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ME118/218N, Spring 2024 **Liwei Lin**

Problem Set #7

Due April 11 (Thursday)

Problem 1 (Schodinger Equation)

The 1D time-independent Schrödinger Equation is defined as

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x)$$

Where ψ is the state vector of the quantum system; $\hbar = h/2\pi$ is the reduced Planck constant; m is the mass of a particle; $U(x)$ is the potential energy; and E is the system energy. To solve the 1D Schrödinger Equation, consider an electron, which is confined to move back and forth between rigid walls that are a distance, ℓ, apart from one another. Consider also de Broglie's *wave particle duality*: the electron and its motion can be described by a wave function, ψ. Classically, this situation is analogous to standing-wave oscillations of a stretched string clamped at each end between supports at a distance ℓ apart. The supports constrain the vibrating string such that the nodes are always at these points, thus limiting the possible wavelengths of the standing waves.

a) What are the possible wavelengths of the standing waves in the string?
b) Each point on the stretched string oscillations with simple harmonic motion. Let $y_{\max}$ be the maximum amplitude anywhere along the string. Derive the amplitude function of the standing wave? I.e. Show that $y_n(x)$ depend on x, n, and ℓ, where n= 1, 2, 3, …?
   Does your function show nodes at x =0 and $x = \ell$?
c) The string is analogous to an electromagnetic wave trapped between two perfectly reflecting mirrors that are separated by a distance ℓ. The electromagnetic wave will also exhibit a standing wave pattern. What is the amplitude function $E_n(x)$ of the electromagnetic wave?
d) Now go back to our original goal which is to solve the 1D time-independent Schrödinger Equation. What is ψ?
e) Quantization of the wavelength of a particle trapped between rigid walls leads to the quantization of its kinetic energy. Show that

$$E_n = n^2\frac{h^2}{8m\ell^2}$$

f) Plot E_n (in normalized units, assuming the same mass) vs n for ℓ = 5 µm and 5 nm

Problem 2 (QD Synthesis)

Name conditions that can control the size of quantum dots and also the trends that relate to quantum dot size.

Problem 3 (QD Application)

Describe five applications of bio-conjugated quantum dots. Why are quantum dots preferred?