broadly shown in Fig. 1.

$F_\text{b}$: "preload" produced by tightening nut

members are "compressed" by force $F_\text{b}$

member stretched to give bolt "tensile" force $F_\text{m}$

note stress concentration:

1) junction of bolt head & shank

2) junction of threaded & unthreaded portions of shank

3) first thread inside nut

bolt will usually fail at one of these positions

Let

$F_\text{b}, F_\text{m}$: bolt force, extension

$F_\text{m}$: member force, extension

If there is no external load on the joint

$F_\text{b} = F_\text{i}$, $F_\text{m} = -F_\text{i}$

Now, if we try to pull the joint apart with an external tensile force $P$

$P = (1-c) \text{ portion of } P \text{ acting upon bolt members}$

we want to determine $c$
\begin{align*}
F_b &= F_a + cP \\
F_m &= -F_a + (1-c)P
\end{align*}

Let \( \Delta_b \) = increase in extension of bolt due to \( cP \)

\( \Delta_m \) = decrease in compression of members due to \( (1-c)P \)

\[
\frac{cP}{\Delta_b} = \frac{(1-c)P}{\Delta_m}
\]

so long as \( F_m < 0 \)

\rightarrow the joint remains intact

we must have \( \Delta_m = \Delta_b \)

\rightarrow \quad \frac{C}{\Delta_b} = \frac{(1-c)P}{\Delta_m}

\rightarrow \quad C = \frac{\Delta_b}{\Delta_b + \Delta_m}

C \rightarrow fraction of \( P \) acts on bolt

called "joint constant"

Note: \( C \ll 1 \) if \( \Delta_b \ll \Delta_m \): most extended load

Parts on members

to minimize load on bolt, we desire stiffness of bolt to be small compare to stiffness of members.

Condition for joint to remain intact

\( F_m < 0 \rightarrow \) members will not separate & joint in intact

\[
C = \frac{\Delta_b}{\Delta_b + \Delta_m} \text{ into } F_m = -F_a + (1-c)P
\]

\( F_m < 0 \implies P < \frac{\Delta_b + \Delta_m}{\Delta_m} F_a \text{ max. allow. } P \text{ for preload } F_a \)

\text{or } F_a > \frac{\Delta_m}{\Delta_b + \Delta_m} P \text{ min. preload } F_a \text{ for given } P
Note: if \( P \) is steadily increased to a value above
\[
\text{Percival} = \frac{\sigma_0 + \sigma_m}{\sigma_m} F_0
\]
at \( P = \text{Percival} \), the members "separate" and the bolt must suddenly carry the full load \( P \) instead of just \( CP \), \( \Rightarrow \) a sufficient preload is very important.

Estimate \( K_0 \) & \( \sigma_m \)

\[
F = \frac{R_i \sigma_1}{R_d} = R_2 d^2
\]

\[
\begin{align*}
\delta_1 &= \frac{F}{E_1} & \delta_2 &= \frac{F}{E_2} \\
\delta = \delta_1 + \delta_2 & \Rightarrow F = \frac{E}{E_1} \left( \frac{F}{E_1} + \frac{F}{E_2} \right) \\
\frac{1}{E_1} &= \frac{1}{E_1} + \frac{1}{E_2}
\end{align*}
\]

The stress of a rod

\[
\frac{F}{A} = E \frac{E}{C} \rightarrow R = \frac{F}{A} = \frac{AE}{C}
\]

For bolt
\[
R_b = \frac{F_0}{A_d E} + \frac{F_0}{A_d E}
\]

where \( A_d = \frac{\pi d^2}{4} \), \( d = \) "major diameter"

\( A_t = \) "tensile stress area"

\[
\Rightarrow R_b = \frac{A_d A_t E}{A_d E + A_t E_d}
\]
\[ \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_N} \] 

if there are \( N \) members

Note: if there is any member which is "soft" than others

\[ \Rightarrow \ \frac{1}{k_m} \text{ is determined by } \frac{1}{k_{soft}} \]

\[ \Rightarrow \ \frac{1}{k_m} \text{ is "large"} \]

\[ \Rightarrow \ k_m \text{ is "small"} \Rightarrow \text{we want } k_m \uparrow \]

recall We want \( c << 1 \)

\[ \Rightarrow \ F_b = P \cdot c \text{ is small} \]

\[ \Rightarrow \ F_m = P \cdot (1-c) \text{ is large} \]

\[ \Rightarrow \text{most } P \text{ acts on member} \]

Estimate \( k_m \) is not easy \[ \text{experiments} \]

\[ \text{first element} \]

\[ \text{approximate model based on } \]

"pressure cone" text 8.5

Text 8.5

\[ \text{dw} \text{ diameter of washer face} \]

\[ d \text{ diameter of bolt shank} \]

\[ \alpha = 30^\circ \text{ in general} \]

\[ t \text{ member thickness} \]

\[ \begin{align*}
\Phi &= \frac{\pi}{6} \left( \text{two member case} \right) \\
\Phi &= \frac{\pi}{6} E \tan \alpha \\
\text{ln} \frac{1.15t + d_w - d}{1.15t + d_w + d} (d_{yd}) \\
\end{align*} \]

\( d_w = 1.5d \)

diameter of washer face is about 50% greater than the fastener diameter for standard hexagon-head bolts

\[ k_m = \frac{\Phi}{2} = \frac{0.577\pi E d}{2 \text{ln} \left( \frac{0.577E + 0.5d}{0.577E + 2.5d} \right)} \Rightarrow d_w = 1.5d \]