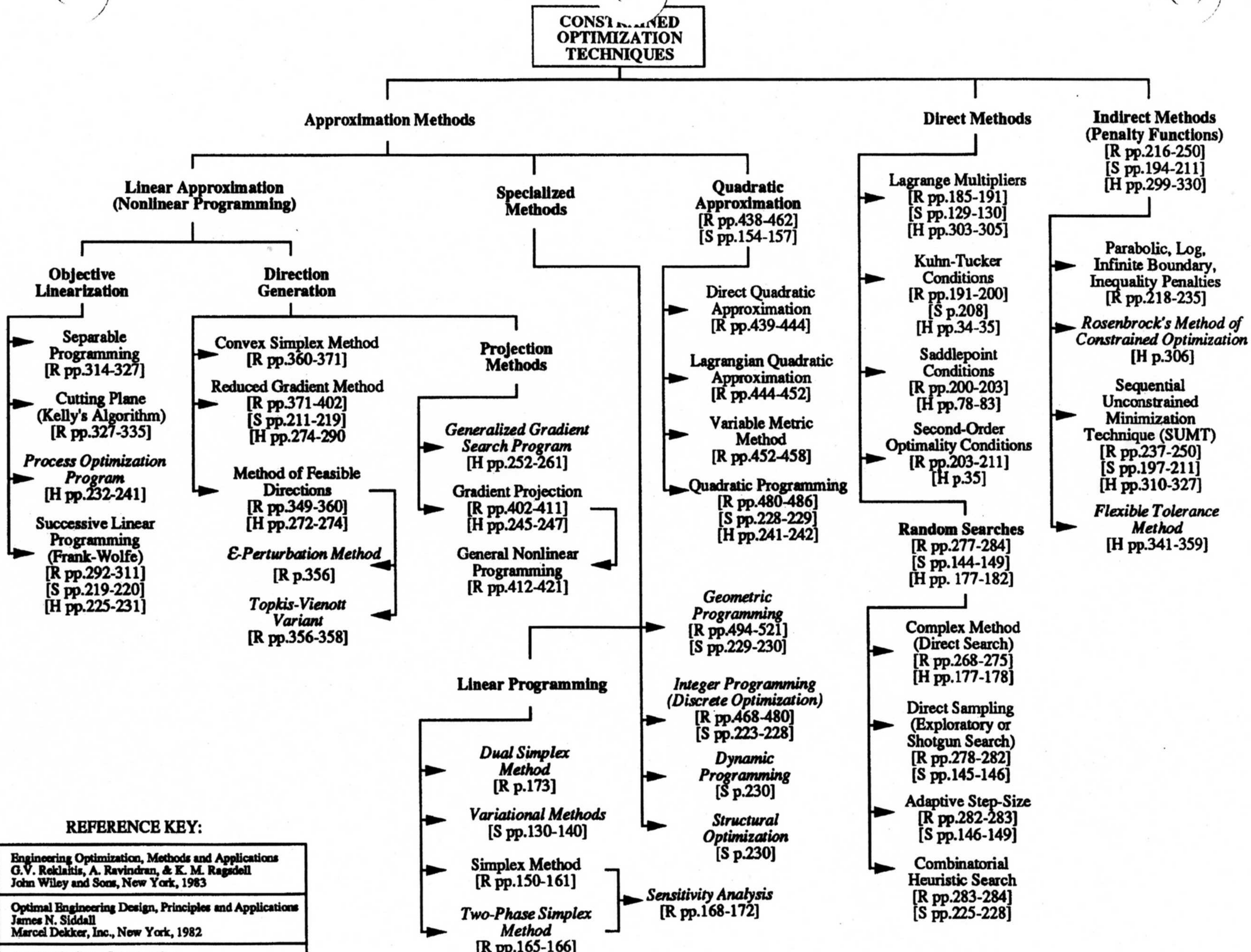



**REFERENCE KEY:**

| | |
|---|---|
| R | Engineering Optimization, Methods and Applications<br>G.V. Reklaitis, A. Ravindran, & K. M. Ragsdell<br>John Wiley and Sons, New York, 1983 |
| S | Optimal Engineering Design, Principles and Applications<br>James N. Siddall<br>Marcel Dekker, Inc., New York, 1982 |
| H | Applied Nonlinear Programming<br>David M. Himmelblau<br>McGraw-Hill, New York, 1972 |

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# DESCRIPTION OF UNCONSTRAINED METHODS

Single Variable Methods:

**Bisection Method (Bolzano Search)** - the minima of a unimodal objective is determined on an interval by region elimination utilizing the derivative of the objective.

**Cubic Search** - the function value and derivative are used to approximate the objective with a cubic. The minima of the approximation is used for the subsequent step.

**Equal Interval Search** - reduces the interval containing the optimum of a unimodal objective function by 0.5 with two function evaluations each step.

**Golden Section Search** - reduces the interval containing the optimum of a unimodal objective by 0.38297 with one function evaluation each step.

**Newton-Raphson** - the root of a linear approximation of the derivative of the objective function determines the next step. The second derivative is used in the approximation.

**Secant Method** - combines Newton's method with region elimination. The root of a secant approximation of the derivative determines the next step.

**Successive Quadratic Estimation** - approximates the objective function as a quadratic on an interval. The minima of the approximation is used for the subsequent step.

**Swann's Method** - an heuristic expanding pattern search to bracket the optimum.

Multi-variable Methods:

**Broyden-Fletcher-Shanno Method** - extension of Davidon-Fletcher-Powell method which eliminates the one-dimension search at each step but requires more complicated matrix calculations.

**Cauchy's Method** - classic method of local steepest descent. First derivative information of the objective is used to determine search direction for the next step. Effectively uses successive linear approximation of the objective.

**Conjugate Gradient Method (Fletcher-Reeves)** - quadratically convergent using only first order information. Quadratic approximation and conjugate directions are used. Easily implemented.

**DSC Method (Davies-Swann-Campy)** - extension of Fletcher's method which locates the minimum of the objective in each of the orthonormal directions like the Davidon-Fletcher-Powell method.

**Fletcher's Method** - combines Broyden-Fletcher-Shanno and Davidon-Fletcher-Powell. Eliminates one dimensional search at each stage and uses factorizing of the inverse hessian matrix.

**Flexible Polyhedron Search (Nelder-Mead)** - same as simplex search but allows irregular polyhedron vertex spacing to improve performance.

**Goldstein-Price Method** - approximates the hessian using a half factorial design and performs a matrix inversion. Similar performance to Davidon-Fletcher-Powell Method.

**Hooke-Jeeves Pattern Search** - combines exploratory and pattern moves with heuristic rules.

**Jacobson-Oksman Method** - Similar to Fletcher's method. Models objective as homogeneous function yielding an optimum in N+2 steps for an N dimensional homogeneous objective.

**Marquardt's Method** - combines Newton's and Cauchy's methods, requiring second-order information. Effectively uses Cauchy far from the solution and Newton close to the solution.

**Multiple Parameter Adjustment Method** - modification of Conjugate Gradient method which provides for more optimal search directions for non-quadratic objective functions.

**Newton's Method** - first and second derivative information is used to implement successive quadratic approximation of the objective. Minimum of approximation determines next step.

**Parallel Tangent Method** - a parallel to the tangent to the objective at the last search step is used as the search direction for the current search.

**Powell's Conjugate Direction Method** - approximates the objective function as a quadratic. A set of transformation vectors (conjugate directions) are calculated which will determine the optimum of a truly quadratic objective in exactly N searches for an N dimensional objective.

**Quasi-Newton Method (Davidon-Fletcher-Powell)** - uses a changing metric matrix at each step to determine the search direction. A robust technique but requires storage of a large matrix.

**Rosenbrock's Method** - similar to Hooke-Jeeves pattern search but the orthonormal search directions are aligned with the quadratic approximation to the objective at each step.

**Simplex Search** - utilizes a set of N+1 search points arranged on a polyhedron equidistantly (a regular simplex) for an N dimensional objective. The function is evaluated at each vertex. The worst is reflected through the centroid to determine the new search direction.

**Zoutendijk's Projection Method** - utilizes a projection matrix to calculate conjugate directions.

# DESCRIPTION OF CONSTRAINED METHODS

Approximation Methods:

**Convex Simplex Method** - generalizes the linear simplex method to linearly constrained nonlinear objective functions.

**Cutting Plane** - uses successive linear programming on decreasing subinterval which is reduced via cutting planes.

**Direct Quadratic Approximation** - similar to successive linear programming but uses quadratic approximation of each nonlinear function and solves resulting series of approximating subproblems.

**Dual Simplex Method** - modified simplex method incorporating Duality Theory.

**Dynamic Programming** - suited for allocation problems and sequential decision problems.

**ε-Perturbation Method** - extension of method of Feasible Directions which uses an iterative adjustment on the positive tolerance parameter, ε, which is used in forming the active constraint set.

**General Nonlinear Programming** - extension of Gradient Projection to nonlinear constraints.

**Generalized Gradient Search Program** - uses steepest descent search interior to a feasible region, forward difference approximation of objective partial derivatives, and history of successful steps.

**Geometric Programming** - constrained optimization of problems involving a special class of polynomial functions, *posynomials*, in the objective and constraints.

**Gradient Projection** - directly calculates the projection of a linearly constrained objective gradient onto the constraint surfaces avoiding some unsatisfactory features of the Reduced Gradient method.

**Integer Programming** - linear programming with some discrete objective variables.

**Lagrangian Quadratic Approximation** - utilizes the Lagrangian function to extend Newton's method to accommodate constraints.

**Method of Feasible Directions** - utilizes a metric matrix which leads to a feasible direction that gives the greatest improvement in the objective without violating any constraint.

**Process Optimization Program** - similar to successive linear programming. Linearization of the problem is repeated at each iteration.

**Quadratic Programming** - an optimization problem involving a quadratic objective function with linear constraints.

**Reduced Gradient Method** - the gradient of the objective function after transformation into a reduced space is used to determine the search direction.

**Sensitivity Analysis** - study of how changes in input data (uncertain coefficients) affects the simplex (linear programming) solution.

**Separable Programming** - certain types of nonlinear constrained optimization problems can be solve by reformulation into equivalent problems involving only linear functions.

**Simplex Method** - method of sequential steps for iterative solution of linear programming problems.

**Structural Optimization** - structural design methods using knowledge of modes of failure.

**Successive Linear Programming** - objective and constraint functions are linearly approximated at each iteration. Descent directions are determined from the approximation.

**Topkis-Vienott Variant Method** - extension of method of Feasible Directions which eliminates active constraint formulation and substitutes an alternative direction finding subproblem avoiding jamming.

**Two-Phase Simplex Method** - extension of simplex method to handle introduction of artificial variables needed to put a problem in the proper form for simplex method solution.

**Variable Metric Method** - approximates second derivative matrices using Quasi-Newton update formulas requiring only differences of gradients.

**Variational Methods** - use of methods similar to Variational Calculus which give the from of a continuous solution directly. Variational problems arise from the nature of the formulation.

Indirect Methods:

**Flexible Tolerance Method** - utilizes information from near-feasible as well as feasible points. The restriction on feasible points is gradually tightened. Many unconstrained methods can then be applied.

**Parabolic, Log, Infinite Boundary, Inequality Penalties** - forms of various penalty functions. The penalty terms enforce constraints by penalizing the objective when constraints are violated.

**Rosenbrock's Method of Constrained Optimization** - objective is modified so that it vanishes outside the feasible region and assumed negative in the interior. Then Rosenbrock's method for unconstrained minimization is applied.

**Sequential Unconstrained Minimization Technique (SUMT)** - classic penalty function algorithm. An unconstrained minimization is done in each variable direction of the augmented (penalized) objective function.

Direct Methods:

**Adaptive Step-Size** - random trial points are used to generate a search direction while step length in that direction is based on previous history of successes and failures.

**Combinatorial Heuristic Search** - utilizes a random search over a discretized finite grid with a heuristic termination algorithm.

**Complex Method** - modification of the simplex method which utilizes a set of randomly generated vertices. Infeasible points are retracted towards the centroid of previously generated points until feasible.

**Direct Sampling** - a series of random trial points are generated. The objective value of each feasible point is compared to the current best objective value.

**Kuhn-Tucker Conditions** - optimality conditions for the general nonlinear objective problem with (or without) equality and inequality constraints.

**Lagrange Multipliers** - Lagrange multipliers are parameters used to combine equality constraints into the objective function. Unconstrained optimization can then proceed.

**Saddlepoint Conditions** - optimality conditions for non-differentiable functions.

**Second-Order Optimality Conditions** - optimality conditions for twice differentiable functions.

## COMPARISON OF BOOKS

**Relkaitis, Ravindran, & Ragsdell** - most complete with clear detailed derivations and discussions.

**Siddall** - less clear derivations, fewer methods covered, more application oriented.

**Himmelblau** - older methods and terminology covered.