Problem 1 (Monotonicity Analysis)

A simple MEMS resonator is constructed from a single cantilever beam and a single lumped proof mass. The resonator motion is in the x-direction. The resonator is presumed to move perfectly in single-axis motion. The figure at the end of this document shows the design as well as some of the variables.

The design objective for this resonator is to drive the natural frequency as low as possible. There are constraints on how large the die size may be, on how large the stresses may become and on how small a feature width the fabrication technology can achieve. Please perform a monotonicity analysis of the following resonator design problem.

Minimize the objective function, \( obj \), which is the square of the natural frequency of the resonator:

\[
obj = \frac{k}{m}
\]

Subject to inequality constraints:

| \( g_1 \) | \( h \geq h_{\text{min}} \) | Fabrication-limited beam width |
| \( g_2 \) | \( y_D \geq y_M + L \) | Mass and flexure fit inside die |
| \( g_3 \) | \( x_D \geq x_M \) | Mass fits inside die |
| \( g_4 \) | \( \sigma_{\text{max}} \geq \sigma \) | Stress limited to working stress |
| \( g_5 \) | \( \delta \geq \delta_{\text{min}} \) | Minimum displacement of Mass |
| \( g_6 \) | \( \delta \leq 0.5 (x_D - x_M) \) | Maximum displacement of Mass |

And subject to equality constraints:

| \( h_1 \) | \( k = \frac{3EI}{L^3} \) | Spring constant of flexure |
| \( h_2 \) | \( I = \frac{bh^3}{12} \) | Definition of second moment of area |
| \( h_3 \) | \( M = x_M y_M b \rho \) | Definition of mass of resonator |
| \( h_4 \) | \( \sigma = \frac{3Eh\delta}{2L^2} \) | Stress formula |

In this problem, the following parameters are held constant and their values are not subject to the wishes of the designer:

- \( b \) Thickness of all structures
- \( \rho \) Density of material of all structures
- \( h_{\text{min}} \) Minimum flexure height due to fabrication limits
- \( E \) Elastic modulus of material for all structures
δ_{\text{min}} \quad \text{Minimum lateral deflection imposed on the resonator}

\sigma_{\text{max}} \quad \text{Maximum allowable working stress in material}

x_D \quad \text{Maximum x-dimension of die on which resonator fits}

y_D \quad \text{Maximum y-dimension of die on which resonator fits}

The following are among the variables in the problem and may be freely adjusted by the designer. (Are there other variables not mentioned here? Define them and put them in. – hint: you should have k, m for the objective function, I and \sigma from the equality constraints)

x_M \quad \text{X-dimension of proof mass}

y_M \quad \text{Y-dimension of proof mass}

L \quad \text{Length of flexure}

h \quad \text{Height of flexure}

\delta \quad \text{Deflection of proof mass}

For this specific optimization problem, please follow the full problem format and be sure to:

1. Formulate a statement of all constraints in the standard form, \( g_i = h_{\text{min}} - h \leq 0 \)
2. Formulate a monotonicity table with initial entries as was done in class. Work to avoid any regional monotonicities.
3. Formulate cases and sub-cases, each with their own monotonicity table, as was done in class. Take care to proceed in a way that generates the minimum number of cases.
4. Formulate a logic table that summarizes all the design cases as was done in class.