Empirical fatigue data

R. R. Moore rotating-beam fatigue testing machine

\[ F \rightarrow \delta A \rightarrow \delta C \rightarrow N \]

section is in pure bending (no shear force)

\[ \text{time } t \quad \text{time } t + \frac{1}{2} \text{ cycle} \]

completely reversed bending stress

\[ \text{tensile stress} \]

\[ S_u \quad S_e \]

ultimate tensile strength

seamless experiments did not fracture

\[ N \]

\[ S_e = \text{endurance limit} \]

\[ S_e' = \text{unmodified endurance limit} \]

if \( S < S_e' \) \( \rightarrow \) specimen never fracture even as \( N \rightarrow \infty \)

NOTE: both axes are log scale

\[ 10^6 \quad 10^7 \]

relationship of \( S_e' \) and \( S_u \)

\[ S_e' = \frac{1}{2} S_u \quad \text{if } S_u < 212 \text{ kpsi} \]

\[ 212 \text{ kpsi} \quad \text{if } S_u > 212 \text{ kpsi} \]

(1480 MPa) (1460 MPa)

NOTE:

existence of an endurance limit is specific to

ferrous materials (containing iron)

(e.g., aluminum will fracture at \( S \) at any \( N \) (however low)

- if we run for sufficiently

large \( N \))

3 regions

low cycle \((N < 10^3)\):

use static failure theory

at \( N = 10^3 \), fracture at \( S \approx 0.9 S_u \)

(usually less than \( S_y \))

fatigue life \((N > 10^6)\):

if specimen survive, must be \( S < S_e' \)

\( \rightarrow \) it will survive indefinitely

finite life \((10^3 < N < 10^6)\)

characterize by a power law: \( S_p(N) = AN^b \)

matching condition \( S_p(10^3) = 0.9 S_u \)

\( S_p(10^6) = S_e' \)
Fatigue under non-zero mean stress

\[ \text{R.R. Moore} \rightarrow \text{zero mean stress} \rightarrow t \]

\[ \text{non zero mean stress} \]

\[ \text{Fatigue strength} \rightarrow (S_e) \]

Empirical results:
- If \( S_m < 0 \) (compressive), \( S_f \) is the same as R.R. Moore test.
- If \( S_m > 0 \) (tensile), \( S_f \) is less than R.R. Moore test.

Safe region on plot of \( S_a \) vs. \( S_m \):

\[ S_a \text{ always } > 0 \]

\[ \text{infinite life} \] (Goodman line)

\[ S_f < S_m \]

\[ \frac{S_m}{S_e} + \frac{S_m - S_a}{S_u} < 1 \]

\[ S_m < 0 \]

\[ S_a < S_e \]

- If also want to guard against yielding:
  must have

\[ S_a + |S_m| < S_y \]

or

\[ \frac{S_a + |S_m|}{S_y} < 1 \] (both \( S_m > 0 \) or \( S_m < 0 \))

\[ \text{Region I: immediate failure} \]
\[ \text{Region II: finite life} \]
\[ \text{Region III: infinite life} \]
- Safe \((S_a, S_m)\) region for a particular life \((N\) cycles\)
  \[
  \rightarrow \text{compute fatigue strength } \quad S_f = aN^b
  \]
  \[
  \rightarrow \text{draw regions defined by}
  \]
  \[
  \begin{cases}
  S_m > 0, & \frac{S_a}{S_f} + \frac{S_m}{S_{ut}} < 1 \\
  S_m < 0, & \frac{S_a}{S_f} < 1
  \end{cases}
  \]

- Some alternative to Goodman line:
  - Soderberg line — uses \(S_y\) instead of \(S_{ut}\)
    \[
    \frac{S_a}{S_e} + \frac{S_m}{S_y} < 1
    \]
  - Gerber parabola — assumes quadratic dependence on \(S_m\)
    \[
    \frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 < 1
    \]