

University of California at Berkeley
College of Engineering
Mechanical Engineering Department

ME118, Spring 2012

Liwei Lin

Problem Set #7
Due April 25 (Wednesday)

Problem 1 (Schrödinger Equation)

The 1D time-independent Schrödinger Equation is defined as

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

where \hbar is Planck's constant $h/2\pi$, m is the mass of a particle, $U(x)$ is the potential energy, and E is the system energy. To solve the 1D Schrödinger Equation, consider an electron, which is confined to move back and forth between rigid walls that are a distance, ℓ , apart from one another. Consider also de Broglie's *wave particle duality*: the electron and its motion can be described by a wave function ψ . Classically, this situation is analogous to standing-wave oscillations of a stretched string clamped at each end between massive supports, which are a distance ℓ apart. The supports constrain the vibrating string such that the nodes are always at these points, thus limiting the possible wavelengths of the standing waves.

- a) What are the possible wavelengths of the standing waves in the string?
- b) Each point on the stretched string oscillates with simple harmonic motion. Let y_{\max} be the maximum amplitude anywhere along the string. Derive the amplitude function of the standing wave? I.e. Show that $y_n(x)$ depend on x , n , and ℓ , where $n=1, 2, 3, \dots$?
Does your function show nodes at $x=0$ and $x=\ell$?
- c) The string is analogous to an electromagnetic wave trapped between two perfectly reflecting mirrors that are separated by a distance ℓ . The electromagnetic wave will also exhibit a standing wave pattern. What is the amplitude function $E_n(x)$ of the electromagnetic wave?
- d) Now go back to our original goal which is to solve the 1D time-independent Schrödinger Equation. What is ψ ?
- e) Quantization of the wavelength of a particle trapped between rigid walls leads to the quantization of its kinetic energy. Show that

$$E_n = n^2 \frac{\hbar^2}{8m\ell^2}$$

- f) Plot E_n (in normalized units, assuming the same mass) vs n for $\ell = 5 \mu\text{m}$ and 5nm

Problem 2 (QD Synthesis)

Name conditions that can control the size of quantum dots and also the trends that relate to quantum dot size.

Problem 3 (QD Application)

Describe five applications of bio-conjugated quantum dots. Why are quantum dots preferred?