Problem 1 (Finite Element Practice)
Please solve the model problem \(-u'' + u = x\) with boundary conditions \(u(0) = u(1) = 0\) (same as in the class) using the basic finite element steps. Let \(N = 3\) and choose \(\phi_i = \sin \frac{i\pi x}{2}, i = 1, 2, 3\). Calculate \(K_{ij}\) and \(F_i\), solve for the coefficient \(\alpha_i\) and construct the approximate solution. Plot the exact solution (you should be able to solve the equation and get the exact solution) and the approximate solution and comment on the accuracy of your approximation. You will need to use computer to draw your solutions on a paper to compare them between 0 and 1.

Problem 2 (Finite Element Solution Practice)
Redo the same model problem but construct the basic function \(\phi_i, i = 1, 2, 3\), that are polynomials of degree \((i+1)\) and satisfy the boundary conditions of the model problem. Repeat the process in Problem 1 for this choice of basis functions.

Problem 3 (Finite Element Derivatives)
For the piecewise-linear finite element approximation of the model problem in which 4, 6 and 8 elements are used, the nodal point values of \(u(x)\), are:
\[
U^T = [0, 0.0353, 0.0569, 0.0505, 0] \text{ for 4 elements}
\]
\[
U^T = [0, 0.0242, 0.0445, 0.0567, 0.0565, 0.0394, 0] \text{ for 6 elements}
\]
\[
U^T = [0, 0.0184, 0.0351, 0.0484, 0.0567, 0.0579, 0.0503, 0.0318, 0] \text{ for 8 elements}
\]
(a) Plot \(u(x)\) on the same graph for four-, six-, and eight-element solutions. Note the location and value of the maximum deflection.
(b) Plot curves through the element values of \(u'(x)\) for the three solutions. Extrapolate to the boundary to find the corresponding maximum shear stress equivalent value as predicted by each solution.