

Vortex ring structure at late stages of formation

Drazen Fabris^{a)}

Department of Mechanical, Materials, and Aerospace Engineering, Illinois Institute of Technology, Chicago, Illinois 60616

Dorian Liepmann

Department of Mechanical Engineering, University of California, Berkeley, California 94720-1740

(Received 17 January 1997; accepted 13 May 1997)

High resolution DPIV measurements are made of a vortex ring formed by a piston/cylinder mechanism (Reynolds number 3700 and 7500). A complex ring structure is identified. The stopping condition of the piston leads to vortical fluid collecting near the forward stagnation point of the ring. In addition, the roll-up of the core produces a vorticity distribution that departs from a simple Gaussian profile. © 1997 American Institute of Physics. [S1070-6631(97)01809-6]

Vortex rings have been investigated since the early work of Lord Kelvin and are considered a fundamental type of vortical flow.^{1,2} Although much has been understood there still exist discrepancies between theoretical modeling of the ring structure and experimentally formed vortex rings. Recently, the ability to more accurately measure vorticity has uncovered a more complex physical ring structure.

The basic theoretical model considers a vortex ring moving in an unbounded medium that is at rest at infinity. In a reference frame moving with the ring, the flow is characterized by three regions: a toroidal core of vortical fluid (I), an external potential flow (II), and an intermediate region of irrotational fluid that is entrained and moving with the ring (III). For rings with large cores, the intermediate region forms a ball that is characterized by forward and leeward stagnation points (Fig. 1). The core of the ring is hypothesized to have a Gaussian distribution of vorticity.^{3,4}

Physically, this flow is typically generated by the ejection of a slug of fluid through a nozzle or orifice. The mechanics of the ejection process and the presence of physical boundaries modifies the above picture of the vortex ring structure. Saffman considered the piston velocity profile in calculating ring properties based on his analytic model.⁵ The numerical studies of Nitsche and Krasny^{6,7} have shown two self-similar components for the axial motion of the vortex core dominant at either early or late times. Measuring the velocity distribution at the exit plane of the nozzle Didden has shown a linear entrainment of vorticity into the ring after an initial larger entrainment rate.⁸ Wakelin and Riley simulate the formation of vortex rings from orifices in a plane wall analyzing the influence of a stopping vortex structure.⁹ In addition, dye visualizations have shown a rolled up spiral forming from the edge of the nozzle (Fig. 2) that is commonly associated with a sheet of vorticity (also see Dahm *et al.*¹⁰). Unfortunately dye visualization fails to capture some of the essential properties of vorticity, stretching, diffusion, and cancellation; and therefore, the true vortical structure can only be inferred.

The application of DPIV permits the measurement of time resolved velocity fields that can then be differentiated to determine the vorticity field. It removes the conjectures associated with dye visualization and uncovers experimental vortex rings with a complex structure. The two cases pre-

sented here show the late stages of the vortex ring formation.

Briefly, the experiment consists of a piston driven in a cylinder (diameter of 10 cm) for a stroke of two diameters. The piston velocity profile is trapezoidal with minimal acceleration and deceleration times and the piston stops flush with the nozzle edge. Silver coated glass spheres ($40\ \mu$ in diameter) are illuminated with a 10 Watt argon-ion laser, imaged onto a Texas Instruments camera (480×1134 pixels), and recorded on a Sony Laser Video Recorder. The DPIV processing uses a new technique based on the work of Willert and Gharib¹¹ the details of which are discussed elsewhere.¹² The resulting measurements produce a velocity vector field of 119 by 127 resolution with 1%–2% uncertainty in velocity and 5%–6% in vorticity¹¹ (although peak values of velocity and vorticity fields are degraded through spatial averaging). The technique measures the vorticity component normal to the plane of illumination in a cross-section through the center of the ring. The structure remains primarily two dimensional at the stages considered.

Four instantaneous azimuthal vorticity fields during the

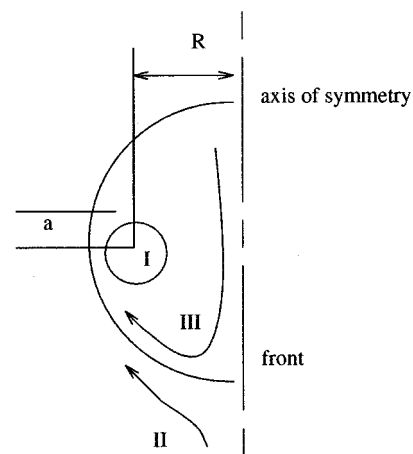


FIG. 1. The simplest steady theoretical model of a vortex ring. The core region is denoted I, the external fluid II, and the entrained irrotational fluid III. The streamlines in the reference frame moving with the ring are shown separating the three regions. The motion of the fluid is indicated with the arrows.



FIG. 2. Dye entrained into a vortex ring (Reynold's number of 4000).

development of a vortex ring are shown in Fig. 3 with the nozzle of the generator protruding into the field of view at the top. The first data set shows the initial formation of the ring from the axisymmetric shear layer that develops from the edge of the nozzle. The piston motion stops at the time corresponding to Fig. 3(a), the shear layers detach from the edges of the nozzle, and the vortex ring advects away from the vortex generator. As a consequence of stopping the piston rapidly, there is a strong radial convergence of the fluid near the end of the cylinder and entrainment into the rear centerline of the ring. This stage of entrainment advects the tail end of the shear layers towards the centerline of the ring. The induced velocity of the vortex pulls the shear layers forward and towards the forward stagnation point [Fig. 3(b) and (c)]. As the shear layers advect through the center of the ring, there is some cancellation of vorticity but the motion is rapid enough so that most of the vortical fluid collects at the front of the ring. Here, near the front stagnation point of the

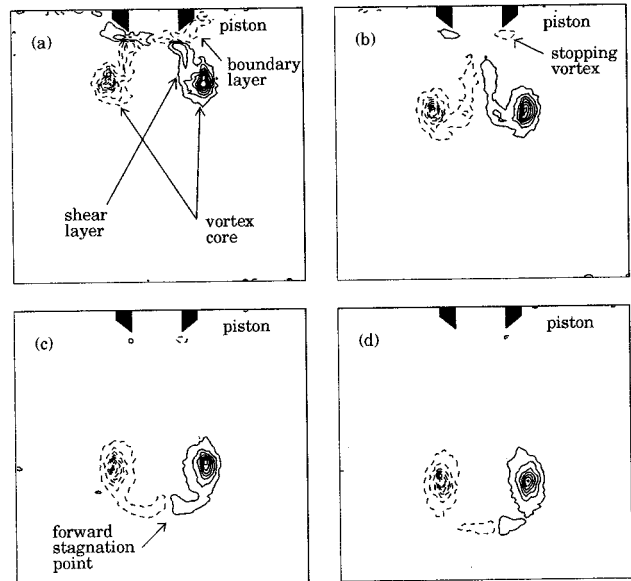


FIG. 3. Temporal development of the vortex ring, vorticity contours every $1 \text{ (s}^{-1}\text{)}$ centered about zero with the negative vorticity contours dashed, measurement points are spaced every 5 mm. The Reynolds number is about 7500 (γ/ν) generated with a piston stroke of 20.4 cm, nozzle diameter of 10.2 cm, piston velocity of 5 cm/s and acceleration/deceleration of 75 cm/s^2 . Frame (a) corresponds to the end of the piston motion, (b) follows by one second [frame (c), 3 s; (d), 4 s].

vortex ring, the local fluid velocity approaches zero (in the moving frame of reference) and the resulting advection of the local structures becomes relatively slow. As a result, vorticity remains at the front of the ring for a relatively long time. Figure 3(d) shows the ring 2.5 diameters from the nozzle. Because of the nature of the stagnation flow, vortical fluid farther from the centerline is drawn off and entrained into the core more quickly than fluid nearer the centerline,

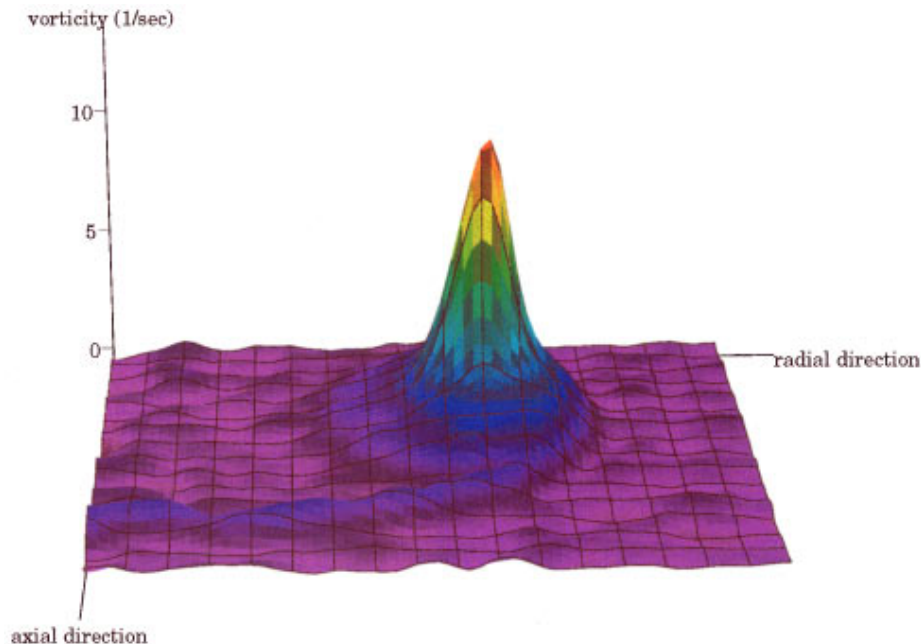


FIG. 4. Vorticity distribution for a ring of Reynold's number 3700 (same stroke, piston velocity 2.5 cm/s, and impulsive acceleration, measurement points are spaced ever 2 mm). Only the right core is shown. The ring propagates forward and the centerline is at the left.

and a local extrema of vorticity forms. There exists some variability in the position of the front stagnation point relative to the geometric center of the ring [Fig. 3(d)]. This is consistent with earlier experimental work.¹³

Figure 4 shows another perspective of the vortex core similar to Fig. 3(c) under higher experimental resolution. The vorticity in the shear layer has now advected to the front of the ring. In addition, the core distribution shows a central Gaussian peak that is surrounded by a vorticity distribution characteristic of the wrap-up of the shear layer. The center of the core is formed at early times relative to the outer distribution. The strength of the early production forms a strong central core that is diffusing through viscosity while the wrap-up of the shear layer ingests more external fluid weakening the magnitude of the vorticity in the outer core. The twofold core structure is the manifestation of the variation of circulation production rates measured by Didden⁸ and computed by Nitsche and Krasny.⁶

Both effects, the advection of the shear layer through the center of the ring and its wrap-up around the the vortex core, produce a vortex ring with a more complex distribution than expected. These variations in the vorticity distributions of the vortex cores may affect the stability characteristics and downstream behavior of the vortex ring, shedding and growth; and are important for accurate simulation and understanding of vortex ring dynamics.

ACKNOWLEDGMENTS

The authors would like to thank Dr. T. Maxworthy, Dr. G. Broadwell, and Dr. J. Koseff for helpful discussions, and

M. DeBar for the flow visualization. This work has been supported in part by the U.S. Department of Energy HPCCP Grant No. DE-FG03-92ER25140.

^{a1}Corresponding author: IIT, 10 W. 32nd St., Eng 1 Bldg, Chicago Illinois, 60616; Electronic mail: fabris@mmae.iit.edu; Telephone: (312)567-3877; Fax: (312)567-7230.

¹H. Lamb, *Hydrodynamics*, 6th ed. (Cambridge University Press, Cambridge, UK, 1932).

²K. Shariff and A. Leonard, "Vortex rings," *Annu. Rev. Fluid Mech.* **24**, 235 (1992).

³P. G. Saffman, "The velocity of viscous vortex rings," *Stud. Appl. Math.* **49**, 371 (1970).

⁴C. Tung and L. Ting, "Motion and decay of a vortex ring," *Phys. Fluids* **10**, 901 (1967).

⁵P. G. Saffman, "On the formation of vortex rings," *Stud. Appl. Math.* **54**, 261 (1975).

⁶M. Nitsche and R. Krasny, "A numerical study of vortex ring formation at the edge of a circular tube," *J. Fluid Mech.* **276**, 139 (1994).

⁷M. Nitsche, "Scaling properties of vortex ring formation at a circular tube opening," *Phys. Fluids* **8**, 1848 (1996).

⁸N. Didden, "On the formation of vortex rings: Rolling-up and production of circulation," *J. Appl. Math. Phys. Z. Angew. Math. Phys.* **30**, 101 (1979).

⁹S. L. Wakelin and N. Riley, "On the formation and propagation of vortex rings and pairs of vortex rings," *J. Fluid Mech.* **332**, 121 (1997).

¹⁰W. J. A. Dahm, C. M. Scheil, and G. Tryggvason, "Dynamics of vortex interactions with a density interface," *J. Fluid Mech.* **205**, 1 (1989).

¹¹C. E. Willert and M. Gharib, "Digital particle image velocimetry," *Exp. Fluids* **10**, 181 (1991).

¹²D. Fabris, "Combined experimental and numerical investigations of a vortex ring impinging on a wall," Ph.D. thesis, University of California, Berkeley, 1996.

¹³A. Glezer, "An experimental study of a turbulent vortex ring," Ph.D. thesis, California Institute of Technology, 1981.