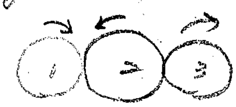
· gear train fundamental

① $\left|\frac{W_1}{W_2}\right| = \left|\frac{d_2}{d_1}\right| = \left|\frac{N_2}{N_1}\right|$

② simple gear train



example ~~$N_1 < N_2 < N_3$~~ $N_1 < N_2 < N_3$

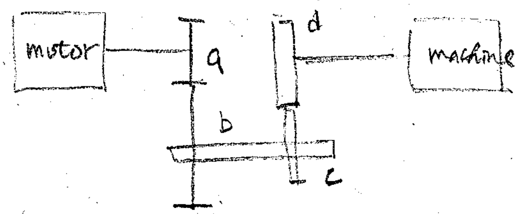
N_1 → engine motor

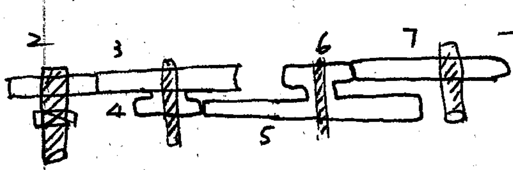
N_3 → output torque

$$W_3 = -W_2 \frac{N_2}{N_3} = -\frac{N_2}{N_3}\left(-W_1 \frac{N_1}{N_2}\right) = W_1 \frac{N_1}{N_3}$$

ex: machine (gear) slow increase torque ← reduction in speed, increase torque ← engine, motor (pinion) fast

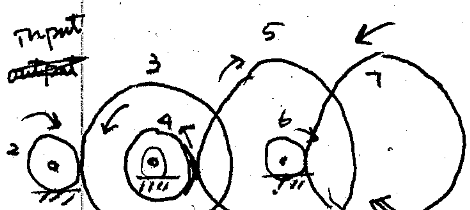
ex) (double reduction)





$$\frac{W_a}{W_d} = \frac{W_a}{W_b} \cdot \frac{W_b^{=W_c}}{W_d} = \left(-\frac{d_b}{d_a}\right)\left(-\frac{d_d}{d_c}\right)$$

W — rad./sec $\qquad = \frac{d_b d_d}{d_a d_c} = \frac{N_b N_d}{N_a N_c}$



train ratio $= \frac{W_d}{W_a} = e$

$W_{last} = W_{first} \cdot e$

$(W_L = W_F \cdot e)$

ex) a design calls for angular velocity ratio of 60:1 (speed reduction of 60:1)

$$\frac{W_7}{W_2} = \frac{1}{60} = \frac{W_7}{W_6} \cdot \frac{W_6}{W_5} \cdot \frac{W_5}{W_4} \cdot \frac{W_4}{W_3} \cdot \frac{W_3}{W_2}$$

$$= \left(-\frac{N_6}{N_7}\right)\left(-\frac{N_4}{N_5}\right)\left(-\frac{N_2}{N_3}\right)$$

much better control

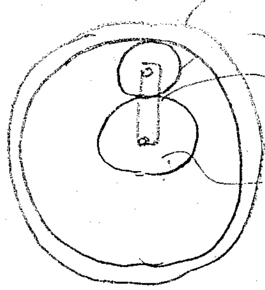
choose, for example $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ $\Rightarrow \frac{W_7}{W_2} = \frac{1}{60}$

③ planetary gear trains (" imagining that arm is fixed")

most general formula

$$\underline{\text{train ratio}}\ e = \frac{\omega_L - \omega_A}{\omega_F - \omega_A}$$ → you are sitting on the arm and observing the moves of others

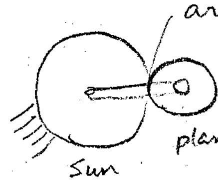
ω_F — first gear angular velocity
ω_A — arm " "
ω_L — last " " "



write it down at hand outs

(actual planetary gear trains involve two or more equally spaced planets to balance forces)

ex) no ring → external mesh



if a known speed ω_a is applied to $\underline{\text{arm}}$, what is the absolute rotation of the planet ?

$\omega_p = \omega_a + \omega_{p/a} \rightarrow (\omega_p - \omega_a)$

planet speed w.r.t. arm

$\Rightarrow \dfrac{\omega_p}{\omega_a} = 1 + \dfrac{\omega_{p/a}}{\omega_a}$

sun is fixed $\Rightarrow \omega_a = \omega_s + \cancel{\omega_{a/s}}\ \omega_{a/s}$

$\Rightarrow \dfrac{\omega_p}{\omega_a} = 1 + \dfrac{\omega_{p/a}}{\omega_{a/s}} = 1 - \boxed{\dfrac{\omega_{p/a}}{\omega_{s/a}}}$ e

$\dfrac{\omega_{p/a}}{\omega_{s/a}} = -\dfrac{N_s}{N_p}$

note the sign changes $\Rightarrow \omega_p = \omega_a \left(1 + \dfrac{N_s}{N_p}\right)$ when arm moves 1 rev, planet moves $\left(1 + \dfrac{N_s}{N_p}\right)$ rev.

✓ ① inputs $\Rightarrow \omega_a$, $\omega_s = 0$

✓ ② $\omega_a = \omega_a$

③ $\dfrac{\omega_{p/a}}{\omega_{s/a}} = -\dfrac{N_s}{N_p}$

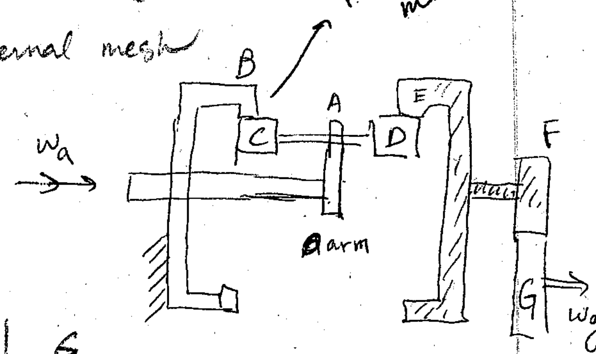
$\omega_p - \omega_a = (\overset{0}{\omega_s} - \omega_a)\left(-\dfrac{N_s}{N_p}\right)$

$\omega_p = \omega_a \left(1 + \dfrac{N_s}{N_p}\right)$

☆ In general if gear i meshes with gear j

then $\quad \omega_{j/a} = \text{sing}\; \omega_{i/a} \dfrac{N_i}{N_j}$

$$\text{sing} = \begin{cases} + & \text{internal mesh} \\ - & \text{external mesh} \end{cases}$$



ex) hand out

	A	B	C	D	E	F	G
ω_{arm}	ω_a	ω_a	ω_a	ω_a	ω_a	ω_a	ω_a
$\omega_i/_{arm}$	0	$-\omega_a$	$-5\omega_a$	$-5\omega_a$	$-\frac{25}{21}\omega_a$	$-\frac{25}{21}\omega_a$	$-\frac{N_F}{N_G}\cdot\left(-\frac{25}{21}\omega_a\right)$
$\omega_i = \omega_a + \omega_i/_{arm}$	ω_a	0	$-4\omega_a$	$-4\omega_a$	$-\frac{4}{21}\omega_a$	$-\frac{4}{21}\omega_a$	

Q: spur gear ratio?

$B = 100\,T$
$C = 20\,T$
$D = 25\,T$
$E = 105\,T$
$\omega_a = 1000$ rpm
$\omega_g = 20$ rpm

B $\rightarrow$ grounded, $\omega_B = 0$

C $\rightarrow$ B & C meshing gears, $\dfrac{\omega_{C/a}}{\omega_{B/a}} = +\dfrac{N_B}{N_C} = \dfrac{100}{20} = 5$ (internal meshing)

$\Rightarrow \omega_{C/a} = +5\,\omega_{B/a} = -5\omega_a$

D $\rightarrow$ same as C (same shaft)

E $\rightarrow$ $\dfrac{\omega_{E/a}}{\omega_{D/a}} = +\dfrac{N_D}{N_E} = \dfrac{25}{105}$, $\Rightarrow \omega_{E/a} = -5\cdot\dfrac{25}{105}\omega_a$

F $\rightarrow$ same as E

① $\omega_F = -\frac{4}{21}\omega_a = -\frac{4000}{21}$ rpm

② F, G spur gears

$$\left|\frac{\omega_F}{\omega_G}\right| = \cancel{-}\frac{N_G}{N_F}, \quad N_G = \frac{\cancel{-}\frac{4}{21}\omega_a N_F}{\omega_G} = \frac{4}{21}\cdot\frac{1000}{20}N_F = \frac{200}{21}N_F$$

choose $N_F = 21 \Rightarrow N_G = 200$

③ ω_F & ω_a reverse

ω_F & ω_G reverse $\Rightarrow$ ω_a & ω_G same direction

④ ~~read example 13.3, 13.4 yourself~~