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# $H_\infty$ CONTROL DESIGN FOR SYSTEMS WITH PERIODIC IRREGULAR SAMPLING USING OPTIMAL $H_2$ REFERENCE CONTROLLERS

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## ABSTRACT

*This paper presents a novel robust $H_\infty$ control design for linear systems with periodic irregular sampling and regular actuation rates. A three-step design algorithm is developed to design a controller that achieves high robustness in terms of disk margin, and high performance in terms of root mean square (RMS) of $3\sigma$-value of performance signal. The proposed method is exploited to design a track-following controller for a hard disk drive (HDD) with 30% sampling time irregularity. The simulation study presented in this paper shows the effectiveness of the proposed control design for high-order systems with large periods.*

## 1 INTRODUCTION

Optimal $H_\infty$ control design is one of the most popular methodologies for synthesizing control systems that achieve robust performance or stabilization. In [1] an explicit solution for optimal $H_\infty$ control synthesis via discrete Riccati equations for LPTV systems is proposed. The authors provided an $H_\infty$ control synthesis based on loop-shaping methods. The performance and robustness of the controller designed by this method strongly depends on the selection of appropriate loop-shaping weighting functions, which is not generally a trivial process and needs several manual iterations. In this paper, we use a novel discretization method proposed in [2] to find an LPTV model for a system with periodic irregular sampling. The control design algorithm in this paper consists of two main steps. In the first step, a linear quadratic Gaussian (LQG) control problem with a constraint on variance of the control signal is solved using the results of [2]. Once the LQG controller is designed, it will be exploited in the second step of the control design algorithm, in which an optimal LPTV $H_\infty$ controller will be designed.

Since the system is LPTV, it is not possible to use classical gain and disk margins as the measures of robustness. Instead, we characterize the robustness of this type of systems by using an idea known as "disk margin", which can be calculated by computing the $H_\infty$ norm of a modified closed loop system. Therefore, the controller design can be formulated as an optimal $H_\infty$ problem that aims to achieve two objectives. The first objective is the minimization of the $H_\infty$ norm associated with the disk margin of the system, and the second objective is to minimize the $H_\infty$ norm

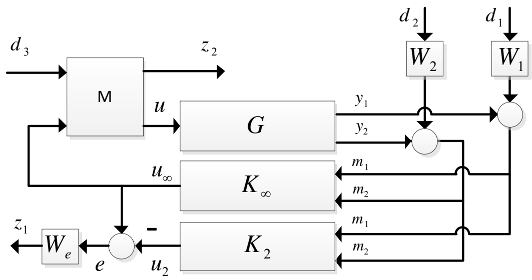


**FIGURE 1**: ARCHITECTURE USED FOR CONTROL DESIGN

of the error between the outputs of the constrained LQG and the $H_\infty$ controller that is being designed. This multi-objective optimization problem is replaced by a single-objective $H_\infty$ optimization problem, which is solved by periodic Riccati equations.

## CONTROL DESIGN

The control objective throughout this paper is to maximize the robustness of the closed-loop system while maintaining good performance. We use the method proposed in [3] to calculate the disk margin of the system as a measure of robustness for LPTV systems. The architecture we use for control design is shown in Fig. 1. The blocks $G$, $K_2$ and $K_\infty$ are respectively the plant model, optimal LQG controller and optimal $H_\infty$ controller that will be calculated through the steps of design algorithm. Block $M$ is a constant matrix and defined by $M := [[1,\sqrt{2}]^T,[\sqrt{2},1]^T]$.

The $H_\infty$ norm from $d_3$ to $z_2$ gives the disk margin of the closed-loop system. It should be noticed that the output of $K_2$ does not enter the closed loop system. Hence, for a fixed $K_\infty$ and $K_2$ the performance and robustness of the system are only functions of $K_\infty$. Although the actual plant is single-input single-output (SISO), the particular discretization method used here introduces two signals, $y_1$ and $y_2$, to model the output dynamics (more details in subsection 1.1). Signal $e$ in Fig. 1 corresponds to the error between the output of the optimal $H_\infty$ controller and $H_2$ controller. Hence the $H_\infty$ norm of the closed loop system from all the inputs to this output will be small when the dynamics of $K_\infty$ controller is close to the dynamics of $K_2$. Finally, signals $d_1$ and $d_2$ are two fictitious disturbances added to the outputs of the plant to satisfy the regularity conditions required for the solution of $H_\infty$ control design [1].

### 1.1 Constructing the plant model

We use the method proposed in [3] to find an LPTV model for a system with periodic irregular sampling. For such a system control update can be either clock-driven (i.e. regular in time) or event-driven (e.g. updating control as soon as obtaining a measurement). As illustrated in [2] the former scheme can result in better closed-loop performance for the type of systems studied in this work. Therefore, we use a clock-driven control update scheme throughout this paper. In this control scheme, there is no fixed relationship between the times at which measurements are obtained and the times at which the control is updated. Hence, the number of samples in the time interval $S_k$ (c.f. 3) is not necessarily constant over $k$. For simplicity, we will consider a situation in which 0, 1, or 2 measurements may be made in any time interval $S_k$. Regardless of how many measurements are placed in a time interval $S_k$, we assume that the system has always 2 outputs, which are denoted as $y_1$ and $y_2$ in Fig. 1 (e.g. $y := [y_1, y_2]^T$ where $y$ is the output vector). When there is just one available output, say $y_1$, the other element of the output signal, $y_2$, will be zero. Similarly when there is no available outputs, both of the elements of the output vector will be zero. It is noteworthy that this assumption makes the system structure time invariant.

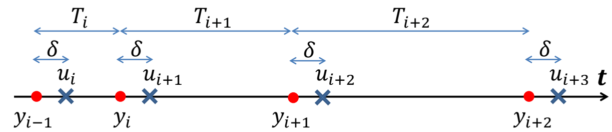


**FIGURE 2**: IRREGULAR SAMPLING (CIRCLES) AND IRREGULAR ACTUATION (CROSSES) RATES

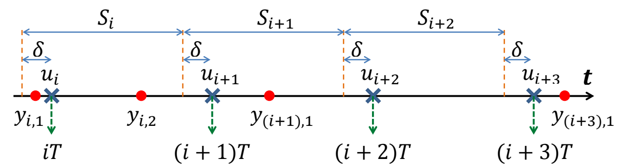


**FIGURE 3**: IRREGULAR SAMPLING (CIRCLES) AND REGULAR ACTUATION (CROSSES) RATES

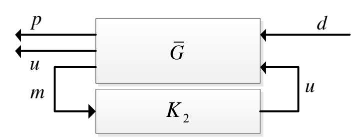


**FIGURE 4**: ARCHITECTURE USED FOR DESIGNING $H_2$ REFERENCE CONTROLLER

### 1.2 Designing optimal LQG controller ($K_2$)

We design an optimal LQG controller subject to a constraint on variance of the control signal for the plant model described in subsection 1.1. This optimal control problem is presented in [2] and the control design architecture is shown in Fig. 4. The block $\bar{G}$ in that figure represents the dynamics of the plant and disturbances. Indeed, $\bar{G}$ is an LPTV state space system from the disturbances and control signal to the performance ($p$), control signal ($u$) and measurement vector ($m$). The disturbance dynamics is modeled and implanted in $\bar{G}$ such that the signal $d$ is white noise with identity covariance. This controller will achieve the minimum RMS 3-$\sigma$ value of $p$ when the mean variance of $u$ is constrained to be less than or equal to a required value. This value will be chosen by the designer and corresponds to the maximum reasonable power to the actuator.

Since the robustness of the closed loop system is not considered in the design of $K_2$, we cannot guarantee a lower bound on the robustness of the closed loop system. To solve this problem and increase the robustness of the system we consider the next step.

### 1.3 Designing optimal $H_\infty$ controller ($K_\infty$)

Once the $K_2$ controller is designed, we choose three static values for the weighting parameters, $W_1, W_2$ and $W_e$, and form the interconnection shown in Fig. 1 to calculate $K_\infty$. For large values of $W_e$ the effect of system dynamics from the inputs to $z_2$ will be negligible. In the absence of $z_2$, the $H_\infty$ norm of the closed loop system will be minimum and equal to zero when $K_\infty = K_2$. In this case the system will have the highest achievable performance. In contrast, when $W_e$ is close to zero, the minimization of the $H_\infty$ norm of the system will result in large disk margin, because the $H_\infty$ norm from $d_3$ to $z_2$ will be small which is conversely related to the robustness of the closed-loop system. Accordingly, increasing and decreasing the value of $W_e$ will respectively result in increasing the performance and robustness of the system.

The other two scaling parameters, $W_1$ and $W_2$, are just considered to make the feed-through term form uncontrollable inputs to measurements in the dynamics of the open loop system non-zero; this is one of the regularity conditions required for the solution of the optimal $H_\infty$ problem [4]. Once all the design parameters are chose, $K_\infty$ can be calculated by the method proposed in [1].

## SIMULATION STUDY

To show the effectiveness of the proposed method, a robust $H_\infty$ controller is designed for track-following in a hard disk drive. Sampling intervals for hard disk drive servo systems are not always equidistant over a revolution of the disk. Since HDDs are naturally periodic, with period equal to the time for one rotation of the disk, the servo system from the control input to the position error signal (PES) can be represented by an LPTV system whose period is the number of servo sectors, $N$. The order of the plant model we use is 18 and the period ($N$) is 400. We assume that the sampling time profile is sinusoidal with period equal to one

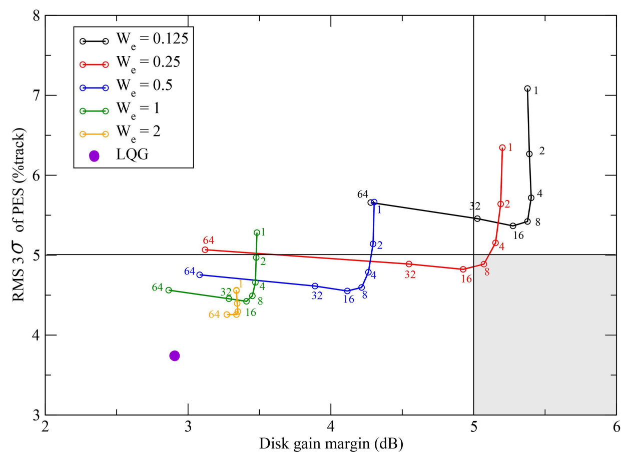


**FIGURE 5**: PERFORMANCE VS ROBUSTNESS OF HDD

revolution of the disk and the variation from minimum to maximum sampling interval is 60% of the nominal sampling time. The performance (in terms of RMS of 3$\sigma$ value of PES) versus the robustness (in terms of disk gain margin) is shown in Fig. 5. The point denoted as LQG represents the performance and robustness characteristics of the $H_2$ reference controller (designed in step 2), which has the best performance but inadequate robustness. Each plot corresponds to a fixed value of $W_e$ and consists of seven points corresponding to different values of $W$, where $W = W_1 = W_2$. The values of $W$ are shown next to each data point. As expected, for fixed values of $W$, increasing $W_e$ increases the robustness; in contrast, decreasing this value boosts the performance of the closed loop system. As a result, when 5 $dB$ of gain disk margin and 5% $track$ of PES 3$\sigma$ are required (shaded region in Fig. 5) the appropriate controller can be designed by choosing $W_e = 0.25$ and $W = 8$.

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