Problem 1 (Lagrangian Function)
A chain is suspended from two twin hooks that are 8 feet apart on a horizontal line. The chain itself consists of 10 links. Each link is one foot in length. We wish to determine the equilibrium shape of the chain. The equilibrium shape is the shape that has the minimum potential energy. We let each link $i$ span a horizontal distance of $x_i$ and a $y$ distance of $y_i$ measured with respect to the start of each link (the value could be negative). Assuming unit weight, the potential energy (objective function) is characterized by:

$$\frac{1}{2} y_1 + \left( y_1 + \frac{1}{2} y_2 \right) + \left( y_1 + y_2 + \frac{1}{2} y_3 \right) + \ldots + \left( y_1 + y_2 + y_3 + \ldots + y_{n-1} + \frac{1}{2} y_n \right) = \sum_{i=1}^{n} \left( n - i + \frac{1}{2} \right) y_i$$

The chain is subject to two constraints. The total $y$ displacement is zero and the total $x$ displacement is 8. Therefore, the problem is formulated as:

Minimize \( \sum_{i=1}^{n} \left( n - i + \frac{1}{2} \right) y_i \)

Subject To

\( \sum_{i=1}^{n} y_i = 0 \)

\( \sum_{i=1}^{n} \sqrt{1 - y_i^2} = 8 \)

n=10

1. Write the Lagrangian function for the minimization problem.
2. Write the first order necessary conditions for the problem.
3. Write the second order necessary conditions for the problem.

Problem 2 (Optimization Practice)
A cardboard box for packing is to be manufactured. The top, bottom and front faces must be of double weight, i.e., two pieces of cardboard, as shown in the figure below. Find the dimensions of such a box that maximizes the volume for a given amount of cardboard, equal to 72 sq. ft.

a) The objective is to maximize the volume $V=xyz$. The constraint is expressed as $4xy + 3xz + 2yz - 72 = 0$.
b) Find $x$, $y$, $z$.
c) (optional) Verify the second order conditions.