ABSTRACT

The stability analysis of an adaptive control scheme for robotic manipulators, originally introduced by Horowitz and Tomizuka (1980), is presented in this paper. In the previous stability proof it was assumed that the manipulator parameter variation is negligible compared with the speed of adaptation. It is shown that this key assumption can be removed by introducing two modifications in the adaptive control scheme: 1) Reparameterizing the nonlinear terms in dynamic equations as linear functions of unknown but constant terms. 2) Defining the Coriolis compensation term in the control law as a bilinear function of the manipulator and model reference joint velocities, instead of a quadratic function of the manipulator joint velocities. The modified adaptive control scheme is shown to be globally asymptotically stable.

1. Introduction

Robotic manipulators are mechanical systems with nonlinear dynamic characteristics. Furthermore, their inertia characteristics and disturbance characteristics, such as the one coming from gravity, vary during operation and they are not necessarily predictable in advance. Adaptive control techniques have been suggested as an effective method for decoupling and linearizing the manipulator dynamic characteristics.

The earliest work on model reference adaptive control for manipulators (Dubowsky and DesForges, 1979) is based on a linear decoupled model. The steepest descent technique is utilized for parameter adaptation. The first work on adaptive controls of mechanical manipulators based on stability theories was reported by Horowitz and Tomizuka (1980). They utilized the hyperstability theory for adaptively linearizing and decoupling the nonlinear manipulator dynamic equations. Preliminary experimental evaluations of this approach have been reported by Anex and Hubbard (1984) and Tomizuka et al. (1985b), (1986). A Lyapunov stability based adaptive control approach for trajectory following has been proposed by Takegaki and Arimoto (1981). The Lyapunov stability based adaptive control approach has been used as in Craig et al. (1986).

Recent works in this field include those by Balesino et al. (1983), Nicosia and Tomei (1984) and Craig et al. (1986). One trend in these recent works is to assure the stability of the overall system in spite of the nonlinear nature of the parameters in the manipulator dynamic equations. This is achieved by either of the following two methods: 1) Decomposing the nonlinear parameters in the manipulator dynamic equations into the product of two quantities: one constant unknown quantity, which includes the numerical values of the masses and moments of inertia of the links and the payload, and link dimensions, the other a known nonlinear function of the manipulator structural dynamics. The nonlinear functions are then assumed to be known and calculable. The parameter adaptation law is only used to estimate the unknown constant quantities (e.g. Craig et al. (1986)). 2) Utilizing nonlinear switching parameter adaptation laws, making use of the knowledge of upper bounds of the parameters in the manipulator dynamic equations. This parameter adaptation law belongs to the class of variable structure control schemes. (e.g. Balesino et al. (1983)). These approaches are all based on the continuous time control theory.

In this paper it is demonstrated that, by modifying the parameter adaptation law using method 1) outlined above, the adaptive control scheme introduced by Horowitz and Tomizuka (1980) is globally asymptotically stable. Moreover in this control scheme only joint position and velocity feedback information is required (no joint acceleration is used, and no matrix inversion is used as in Craig et al. (1986)).

Adaptive control approaches have also been proposed based on the discrete time control theory. Koivo and Gue (1983) proposes a self tuning regulator approach based on a functional linear difference equation model. Dubowsky (1981) has investigated the discrete time implementation of the model reference adaptive control approach (Dubowsky and DesForges, 1979). Horowitz and Tomizuka (1982) proposed a discrete time adaptive control scheme for mechanical manipulators based on the adaptive control approach by Landau and Lozano (1981). The Landau and Lozano method is also the basis for a more recent decentralized adaptive control scheme (Sundareshan and Koenig, 1985).

This paper is organized as follows: In Section 1., the manipulator's dynamic equations of motion are presented. Section 2., contains the non-adaptive non-linearly compensation decoupling control law. In sec-

*The authors have recently learned of a similar stability analysis by Slotine(1986), after the submission of this paper.
tion 4., the adaptive control law is presented and a global asymptotic stability theorem is proven. Section 5. contains the conclusions and the references are in section 6.

2. Dynamic Model of a Robotic Manipulator

In this paper we consider robotic manipulator composed of a serial open chain of rigid links connected with either revolute or prismatic joints. The dynamic equations of motion for the manipulator can be expressed in the following form:

$$\frac{d}{dt}x_p(t) = x_v(t)$$

$$M(x_p) \frac{d}{dt}x_v(t) = q(t) - v(x_p, x_v) - g(x_v) - c(x_p, x_v)$$

where

- $x_p$ is the $n \times 1$ vector of joint positions,
- $x_v$ is the $n \times 1$ vector of joint velocities,
- $M(x_p)$ is the $n \times n$ symmetric and positive definite matrix. Also called the generalized inertia matrix,
- $q(t)$ is the $n \times 1$ vector of joint torques or forces supplied by the actuators,
- $v(x_p, x_v)$ is the $n \times 1$ vector due to Coriolis and centrifugal accelerations,
- $g(x_v)$ is the $n \times 1$ vector due to gravitational forces and
- $c(x_p, x_v)$ is the $n \times 1$ vector due to friction forces.

$v(x_p, x_v)$ can be expressed in the following form

$$v(x_p, x_v) = \begin{bmatrix}
    x_v^T N(x_v) x_v \\
    \vdots \\
    x_v^T N(x_v) x_v
\end{bmatrix}$$

where the matrices $N_i$'s are symmetric.

The following relation is satisfied between the matrices $N_i$'s and the generalized inertia matrix $M$.

$$N_i(x_p) = \left[ \frac{\partial m_i}{\partial x_p} - \frac{1}{2} \frac{\partial M}{\partial x_p} \right]$$

where $m_i$ is the $i$th row (or column) of $M$ and $x_p$ is the $i$th element of $x_p$. The proof is found in Appendix 1.

The $i$th element of the friction force vector $c(x_p, x_v)$ can be expressed as

$$c_i(x_p, q_i) = c_{ci}(x_{pi}, q_i) + c_{ri} x_{pi}(t)$$

where $c_{ci}(x_{pi}, q_i)$ represents the Coulomb friction component and $c_{ri}$ represents the linear friction component.

$$c_{ci}(x_{pi}, q_i) = \begin{cases}
    c_{emi} \text{sign}(x_{pi}(t)) & \text{if } |x_{pi}(t)| > 0 \\
    c_{emi} \text{sign}(q_i) & \text{if } |x_{pi}(t)| = 0 \\
    q_i(t) & \text{if } |x_{pi}(t)| = 0 \\
    q_i(t) & \text{and } |q_i(t)| > c_{emi}
\end{cases}$$

where $c_{emi}$ is the magnitude of the friction force.

The values of friction force magnitudes $c_{emi}$ and $q_i$ in general do not vary substantially with manipulator motion or payload variation. Thus, they can be considered as constant and can be estimated, if desired, by off-line experiments, as shown by Tomizuka et al. (1985a), Kubo et al. (1986) and Anwar et al. (1986).

The elements of the matrices $M(x_p)$, $N(x_v)$ and of the vector $g(x_v)$ are in general highly nonlinear functions of the position vector $x_p$. They are also a function of the link and payload masses and moments of inertia, which may not be precisely known or may vary during the manipulator task. The variation of these parameters is especially significant in the dynamic response of direct drive robotic arms.

2.1. Dynamic Equations Reparametrization

As discussed in Craig et al. (1986) the nonlinear parameters in $M(x_p)$, $N(x_v)$ and $g(x_v)$ can be decomposed into products of constant terms, which are functions of the inertia characteristics of the links and the payload, and known nonlinear functions of the joint positions.

One method of reparametrizing the manipulator's dynamic equations consists in decomposing each element of the matrices $M(x_p)$, $N(x_v)$ and the vector $g(x_v)$ into products of unknown constant terms and known functions of the joint displacement vector $x_p$ i.e.

$$m_{ij}(x_p) = \sum_{r=1}^{r=rmij} m_{rij} f_{nrij}(x_p)$$

where $m_{ij}$ is the $ij$th element of $M(x_p)$, $m_{rij}$'s are constant parameters, which are assumed unknown, and $f_{nrij}(x_p)$'s are known functions of $x_p$.

Similarly,

$$n_{ikj}(x_v) = \sum_{r=1}^{r=rmij} n_{ikj} f_{nrij}(x_p)$$

where $n_{ikj}$ is the $ikj$th element of $N$ and $n_{ikj}$'s are constant unknown quantities and $f_{nrij}(x_p)$'s and $f_{grij}(x_p)$'s are known functions.

A second method, which is the one proposed in Craig et. al. (1986), consists in the reparametrization of Eq. (1) into the product of unknown constant vector, which is function of the unknown masses and moments of inertia of the links and the payload, and a matrix formed by known functions of $x_p$, $x_v$ and $x_{qi}$, where $x_q = \frac{d}{dt} x_v$ are the joint accelerations.

$$M(x_p) x_v + v(x_p, x_v) + g(x_v) = W(x_p, x_v, x_{qi}) \Theta$$

where $W(x_p, x_v, x_{qi})$ is an $n \times r$ matrix and $\Theta$ is an $r \times 1$ vector of the unknown constant parameters. Notice that $W(x_p, x_v, x_{qi})$ is a linear function of $x_v$.

By using this method, a substantial reduction in the number of parameters that need to be estimated is achieved.

3. Nonlinearity Compensation and Decoupling Control

In order to dynamically decouple and linearize the manipulator equations of motion (1), the following controller is proposed (Horowitz and Tomizuka 1980):
\[ q(t) = \ddot{\mathbf{q}}(t)u(t) + \dot{\varphi}(t) + \ddot{\mathbf{g}}(t) + \ddot{\mathbf{e}}(t) \]
\[ + \mathbf{F}_p[\mathbf{x}_p(t) - \mathbf{x}_q(t)] + \mathbf{F}_v[\mathbf{x}_v(t) - \mathbf{x}_q(t)] \]
(10)

where \(u(t)\) is the acceleration input, and \(\mathbf{q}_p(t)\) and \(\mathbf{q}_v(t)\) are the reference velocities and position vectors:
\[ \frac{d}{dt}\mathbf{q}_p(t) = \dot{\mathbf{q}}_p(t) \]
(11)
\[ \frac{d}{dt}\mathbf{q}_v(t) = u(t) \]
(12)

\(\mathbf{F}_p\) and \(\mathbf{F}_v\) are constant diagonal positive definite gain matrices and the functions \(\mathbf{q}_p, \dot{\mathbf{q}}_p\) and \(\mathbf{q}_v, \dot{\mathbf{q}}_v\) are the estimates of \(\mathbf{q}(x_p, x_q), \dot{\mathbf{q}}(x_p, x_q)\) and \(\mathbf{q}(x_v, x_q)\) respectively.

Note that the assumption that
\[ \ddot{\mathbf{q}}(t) = \mathbf{M}(x_p), \quad \dot{\mathbf{q}}(t) = \mathbf{v}(x_p, x_q), \quad \mathbf{g}(t) = \mathbf{g}(x_p) \]
and \(\mathbf{c}(x_v, x_q)\) respectively.

In order to complete the controller design, an outer loop consisting of a proportional, integral and derivative (PID) action is used for trajectory tracking purposes:
\[ u(t) = \bar{\mathbf{u}}_p(t) = \mathbf{K}_p[\mathbf{x}_d(t) - \mathbf{x}_p(t)] + \mathbf{K}_v[\mathbf{x}_d(t) - \mathbf{x}_v(t)] \]
(16)
\[ \frac{d}{dt}u(t) = \mathbf{K}_p[\dot{\mathbf{x}}_d(t) - \dot{\mathbf{x}}_p(t)] + \mathbf{K}_v[\dot{\mathbf{x}}_d(t) - \dot{\mathbf{x}}_v(t)] \]
(17)

where \(\mathbf{x}_d(t)\) and \(\dot{\mathbf{x}}_d(t)\) are respectively the position, velocity and acceleration of the desired trajectory, and the matrices \(\mathbf{K}_p, \mathbf{K}_v\) and \(\mathbf{K}_p\) are the PID gains which are selected such that the characteristic equation:
\[ s^2\mathbf{K}_p + \mathbf{s}\mathbf{K}_v + \mathbf{K}_p = 0 \]
has its root in the left hand side of the complex plane.

The friction force estimate \(\tilde{\mathbf{g}}(t)\) can be calculated by Eqs. (4) and (5), replacing the magnitudes \(\tau_\theta\)'s and \(\tau_{cm}\)'s by their estimates. Since these quantities can be successfully estimated off-line, we will assumed that adequate friction compensation is implemented and neglect the effect of the friction forces altogether.

4. Adaptive Controller
In this section we discuss an adaptive control scheme for updating the control parameters \(\ddot{\mathbf{q}}(t), \dot{\varphi}(t)\) and \(\ddot{\mathbf{e}}(t)\) in the control law given by Eq. (10). The scheme presented here is a modification of the parameter adaptation law originally introduced by Horowitz and Tomizuka (1980). Thus, we first introduce the original scheme which we call the "Constant Plant Parameter Adaptive Control (CPPAC)."

4.1. Constant Plant Parameter Adaptive Control (CPPAC)
\[ q(t) = \ddot{\mathbf{q}}(t)u(t) + \dot{\varphi}(t) + \ddot{\mathbf{e}}(t) \]
\[ + \mathbf{F}_p[\mathbf{x}_p(t) - \mathbf{x}_q(t)] + \mathbf{F}_v[\mathbf{x}_v(t) - \mathbf{x}_q(t)] \]
(18)

where \(\mathbf{v}\) is defined by
\[ \mathbf{v}(x_v, t) = \begin{bmatrix} \mathbf{x}_p^T \tilde{\mathbf{q}}_v(t) \\ \mathbf{x}_p^T \tilde{\mathbf{q}}_v(t) \\ \mathbf{x}_v^T \tilde{\mathbf{q}}_v(t) \end{bmatrix} \]
(19)

and the elements of the parameters \(\ddot{\mathbf{q}}(t), \tilde{\mathbf{q}}^r(t)\)'s and \(\tilde{\mathbf{e}}(t)\) are updated as follows
\[ \frac{d}{dt}\mathbf{m}_i = k_{mi}[\mathbf{v}(t)u_i] \]
(20)
\[ \frac{d}{dt}\tilde{\mathbf{q}}^r_i = k_{q_i}[\mathbf{y}_i\mathbf{x}_q u_i] \]
(21)
\[ \frac{d}{dt}\hat{\mathbf{g}}_i = k_p y_i \]
(22)

where \(\mathbf{m}_i\) is the \(i\)th element of \(\ddot{\mathbf{q}}\), \(\tilde{\mathbf{q}}^r_i\) is the \(i\)th element of \(\tilde{\mathbf{q}}\), \(\hat{\mathbf{g}}_i\) is the \(i\)th element of \(\mathbf{g}\), and the vector \(\mathbf{y}(t)\) is defined as
\[ \mathbf{y}(t) = [y_1, ..., y_n]^T = \mathbf{C}_p\mathbf{x}_p(t) + \mathbf{C}_v\mathbf{x}_v(t) \]
(23)

where \(k_{mi} > 0, k_{q_i} > 0, k_p > 0\).
(24)
\[ \mathbf{F}_p = \mathbf{F}_v = \rho_\theta \mathbf{I}, \quad \mathbf{C}_p = \mathbf{C}_v = \mathbf{C}_o = \mathbf{C}_s \]
(25)

is the identity matrix and \(\rho_\theta, \rho_v, \sigma_\theta, \sigma_v\) satisfy
\[ \sigma_\theta > \sigma_v > \sigma_\theta \rho_\theta \quad \sigma_v > \sigma_v \rho_v \quad \sigma_v \mathbf{M}_{max} > 0 \]
and \(\sigma_v \mathbf{M}_{max} > 0 \quad \mathbf{M}_{max} = \max \{\mathbf{M}(x_p)\} \). 

Note that conditions (25) are satisfied when
\[ \mathbf{y}(t) = \mathbf{x}_v(t) \]
(22a)

As shown in Horowitz and Tomizuka (1980), the control law in Eqs. (18) and (19), and the parameter adaptation laws in Eqs. (19) - (22) guarantee the asymptotic tracking objective:
\[ \lim_{t \to \infty} [\mathbf{E}_p(t) - \mathbf{E}_q(t)] = 0 \quad \text{and} \quad \lim_{t \to \infty} [\mathbf{E}_v(t) - \mathbf{E}_q(t)] = 0 \]
under the assumption that the parameters \(\mathbf{M}, \mathbf{N}_p\)'s and \(\mathbf{g}\) remain constant during the adaptation.

Notice that in this adaptive control scheme no joint acceleration feedback or matrix inversion is required as in Craig et al. (1965).*
4.2. Modified Adaptive Control Scheme

In order to remove the assumption that the parameters \( M \), \( N \)'s and \( g \) remain constant during adaptation, the following modifications to the control scheme in Eqs. (19) - (25) should be made:

4.2.1. Modification in the Control law:

\[
q(t) = \hat{\mathbf{M}}(\mathbf{x}, t)u(t) + \hat{\mathbf{V}}(\mathbf{x}, \mathbf{\dot{x}}, \mathbf{\dot{\dot{x}}}, t) + \hat{\mathbf{g}}(\mathbf{x}, t)
\]

where \( \hat{\mathbf{V}} \) is now defined by

\[
\hat{\mathbf{V}}(\mathbf{x}, \mathbf{\dot{x}}, \mathbf{\dot{\dot{x}}}, t) = \begin{bmatrix}
\mathbf{x}' \mathbf{N}'(\mathbf{x}, t) \mathbf{x}' \\
\mathbf{x}' \mathbf{N}'(\mathbf{x}, t) \mathbf{\dot{x}}' \\
\vdots \\
\mathbf{x}' \mathbf{N}'(\mathbf{x}, t) \mathbf{\dot{\dot{x}}}'
\end{bmatrix}
\]

Note that there are only two differences between Eqs. (18) and (26): 1) In Eq. (18) \( \mathbf{V} \) is a quadratic function of the joint velocity vector \( \mathbf{x}_d(t) \), while in Eq. (27) \( \hat{\mathbf{V}} \) is a bilinear function of the velocity vector \( \mathbf{x}_d(t) \) and the reference model velocity vector \( \mathbf{\dot{x}}_d(t) \). 2) In Eq. (26), only the error between the manipulator velocity vector and the reference model velocity vector, \( \mathbf{e}_v \), is used. The position error, \( \mathbf{e}_p \), between the manipulator and the reference model is not used at all. Thus, Eq. (26) can be viewed as a minor loop velocity feedback compensation control. The use of an adaptive minor loop velocity feedback control and a non-adaptive outer loop in the control system structure has been extensively utilized in experimental studies. (Anwar et. al. (1986) and Tomizuka et. al. (1986)).

4.2.2. Modification in the Parameter Adaptation Law

It is apparent that, in order to guarantee asymptotic convergence of the adaptive control scheme, the parameter adaptation law should track a constant parameter instead of a function of time or the state \( \mathbf{x}_d \). The choice of dynamic equation reparametrization determines the choice of parameter adaptation law.

The reparametrization of \( \mathbf{M}(\mathbf{x}_p), \mathbf{N} \)'s and \( g \) in Eqs. (6) - (8) leads to the following parameter adaptation algorithm (PAA): Define the parameter estimates \( \hat{\mathbf{M}}, \hat{\mathbf{N}} \)'s and \( \hat{\mathbf{g}} \) by

\[
\hat{\mathbf{m}}_{ij}(\mathbf{x}_p, t) = \sum_{r=1}^{m} m_{r_{ij}}(t) f_{m_{r_{ij}}}(\mathbf{x}_p)
\]

\[
\hat{n}^k_{ij}(\mathbf{x}_p, t) = \sum_{r=1}^{n} n^k_{r_{ij}}(t) f_{n^k_{r_{ij}}}(\mathbf{x}_p)
\]

\[
\hat{g}_i(\mathbf{x}_p, t) = \sum_{r=1}^{g} \hat{g}_r(t) f_{g_r}(\mathbf{x}_p)
\]

where \( f_{m_{r_{ij}}}(\mathbf{x}_p)'s, f_{n^k_{r_{ij}}}(\mathbf{x}_p)'s \) and \( f_{g_r}(\mathbf{x}_p)'s \) are known functions, and the parameters \( m_{r_{ij}}'s(t), n^k_{r_{ij}}'s(t) \) and \( \hat{g}_r's(t) \) are updated by

\[
\frac{d}{dt}\hat{m}_{r_{ij}} = k_{m_{r_{ij}}} \sum_{r=1}^{m} m_{r_{ij}}(t) f_{m_{r_{ij}}}(\mathbf{x}_p)
\]

\[
\frac{d}{dt}\hat{n}^k_{r_{ij}} = k_{n^k_{r_{ij}}} \sum_{r=1}^{n} n^k_{r_{ij}}(t) f_{n^k_{r_{ij}}}(\mathbf{x}_p)
\]

\[
\frac{d}{dt}\hat{g}_r = k_{g_r} \sum_{r=1}^{g} \hat{g}_r(t) f_{g_r}(\mathbf{x}_p)
\]

where \( k_{m_{r_{ij}}} > 0, k_{n^k_{r_{ij}}} > 0, k_{g_r} > 0 \),

where \( \hat{e}_v \) is the \( k \)th element of the velocity error vector, \( \mathbf{e}_v \), defined in Eq. (15).

The time varying parameters \( \hat{m}_{r_{ij}}'s, \hat{n}^k_{r_{ij}}'s \) and \( \hat{g}_r's \) are the estimates of the constant parameters \( m_{r_{ij}}'s, n^k_{r_{ij}}'s \) and \( g_r's \) respectively.

If the dynamic equations are reparametrized in the form of Eq. (9), the following PAA should be employed:

From Eq. (26)

\[
\hat{\mathbf{M}}(\mathbf{x}, t)u(t) + \hat{\mathbf{V}}(\mathbf{x}, \mathbf{\dot{x}}, \mathbf{\dot{\dot{x}}}, t) + \hat{\mathbf{g}}(\mathbf{x}, t) = \mathbf{W}(\mathbf{x}, \mathbf{\dot{x}}, \mathbf{\dot{\dot{x}}}, t) \hat{\mathbf{\Theta}}(t)
\]

where \( \mathbf{r} \times 1 \) vector \( \hat{\mathbf{\Theta}}(t) \) is the estimate of the unknown vector \( \mathbf{\Theta} \) in Eq. (9), and the matrix \( \mathbf{W}(\mathbf{x}, \mathbf{\dot{x}}, \mathbf{\dot{\dot{x}}}, t) \) is similar to \( \mathbf{W}(\mathbf{x}, \mathbf{\dot{x}}) \) in Eq. (9), only that \( \mathbf{x} \) is replaced by \( \mathbf{u}(t) \) and \( \mathbf{x}\mathbf{u} \) is replaced by \( \mathbf{x}\mathbf{u}(\mathbf{\dot{x}}) \).

Notice that in the modified parameter adaptation law, the velocity error signal \( \mathbf{e}_v(\mathbf{t}) \) is used in Eqs. (31) - (33) (i.e. \( \mathbf{e}_p = 0 \) in Eqs. (21) and (22)). Also, comparing with the method presented by Craig et. al. (1986), the acceleration input \( \mathbf{u}(t) \) is used in the PAA's instead of the joint accelerations \( \mathbf{\dot{x}} \), which are not measurable in most realistic applications, and no matrix inversion is required in the control algorithm.

Theorem 1

For a mechanical manipulator governed by Eq. (1), given a bounded desired trajectory \( \mathbf{x}_d(t) \) with bounded first and second derivatives \( \mathbf{\dot{x}}_d(t) \) and \( \mathbf{\ddot{x}}_d(t) \), if the adaptive control law given by Eqs. (12),(15),(16),(26) - (33) or by Eqs. (12),(15),(16),(26), (27), (34) and (35) is used, the error between the desired and actual trajectory converges asymptotically to the zero, i.e.

\[
\lim_{t \to \infty} [\mathbf{x}(t) - \mathbf{x}_d(t)] = 0 \quad \text{and} \quad \lim_{t \to \infty} [\mathbf{\dot{x}}(t) - \mathbf{\dot{x}}_d(t)] = 0
\]

Proof

In this section we will present the stability proof of the adaptive control system when the adaptive control law given by Eqs. (26),(27),(34) and (35) is utilized. The proof for the case when the adaptive control law given by Eqs. (26)-(33) is employed is almost identical and will be omitted.

The proof of Theorem 1 will be carried out in two stages. In the first stage it will be shown that, regardless of the acceleration input function \( \mathbf{u}(t) \), the velocity error signal \( \mathbf{e}_v(\mathbf{t}) \) between the reference model \( \mathbf{\dot{x}}_d(t) \) and the manipulator velocity \( \mathbf{x}_d(t) \) remains bounded. In the second stage of the proof it is shown that, given a bounded desired trajectory signal \( \mathbf{x}_d(t) \), all signals in the closed loop system remain bounded, and, as a consequence, the tracking error between the desired trajectory \( \mathbf{x}_d(t) \) and the manipulator trajectory \( \mathbf{x}_d(t) \) converges asymptotically to zero.

We begin by obtaining an expression for the velocity error between the reference model and the manipulator, \( \mathbf{e}_v(\mathbf{t}) \), as a function of the parameter error.

Multiplying Eq. (12) by \( \mathbf{M}(\mathbf{x}_d) \), subtracting Eq. (1) and utilizing the control law (26) we obtain

\[
\mathbf{M}(\mathbf{x}_d) \frac{d}{dt} \mathbf{e}_v = - \mathbf{P}_v \mathbf{e}_v + [\mathbf{M}(\mathbf{x}_d) \cdot \hat{\mathbf{M}}(\mathbf{x}_d, t)] \mathbf{u}(t) + \mathbf{V}(\mathbf{x}_d, \mathbf{\dot{x}}_d, \mathbf{\ddot{x}}_d, t) + \mathbf{g}(\mathbf{x}_d) - \mathbf{\hat{g}}(\mathbf{x}_d, t)
\]
where we have used the definition of \( e \), given in Eq. (15).

By Eqs. (2) and (27), Eq. (36) can be rearranged as follows

\[
\mathbf{M}(x_p) \frac{d}{dt} e_v(t) = -F_v e_v(t) + [\mathbf{M}(x_p) - \hat{\mathbf{M}}(x_p, t)] u(t)
\]

\[
\begin{bmatrix}
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v
\end{bmatrix}
\begin{bmatrix}
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v
\end{bmatrix}
\begin{bmatrix}
\mathbf{e}(x_p, t) - \mathbf{w}(x_p, x_v, x_u) \hat{\mathbf{e}}(t)
\end{bmatrix}
\]

\[
\mathbf{M}(x_p) \frac{d}{dt} e_v(t) = -F_v e_v(t) - \mathbf{v}(x_p, x_v, x_u) - \mathbf{w}(x_p, x_v, x_u, u) \hat{\mathbf{e}}(t)
\]

(37)

where

\[
v(x_p, x_v, x_u) = \begin{bmatrix}
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v \\
\mathbf{x}_v^T \mathbf{N}(x_p) e_v
\end{bmatrix}
\]

and Eqs. (9) and (34) have been utilized. The parameter error vector \( \hat{\mathbf{e}}(t) \) is defined by

\[
\hat{\mathbf{e}}(t) = \hat{\mathbf{e}}(t) - \mathbf{e}.
\]

Lemma 1

Consider the system described by the error equation Eq. (38), and the PAA Eq. (35). Then \( e_v(t) \) and \( \hat{\mathbf{e}}(t) \) are bounded for all \( t \geq 0 \).

a.1)

If, in addition \( \|\mathbf{w}(x_p, x_v, x_u)\| \leq \mathcal{W}_v < \infty \) then

\[
\lim_{t \to \infty} e_v(t) = 0
\]

Proof:

Define the Lyapunov function candidate

\[
V(t) = \frac{1}{2} e_v^T e_v(t) M(x_p(t)) e_v(t) + \frac{1}{2} \hat{\mathbf{e}}^T(t) K^{-1} \hat{\mathbf{e}}(t).
\]

Notice that \( V(t) \) is a legitimate Lyapunov function candidate since (Horowitz and Tomizuka (1980)) \( \mathbf{M}(x_p) \) is symmetric and

\[
0 < \|\mathbf{M}\| \leq \|\mathbf{M}(x_p)\| \leq \|\mathbf{M}_{\text{max}}\| < \infty.
\]

From (41) and (38) we obtain

\[
\frac{d}{dt} V(t) = -\mathbf{e}_v^T F_v e_v + \mathbf{e}_v^T \left[ -\mathbf{v}(x_p, x_v, x_u) - \mathbf{w}(x_p, x_v, x_u, u) \hat{\mathbf{e}} \right] + \frac{1}{2} \mathbf{e}_v^T \mathbf{M}(x_p) e_v + \frac{1}{2} \hat{\mathbf{e}}^T K^{-1} \hat{\mathbf{e}}
\]

\[
\frac{d}{dt} V(t) = -\mathbf{e}_v^T F_v e_v + \mathbf{e}_v^T \left[ \mathbf{M}(x_p) e_v - \mathbf{v}(x_p, x_v, x_u) \right] + \left[ \hat{\mathbf{e}}^T K^{-1} \hat{\mathbf{e}} - \mathbf{e}_v^T \mathbf{w}(x_p, x_v, x_u, u) \hat{\mathbf{e}} \right].
\]

(43)

Notice that, from Eq. (A1.4) in appendix 1 and Eq. (3), the second term in Eq. (43),

\[
e_v^T \left[ \frac{1}{2} \mathbf{M}(x_p) e_v - \mathbf{v}(x_p, x_v, x_u) \right] = -\mathbf{e}_v^T \mathbf{R}(x_p, x_v) e_v = 0.
\]

since the matrix

\[
\mathbf{R}(x_p, x_v) = \begin{bmatrix}
\mathbf{x}_v^T \frac{\partial \mathbf{M}}{\partial \mathbf{x}_v} & \frac{\partial \mathbf{M}}{\partial x_v} \\
\mathbf{x}_v^T \frac{\partial \mathbf{M}}{\partial x_v} & \frac{\partial \mathbf{M}}{\partial x_v}
\end{bmatrix}
\]

is skew symmetric.

The third term in Eq. (43),

\[
\hat{\mathbf{e}}^T K^{-1} \hat{\mathbf{e}} - \mathbf{e}_v^T \mathbf{w}(x_p, x_v, x_u, u) \hat{\mathbf{e}} = 0
\]

by Eq. (35), the PAA.

Thus,

\[
\frac{d}{dt} V(t) = -\mathbf{e}_v^T F_v e_v < 0
\]

(47)

which proves the boundness of \( \|\mathbf{e}_v\| \) and \( \|\hat{\mathbf{e}}\| \).

If condition a.1) is satisfied, then, using standard adaptive control arguments (Narendra and Valvani (1980)), Eq. (40) is satisfied.

Notice that \( \|\mathbf{e}_v\| \) and \( \|\hat{\mathbf{e}}\| \) are bounded regardless of the boundness of \( \|\mathbf{w}(x_p, x_v, x_u, u)\| \).

We will now prove the boundness of \( \|\mathbf{w}(x_p, x_v, x_u, u)\| \).

Lemma 2

Consider the system described by Eqs. (1), (38), the PAA, Eq. (35), and the control law, Eq. (16). U.t.c.

a.2)

If \( x(t) \), \( \dot{x}(t) \), and \( \ddot{x}(t) \) are all bounded signals, and

\[
\begin{bmatrix}
\mathbf{x}_v^T & \frac{\partial \mathbf{M}}{\partial \mathbf{x}_v} & \frac{\partial \mathbf{M}}{\partial x_v} \\
\mathbf{x}_v^T & \frac{\partial \mathbf{M}}{\partial \mathbf{x}_v} & \frac{\partial \mathbf{M}}{\partial x_v}
\end{bmatrix}
\]

(45)

and

where \( \mathbf{E}(s) \) and \( \mathbf{E}_v(s) \) are the Laplace transforms of \( \mathbf{e}(t) \) and \( e_v(t) \) respectively.
By a.3) and Lemma 1, Eqs. (50) and (51) imply the
boundness of |e(t)| and |le(t)|. By a.2) and Eq. (48), the
boundness of |x(t)| and |le(t)| follows. By Lemma 1,
|x(e)| is also bounded.

By Eqs. (16) and (12), since \( K_p, K_q \) and \( K \)
are constant matrices, and |x(t)|, |e(t)| and |le(t)| are
bounded, |u(t)| unbounded would imply that |x(e)| is
also unbounded, which is a contradiction. Since
\( W(x_c, x_u, x_u) \) is an algebraic function of bounded
signals, condition a.1) is verified, and the lemma is proven.

The asymptotic convergence of |e(t)|, |le(t)| and
|le(t)| follows from Lemma 1, Lemma 2 and Eqs. (50) and
(51).

5. Illustrative Example

Consider a two-link planar manipulator on a hor-
izontal plane (i.e. \( g(x_c) = 0 \)) with rotary joints. The links
have height \( l_1 \) and \( l_2 \). The masses and moment of iner-
tias of the links are \( m_1, m_2, I_1 \), and \( I_2 \) respectively (figure
1). It can be shown that the elements of inertia matrix
and Coriolis vector are as follows:

\[
M_1 = I_1 + l_1(25m_1 + m_2)l_1^2 + m_1l_2 \cos(x_c),
\]

\[
M_2 = I_2 + 25m_2l_2 + 5m_1l_2 \cos(x_c),
\]

\[
W_1(x_c, x_u, x_u) = 5m_1l_2 \cos(x_c),
\]

\[
W_2(x_c, x_u, x_u) = 5m_1l_2 \sin(x_c).
\]

Let's define:

\[
\Theta_1 = I_1 + l_1^{2/3} + 25m_1 + m_2 \]

\[
\Theta_2 = I_2 + 25m_2l_2
\]

\[
\Theta_3 = 5m_1l_2
\]

Then by some algebraic manipulation one obtains
the following expression for elements of \( W(x_c, x_u, x_u) \)
matrix:

\[
N_{11} = u_1
\]

\[
N_{12} = u_2
\]

\[
N_{13} = 2u_1 \cos(x_c) + u_3 \cos(x_c) + x_c \sin(x_c),
\]

\[
N_{21} = 0
\]

\[
N_{22} = u_4 + u_2
\]

\[
N_{23} = u_2 \cos(x_c) + x_c \sin(x_c).
\]

6. Computational Efficiency

In order to implement the adaptive controller
described above one needs to calculate the elements of
\( W(x_c, x_u, x_u) \) on-line. As can be seen in the example
above this procedure may be excessively time con-
suming since it involves computations of highly nonlinear
functions of joint position and velocities. Consequently
the real time implementation of such a scheme is rather
difficult. To overcome this difficulty we suggest to
represent \( x_c \) and \( x_u \) with their desired counterparts, namely
\( \hat{x}_c(t) \) and \( \hat{x}_u(t) \), in \( W(x_c, x_u, x_u) \) matrix.

The desired quantities are known in advance and
therefore all their corresponding calculations can be
performed off-line. In a paper, soon to be released, we
have proven that in spite of such modifications the
asymptotic stability of the adaptive system is still
preserved provided:

1) Presence of sufficiently large Velocity and position
feedback gains.

2) Addition of an explicitly defined auxiliary non-
linear feedback to the existing controller.

7. Conclusion

A stable continuous time model reference adaptive
controller for robotic manipulators was presented. This
controller is a modified version of the one originally in-
roduced by Horowitz and Tomizuka (1980). The advan-
tages of the present work are:

1) No slowly time varying assumption about the system
parameters is required to prove asymptotic
stability. This was the main deficiency of the original
scheme.

2) The control scheme requires only joint position
and velocity feedback, no acceleration feedback is
required.

3) The scheme does not involve any matrix inversion
and, therefore, all control parameters can be
updated in parallel.

8. Acknowledgment

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Appendix 1

Proof of Equation (3)

Consider a n degree of freedom robotic manipulator composed by revolute or prismatic joints. The equations of motion for the arm are given by Eq. (1). All the constraints in this system are holonomic scleronomic (Rosenberg (1977)), and the joint position coordinates, \( x_p \)’s constitute a set of generalized coordinates. Assume that there are no gravitational or friction forces acting on the system i.e. \( g(x_p) = 0 \), \( c(x_p, x_v) = 0 \) in Eq. (1).

Defining the kinetic energy of the system by

\[
T = \frac{1}{2} x_{v}^{T} M(x_p) x_{v} + \frac{1}{2} \sum_{i=1}^{n} \frac{\partial M(x_p)}{\partial x_{pi}}^{T} x_{vi} \frac{\partial M(x_p)}{\partial x_{vi}}
\]

Eq. (1) can be obtained by using Lagrange’s Equations.

Expanding Eq. (A1.2),

\[
\frac{d}{dt} \frac{\partial T}{\partial x_{v}} - \frac{\partial T}{\partial x_{p}} = q(t)
\]

Expanding the first term in Eq. (A1.3), and noticing that

\[
M = \begin{bmatrix} m_{1} & m_{2} & \cdots & m_{n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n} & & & m_{n} \end{bmatrix}
\]

\[
\dot{M}(x_p)x_v + \frac{\partial M(x_p)}{\partial x_{p1}} x_{v} + \cdots + \frac{\partial M(x_p)}{\partial x_{pn}} x_{v} = q(t)
\]

where we are using the notation \( \dot{f}(t) = \frac{df}{dt} \).

Combining Eqs. (A1.3) and (A1.4) we obtain the desired result:

\[
\begin{bmatrix} m_{1} & r_{1} \frac{\partial M(x_p)}{\partial x_{p1}} & \cdots & r_{n} \frac{\partial M(x_p)}{\partial x_{pn}} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n} & & & m_{n} \end{bmatrix}
\begin{bmatrix} m_{1} & m_{2} & \cdots & m_{n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n} & & & m_{n} \end{bmatrix} = q(t)
\]