HW#3 Solution

1. Gear Fundamentals

Given

- module $m = 4\ mm$
- velocity ratio $\frac{r_{gear}}{r_{pinion}} = 2.80$
- number of teeth in pinion $N_{pinion} = 20$

(a) Gear pitch diameter d_{gear} is given by

$$d_{gear} = \frac{r_{gear}}{r_{pinion}} d_{pinion} = \frac{r_{gear}}{r_{pinion}} N_{pinion} m = 2.8 \cdot 20 \cdot 4 = 224\ mm$$

(b) Number of teeth on the driven gear N_{gear} is given by

$$N_{gear} = \frac{d_{gear}}{m} = \frac{224}{4} = 56$$

(c) Theoretical center-to-center distance $r_{gear} + r_{pinion}$ is

$$r_{gear} + r_{pinion} = \frac{d_{gear} + d_{pinion}}{2} = \frac{d_{gear} + mN_{pinion}}{2} = \frac{224 + 4 \cdot 20}{2} = 152\ mm$$

2. Contact Ratio & Interference

Given

- module $m = 4\ mm$
- velocity ratio $\frac{r_{gear}}{r_{pinion}} = 3$
- center distance $r_{gear} + r_{pinion} = 4\ inch$
- diametral pitch $P = 6$ T/in
- pressure angle $\phi = 20$ degrees

(a) The pitch radius of gear and pinion are given by solving the following two equations.

$$\frac{r_{gear}}{r_{pinion}} = 3$$

$$r_{gear} + r_{pinion} = 4\ inch$$

Thus $r_{gear} = 3\ inch, \quad r_{pinion} = 1\ inch$

Numbers of teeth are obtained by

$$N_{gear} = Pd_{gear} = P2r_{gear} = 6\cdot 2\cdot 3 = 36$$
$$N_{pinion} = Pd_{pinion} = P2r_{pinion} = 6\cdot 2\cdot 1 = 12$$

(b) Contact ratio cr is

$$cr = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - (r_{pinion} + r_{gear})\sin\phi}{p_b}$$

where

r_{ap}, r_{ag} are addendum radii of the mating pinion and gear which are given by $r_a = r + 1/P$.
r_{bp}, r_{bg} are base circle radii of the mating pinion and gear which are given by $r_b = r\cos\phi$.
and base pitch p_b is

$$p_b = \frac{\pi d_b}{N} = \frac{\pi\cos\phi}{P} \quad (d_b = d\cos\phi \text{ : diameter of the base circle – either gear or pinion})$$

Followings are the calculations.

$$r_{ap} = r_{pinion} + 1/P = 1 + 1/6 = 7/6 = 1.1667\ in$$
$$r_{ag} = r_{gear} + 1/P = 3 + 1/6 = 19/6 = 3.1667 in$$
$$r_{bg} = r_{gear}\cos 20 = 3\cos 20\ in$$
$$r_{bp} = r_{pinion}\cos 20 = \cos 20\ in$$
$$p_b = \frac{\pi\cos\phi}{P} = \frac{\pi\cos 20}{6}$$
$$cr = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - (r_{pinion} + r_{gear})\sin\phi}{p_b} = 6\frac{\sqrt{(7/6)^2 - \cos^2 20} + \sqrt{(19/6)^2 - 9\cos^2 20} - 4\sin 20}{\pi\cos 20}$$
$$= 1.5564$$

(c) We can determine whether there is interference by comparing addendum radii r_a with maximum non-interfering addendum radii $r_{a(\max)}$ which are given by

$$r_{a(\max)} = \sqrt{r_b^2 + (r_{pinion} + r_{gear})^2\sin^2\phi}$$
$$r_{ag(\max)} = \sqrt{r_{bg}^2 + (r_{pinion} + r_{gear})^2\sin^2\phi} = \sqrt{(3\cos 20)^2 + 4^2\sin^2 20} = 3.1355 \text{ in}$$
$$r_{ap(\max)} = \sqrt{r_{bp}^2 + (r_{pinion} + r_{gear})^2\sin^2\phi} = \sqrt{(\cos 20)^2 + 4^2\sin^2 20} = 1.6597 \text{ in}$$

Thus, both addendum radii are greater than the maximum addendum radii. So there is interference.

3. Gear Train

Gear 2 which is attached to the *a* axis is rotating at the speed of $\omega_2 = 600$ rpm in counter-clock wise direction when seen from the left. Accordingly, gear 3 is rotating at the speed of $\omega_3 = \frac{20}{40}\omega_2 = 300\ rpm$ in clockwise direction. Gear 6 is rotating at the same speed as gear 5 since they are attached to the same shaft and gear 5 is rotating at the speed of $\omega_5 = \frac{8}{17}\omega_3 = 141.1765\ rpm$ in counter-clockwise when seen from the front. Gear 6 is rotating at the same speed as gear 5. Finally, gear 7 is rotating at the speed of $\omega_7 = \frac{20}{60}\omega_5 = 47.0588\ rpm$ in clockwise direction when seen from the front.