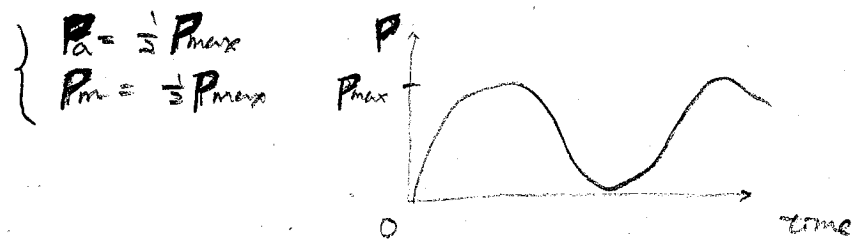


Fatigue loading on pressure vessel

Assume $P \rightarrow$ Force varies between 0 and P_{max} (e.g. pressure vessel)



force on bolt

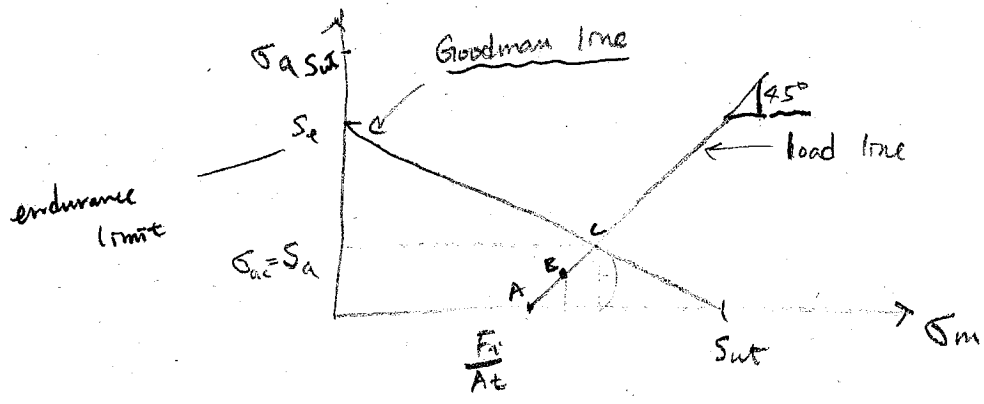
$$F_b = F_i + cP$$

\uparrow fixed \uparrow time dependent

$$\left. \begin{aligned} \sigma_a &= \frac{c P_{max}}{2 A_t} \\ \sigma_m &= \frac{F_i}{A_t} + \frac{c P_{max}}{2 A_t} \end{aligned} \right\}$$

these equations define the "load line" as:

$$\sigma_a = \sigma_m - \frac{F_i}{A_t}$$



Intersection of the load & Goodman lines $\rightarrow \sigma_{ac}$

\rightarrow alternating strength

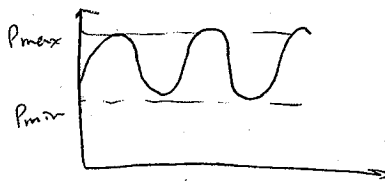
$$\sigma_{ac} = \sigma_a = \frac{S_{ut} - \frac{F_i}{A_t}}{1 + \frac{S_{ut}}{S_e}}$$

$$S.F. = \frac{AC}{AB}$$

$$\rightarrow S.F. = \frac{\sigma_a}{\sigma_m} = \frac{\frac{S_{ut} - \frac{F_i}{A_t}}{1 + \frac{S_{ut}}{S_e}}}{\frac{c P_{max}}{2 A_t}}$$

$$\Rightarrow P_{max} \leq \frac{2}{S.F. \cdot c} \frac{S_{ut} A_t - F_i}{1 + \frac{S_{ut}}{S_e}}$$

• more on Fatigue. $P_{min} \neq 0$



$$\begin{cases} F_{max} = F_i + P_{min} \cdot C \\ F_{min} = F_i + P_{max} \cdot C \end{cases}$$

$$\rightarrow \begin{cases} \sigma_a = \frac{F_{max} - F_{min}}{2A_t} = \frac{C(P_{max} - P_{min})}{2A_t} \\ \sigma_m = \frac{F_{max} + F_{min}}{2A_t} = \frac{F_i}{A_t} + \frac{C(P_{max} + P_{min})}{2A_t} \end{cases}$$

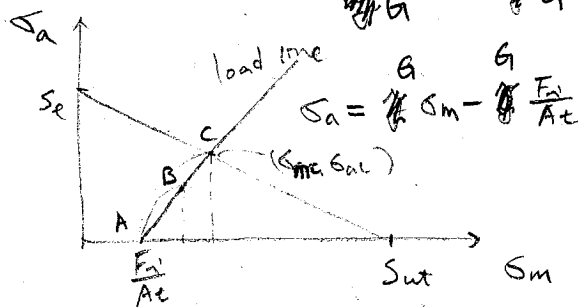
Let $P_{max} = P_0$
 $P_{min} = \alpha P_0 \quad (0 < \alpha < 1)$

$$\Rightarrow \begin{cases} \sigma_a = \frac{C P_0 (1 - \alpha)}{2A_t} = \beta (1 - \alpha) \\ \sigma_m = \frac{F_i}{A_t} + \frac{C P_0 (1 + \alpha)}{2A_t} = \frac{F_i}{A_t} + \beta (1 + \alpha) \end{cases}$$

$$\frac{\sigma_a - 0}{\sigma_m - \frac{F_i}{A_t}} = \frac{\beta(1 - \alpha)}{\beta(1 + \alpha)} = \frac{1 - \alpha}{1 + \alpha}$$

$$\Rightarrow \sigma_a = \frac{1 - \alpha}{1 + \alpha} \sigma_m - \frac{1 - \alpha}{1 + \alpha} \frac{F_i}{A_t}$$

slope $\frac{1 - \alpha}{1 + \alpha}$



$$S.F. = \frac{AC}{AB} = \frac{\sigma_{ac}}{\sigma_a} = \frac{\sigma_m}{\sigma_c}$$

Solve intersection point

$$\Rightarrow \begin{cases} \sigma_{mc} = \frac{Se + \frac{F_i}{A_t}}{1 + \frac{Se}{S_{ut}}} \\ \sigma_{ac} = \frac{S_{ut} - \frac{F_i}{A_t}}{1 + \frac{S_{ut}}{Se}} \end{cases} \Rightarrow S.F. = \frac{\sigma_{ac}}{\sigma_a}$$