

Optimal control of freeway networks based on the Link Node Cell Transmission model.

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Abstract—We present an optimal control approach to freeway traffic congestion control. A Link-Node Cell transmission model (LN-CTM) is used to represent the freeway traffic dynamics. The approach searches for solutions represented by a combination of ramp metering and variable speed control. The optimization problem corresponding to the optimal control problem based on the LN-CTM is non-convex and non-linear. We relax the problem to a linear optimization problem, and propose an approach to map the solution of the linear optimization algorithm to the solution of the original optimal control problem. We prove that the solution derived from this approach is optimal for the original optimal control algorithm. Finally, we use a model predictive framework to demonstrate the optimal control formulation presented in this paper and discuss its potential use.

I. INTRODUCTION

Traffic congestion has increased over the last decade, leading to loss of productivity due to increased time spent on travel and commute. This problem is particularly notorious around cities and metropolitan areas, where the consistent increase in demand has led to increases in both recurrent and non-recurrent congestion. Transportation engineers counter congestion by infrastructure expansions and intelligent operational management of the infrastructure. Traffic control, effected through the use of ramp metering and variable speed control, is one of the tools available for intelligent operational management. In ramp metering, traffic entering the on-ramp is regulated, with the objective of allowing the freeway to operate at maximum efficiency. Speed control is primarily used for improving safety [1], but it has also been found to be useful for congestion alleviation when ramp metering is insufficient [2].

Optimal control methods based on macroscopic models have been extensively reported in literature [3], [4], [5], [2]. For freeway networks, first order models (Cell Transmission model, CTM [6]) and second order models (METANET [3]) are popular for dynamic traffic simulations. Second order models have an advantage over the first order models in incorporating the capacity drop, if its effect is significant. However, the optimization problems based on the second order models are non-linear, computationally intensive and the solutions obtained are usually only locally optimal. The former proves to be a drawback when the controller is embedded as a part of a model predictive framework, since this requires fast optimizations to be executed repeatedly [3].

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In comparison, optimization approaches based on CTM hold more promise in terms of computational efficiency and global optimality. In [4], the authors presented an optimal controller based on the Asymmetric Cell Transmission model (ACTM), a simplified version of the CTM. The authors presented a relaxed version of an optimal ramp metering problem formulated using the ACTM, and proved that the problems are equivalent in terms of the optimal solution trajectory. The relaxed problem is a linear optimization problem, which can be solved efficiently for large scale networks.

We present an optimal controller based on the Link-Node Cell Transmission model (LN-CTM). The LN-CTM uses a more accurate model of link merges as compared to the ACTM. This makes the LN-CTM suitable for simulating onramp merges even when onramp inflows are appreciable (eg. freeway-freeway interconnections). However, this comes at an additional cost of added non-linearity, and therefore the results in [4] cannot be translated in this case. In this paper we present the original nonlinear optimal control problem based on the LN-CTM and prove that an equivalent simplified linear optimal control problem can be used to derive the optimal ramp metering and speed limit profile for the original problem. Another observation is that the authors in [4] highlighted that under constant split ratios (turning proportions), the optimal trajectory does not involve any speed control. However, we observe that in the LN-CTM, the optimal control strategy includes a speed control law, though, only at on-ramp junctions.

The paper is organized as follows. We present the LN-CTM model in section II and continue onto the problem formulation and basic results in Section III. In IV, we present an MPC controller based on the developed framework, and demonstrate the results in IV.

II. LINK NODE CELL TRANSMISSION MODEL

We adopt the Link-Node Cell transmission model to describe the traffic dynamics in the optimal control problem. The Link-Node Cell Transmission Model (LN-CTM) is an extension of the CTM [6], which can be used to simulate traffic in any road network. This model is implemented as a part of Tools for operations planning ([7]) in Aurora, the simulation tool ([8]). Figure 1 shows the directed graph representation of a freeway network for the LN-CTM. Links represent road segments and belong to three types : (a) normal (b) source and (c) sink. Nodes represent link junctions and they are specified with a (possibly) time-varying split-ratio matrix (routing parameters), which represents the portion of traffic moving from a particular input link to an

output link. A fundamental diagram (which specifies the flow-speed-density relationship) is specified for each link, while the source links are also specified with an input demand profile. Table I lists the variables used here. We

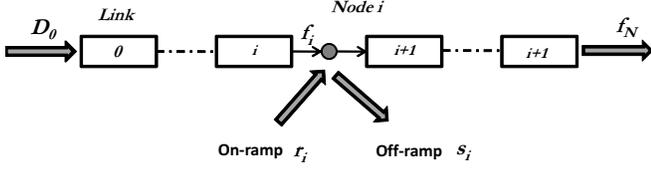


Fig. 1. Freeway with N links. Each Node contains a maximum of one on- and one off-ramp. Note that Node i is upstream of Link i

Symbol	Name	Unit
F_i	Flow capacity of Link i	veh/period
V_i	max. free flow speed of Link i	section/period
W_i	congestion wave speed of Link i	section/period
n_i^j	jam density of Link i	veh/section
k	period number	dimensionless
$\beta_i(k)$	split ratio at node i	dimensionless
$f_i(k)$	flow out of Link i	veh/period
$n_i(k)$	number of vehicles in Link i	veh/period
$s_i(k), r_i(k)$	off-ramp, on-ramp flow in node i	veh/period
$d_i(k)$	demand for on-ramp i	veh/period
$l_i(k)$	queue at on-ramp i	veh/period
$c_i(k)$	Ramp metering rate for on-ramp i	veh/period
C_i	Flow capacity for on-ramp i	veh/period
L_i	Queue capacity for on-ramp i	veh/period
$D_i(k)$	input flow for on-ramp i	veh/period

TABLE I
MODEL VARIABLES AND PARAMETERS.

use a triangular fundamental diagram, characterized by the freeflow speed (V_i), congestion wave speed (W_i) and Capacity F_i (Figure 2). In particular, $0 < V_i, W_i < 1$ is assumed by a proper choice of simulation time step. For typical freeways with minimum link length of 1500ft, a time step of 10s is appropriate. The triangular fundamental diagram defines a demand and supply function for a link. Under nominal conditions without the application of variable speed limits (VSL), the demand function $\bar{D}_i(n_i(k)) = \min(n_i(k) * V_i, F_i)$ defines the number of vehicles available to move out of the link, while the supply function $\bar{S}_i(n_i(k)) = \min(W_i(n_i^j - n_i(k)), F_i)$ defines the maximum number of vehicles that can move into the link. Variable speed limits, specified using a time varying profile $v_i(k)$, affect the demand function $\bar{D}_i(n_i(k)) = \min(n_i(k) * v_i(k), F_i)$, while the supply function is unchanged.

The LN-CTM algorithm is explained in [8]. The model update is executed in two steps - (a) Flow update which calculates the flows at each node using the input link demands, output link supplies and the split ratio matrix. (b) Density update which uses a vehicle conservation equation with the flows calculated in (a). The general model can be simplified for a freeway corridor. Assuming that the off-ramps boundaries are not congested, the model evolution can be described by the following equations.

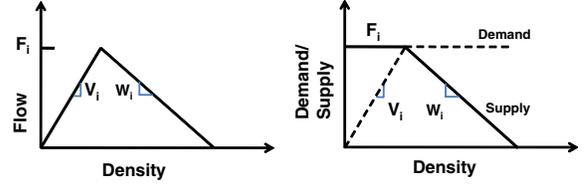


Fig. 2. Fundamental Diagram, Demand and Supply functions.

Mainline/Queue Conservation Equation

$$\begin{aligned} n_0(k+1) &= n_0(k) + D_0(k) - f_0(k) \\ n_i(k+1) &= n_i(k) + f_{i-1}(k)(1 - \beta_{i-1}(k)) + r_{i-1}(k) - f_i(k) \\ l_i(k+1) &= l_i(k) + D_i(k) - r_i(k) \quad i = 1, \dots, N \end{aligned} \quad (1)$$

Flow Equations

$$\begin{aligned} f_N(k) &= \min(n_N(k)v_N(k), F_N) \\ f_i(k) &= \min(n_i(k)v_i(k), F_i) \times \frac{\min(R_i(k), S_{i+1}(k))}{R_i(k)} \\ r_i(k) &= d_i(k) \times \frac{\min(R_i(k), S_{i+1}(k))}{R_i(k)} \quad i = 1, \dots, N \end{aligned}$$

where $d_i(k) = \min(c_i(k), l_i(k))$,

$$S_{i+1}(k) = \min(W_{i+1}(n_{i+1}^j - n_{i+1}(k)), F_{i+1})$$

$$R_i(k) = \min(n_i(k)v_i(k), F_i)(1 - \beta_i(k)) + d_i(k) \quad (2)$$

Here $\beta_i(k) = s_i(k)/f_i(k)$ is a (possibly) time-varying split ratio matrix that describes the portion of vehicles exiting through a particular off-ramp. In the above equations, we have assumed that the last cell is in free-flow. This can usually be done by choosing boundaries appropriately. The supply for Link $i+1$ is given by $S_{i+1}(k)$, while the total demand into Link $i+1$ ($R_i(k)$) is given by the sum of the demand from the previous link i ($\bar{D}_i(n_i(k))(1 - \beta_i(k))$) and the demand from the on-ramp for node i ($d_i(k)$). In the model, the flows are calculated by comparing the demand and supply at each node. In free-flow conditions, $R_i(k) \leq S_{i+1}(k)$ and the flow equals the demand. In congested conditions, $R_i(k) > S_{i+1}(k)$, and the available supply is shared by the flows from the on-ramp and the previous link. In the LN-CTM model, the available supply is shared proportionally to the demands (i.e. $f_i(k)/r_i(k) = \min(n_i(k)v_i(k), F_i)/d_i(k)$). This results in the above closed form expressions for the flow computation at each node. In this paper we assume that the on-ramp demands $d_i(k)$'s can be controlled by manipulating the ramp metering rates $c_i(k)$'s and link free flow speeds $v_i(k)$'s can be controlled through the use of Variable Speed Limits (VSL). VSL control allows us to change the input demand which affect the flows at the nodes. Therefore, the control variables are the ramp metering rates, $c_i(k)$'s and the mainline link speeds, $v_i(k)$'s.

III. OPTIMAL RAMP METERING AND SPEED CONTROL

The optimal controller presented in this section is based on the LN-CTM developed above. The controller is typically designed to minimize an appropriate cost function, for example

Total Travel Time (TTT), Total Delay (TD). In terms of the macroscopic variables, we define the following generalized objective function

$$J = \sum_{k,i} (n_i(k) + l_i(k) - \alpha_i(k)f_i(k) - \bar{\alpha}_i(k)r_i(k)) \quad (3)$$

where $k = 1 \dots K$ denotes the time period and $i = 0 \dots N$ denotes the link ($n_i(k)$) or ramp ($l_i(k)$) index. By choosing values for the parameters $\alpha_i(k) \geq 0, \bar{\alpha}_i(k) \geq 0$, we can represent the following commonly used objective functions, where TTD represents Total Travel Distance.

$$J_a = TTT = \sum_{k,i} (n_i(k) + l_i(k))$$

$$J_b = TTT - aTTD = \sum_{k,i} (n_i(k) + l_i(k)) - a \sum_{k,i} (f_i(k) + r_i(k))$$

$$J_c = TD = \sum_{k,i} \left(n_i(k) + l_i(k) - \frac{1}{V_i} f_i(k) \right)$$

The optimal controller regulates the traffic using a speed limit profile $v_i(k)$ and a ramp metering rate $c_i(k)$. The speed limit profile serves as an indirect control mechanism for regulating flows that exit any particular section of the freeway. The ramp metering rate serves to regulate the flow entering into the freeway through any particular ramp. Based on these control mechanisms, we now define two optimal control problems. The first, which we denote **Problem A**, constitutes the original optimization problem. Its solution involves nonlinear optimization. The second problem, which we denote **Problem B** constitutes a relaxed optimization problem since its solution only involves linear programming. Subsequently we prove that a solution of **Problem B** is also a solution of **Problem A**.

Problem A Original Problem

Min. : J , given by Eq. (3)

S.t. : For $k = 1, \dots, K$

Conservation Equations

Equations (1)

Flow equations

Equations (2)

Constraint equations

$$0 \leq v_i(k) \leq V_i$$

$$0 \leq d_i(k) \leq \min(C_i, l_i(k))$$

$$l_i(k) \leq L_i$$

$$n_i(k), l_i(k), f_i(k), r_i(k) \geq 0$$

with given initial conditions/parameters. (4)

The following parameters and initial conditions must be specified for each link and on-ramp:

- Link i Fundamental Diagram Parameters : Capacity F_i , Max. Free-flow speed V_i and Congestion wave speed W_i
- On-ramp i parameters (Flow capacity and maximum queue length): $C_i, L_i \quad i = 1, \dots, N$
- Off-ramp i split ratios: $\beta_i(k) \quad i = 0, \dots, N, k = 1, \dots, K$
- Initial Conditions : $n_0(0), n_i(0), l_i(0) \quad i = 1, \dots, N$

- Flow Demands : $D_i(k) \quad i = 0, \dots, N, k = 0, \dots, K$

The optimal controller defined in **Problem A** provides a ramp demand profile ($d_i(k)$) for all ramps and a speed limit ($v_i(k)$) profile for all links in the network. This ramp demand profile can be used to extract the ramp metering rate profile ($c_i(k)$). Since $d_i(k) = \min(c_i(k), l_i(k))$ (Eq. (2)), choosing $c_i(k) = d_i(k)$ we get a ramp metering rate profile. We highlight that **Problem A** involves non-linear optimization, primarily due to non-linear equalities in the flow equations.

We now pose an alternate relaxed optimal control problem with a solution that only involves a linear program.

Problem B Relaxed Problem

Min. : J , given by Eq. (3)

S.t. : For $k = 1, \dots, K$

Conservation Equations

Equations (1)

Relaxed Flow equations

$$\bar{f}_i(k) \leq \bar{n}_i(k)V_i \quad i = 1, \dots, N$$

$$\bar{f}_i(k) \leq F_i \quad i = 1, \dots, N$$

$$\bar{f}_i(k)(1 - \beta_i(k)) + \bar{r}_i(k) \leq F_{i+1} \quad i = 1, \dots, N-1$$

$$\bar{f}_i(k)(1 - \beta_i(k)) + \bar{r}_i(k)$$

$$\leq W_{i+1}(n_{i+1}^J - \bar{n}_{i+1}(k)) \quad i = 1, \dots, N-1$$

Constraint equations

$$0 \leq \bar{r}_i(k) \leq \min(C_i, \bar{l}_i(k)) \quad i = 1, \dots, N$$

$$\bar{l}_i(k) \leq L_i$$

with the same initial conditions/parameters. (5)

Notice that we have chosen to use an upper bar to denote the optimization variables in **Problem B** (e.g. $\bar{n}_i(k), \bar{f}_i(k), \bar{r}_i(k)$) in order to distinguish them from their counterparts in **Problem A**. Also, we have indicated that all cells have on-ramps. However, in case on-ramps are absent the corresponding variables are removed from the formulation. In the above problem specification, we do not explicitly consider the link velocity variables (e.g. $\bar{v}_i(k)$) and the onramp demands (e.g. $\bar{d}_i(k)$). We will outline the methodology adopted to convert a solution of **Problem B** to a solution of **Problem A** below.

Let $\bar{n}_i^*(k), \bar{f}_i^*(k), \bar{l}_i^*(k), \bar{r}_i^*(k)$ denote the optimal (or a feasible) solution of **Problem B**. **Algorithm A** given below generates outputs $n_i^*(k), f_i^*(k), l_i^*(k), r_i^*(k), v_i^*(k), d_i^*(k)$.

Algorithm A

For each time period k and link $0 \leq i \leq N$,

$$n_i^*(k) = \bar{n}_i^*(k)$$

$$f_i^*(k) = \bar{f}_i^*(k)$$

$$l_i^*(k) = \bar{l}_i^*(k)$$

$$r_i^*(k) = \bar{r}_i^*(k)$$

For each time period k and link $0 \leq i < N-1$,

$$\text{if } f_i^*(k) = \min(n_i^*(k)V_i, F_i)$$

$$v_i^*(k) = V_i$$

$$d_i^*(k) = r_i^*(k)$$

else if $f_i^*(k)(1 - \beta_i(k)) + r_i^*(k) < S_{i+1}^*(k)$
 $d_i^*(k) = r_i^*(k)$
 $v_i^*(k) = f_i^*(k)/n_i^*(k)$
else if $\frac{r_i^*(k)}{S_{i+1}^*(k)} < \frac{\min(C_i, l_i^*(k))}{\min(n_i^*(k)V_i, F_i)(1 - \beta_i(k)) + \min(C_i, l_i^*(k))}$
 $v_i^*(k) = V_i$
 $d_i^*(k) = r_i^*(k) \times \frac{\min(n_i^*(k)V_i, F_i)(1 - \beta_i(k))}{S_{i+1}^*(k) - r_i^*(k)}$
else
 $v_i^*(k) = \frac{\min(C_i, l_i(k))}{n_i^*(k)(1 - \beta_i(k))} \times \left(\frac{S_{i+1}^*(k)}{r_i^*(k)} - 1 \right)$
 $d_i^*(k) = \min(C_i, l_i(k))$
where $S_i^*(k) = \min(W_i(n_i^J - \bar{n}_i^*(k)), F_i(k))$
and for each time period k
if $f_N^*(k) = \min(\bar{n}_N(k)V_N, F_N)$
 $v_N^*(k) = V_N$
else
 $v_N^*(k) = f_N^*(k)/n_N^*(k)$

The following results will help prove that the variables $n_i^*(k), f_i^*(k), l_i^*(k), r_i^*(k), v_i^*(k), d_i^*(k)$ are feasible and optimal.

Lemma 3.1: Let $A = \{n_i(k), f_i(k), l_i(k), r_i(k), v_i(k), d_i(k)\}$ be the solution derived from $B = \{\bar{n}_i(k), \bar{f}_i(k), \bar{l}_i(k), \bar{r}_i(k)\}$ using **Algorithm A**. Then A is a feasible solution for **Problem A** if B is a feasible solution of **Problem B**.

Proof: It is easy to see that A satisfies the conservation equations and the queue constraints since B satisfies the conservation equations/queue constraints. We need to prove that A satisfies the flow equations and other constraints of **Problem A**. From the constraints of **Problem B** we get,

$$\begin{aligned} f_i(k) &\leq \min(n_i(k)V_i, F_i) \quad i = 0..N \\ f_i(k)(1 - \beta_i(k)) + r_i(k) &\leq S_{i+1} \quad i = 0..N - 1 \\ r_i(k) &\leq \min(C_i, l_i(k)) \quad i = 1..N \\ \text{where } S_{i+1} &= \min(F_{i+1}(k), W_{i+1}(n_{i+1}^J - n_{i+1}(k))) \end{aligned}$$

We define $R_i(k) = \min(n_i(k)v_i(k), F_i)(1 - \beta_i(k)) + d_i(k)$ as before. At each time instant k and for any link i , the following cases completely cover all possibilities:

Case (a) $f_i(k) = \min(n_i(k)V_i, F_i)$:

$$\begin{aligned} v_i(k) &= V_i, r_i(k) = d_i(k) \leq \min(C_i, l_i(k)) \\ \text{and } S_{i+1}(k) &\geq R_i(k) \Rightarrow \\ \min(n_i(k)v_i(k), F_i) \times \frac{\min(R_i(k), S_{i+1}(k))}{R_i(k)} &= f_i(k) \end{aligned}$$

$$d_i(k) \times \frac{\min(R_i(k), S_{i+1}(k))}{R_i(k)} = r_i(k)$$

Case (b) $f_i(k) < \min(n_i(k)V_i, F_i)$ and $f_i(k)(1 - \beta_i(k)) + r_i(k) < S_{i+1}(k)$:

$$\begin{aligned} d_i(k) &= r_i(k) \leq \min(C_i, l_i(k)), \\ v_i(k) &= f_i(k)/n_i(k) \leq V_i, \\ \frac{\min(R_i(k), S_{i+1}(k))}{R_i(k)} &= 1 \Rightarrow \\ \min(n_i(k)v_i(k), F_i) \times \frac{\min(R_i(k), S_{i+1}(k))}{R_i(k)} &= f_i(k) \\ &= \min\left(n_i(k) \frac{f_i(k)}{n_i(k)}, F_i\right) = f_i(k) \\ d_i(k) \times \frac{\min(R_i(k), S_{i+1}(k))}{R_i(k)} &= d_i(k) = r_i(k) \end{aligned}$$

Case (c) $f_i(k) < \min(n_i(k)V_i, F_i)$, $f_i(k)(1 - \beta_i(k)) + r_i(k) = S_{i+1}(k)$ and $\frac{r_i(k)}{S_{i+1}(k)} \leq \frac{\min(C_i, l_i(k))}{\min(n_i(k)V_i, F_i)(1 - \beta_i(k)) + \min(C_i, l_i(k))}$:

$$\begin{aligned} d_i(k) &= r_i(k) \times \frac{\min(n_i(k)V_i, F_i)(1 - \beta_i(k))}{S_{i+1}(k) - r_i(k)}, v_i(k) = V_i \\ \Rightarrow r_i(k) &= d_i(k) \frac{S_{i+1}(k)}{R_i(k)} \end{aligned}$$

$$\begin{aligned} \text{Since } \frac{r_i(k)}{S_{i+1}(k)} &= \frac{d_i(k)}{\min(n_i(k)V_i, F_i)(1 - \beta_i(k)) + d_i(k)} \\ &\leq \frac{\min(C_i, l_i(k))}{\min(n_i(k)V_i, F_i)(1 - \beta_i(k)) + \min(C_i, l_i(k))} \\ \Rightarrow d_i(k) &\leq \min(C_i, l_i(k)) \end{aligned}$$

$$\begin{aligned} \text{Finally } f_i(k) &= \frac{S_{i+1}(k) - r_i(k)}{(1 - \beta_i(k))} \\ &= \min(n_i(k)v_i(k), F_i) \times \frac{\min(R_i(k), S_{i+1}(k))}{R_i(k)} \end{aligned}$$

Case (d) $f_i(k) < \min(n_i(k)V_i, F_i)$, $f_i(k)(1 - \beta_i(k)) + r_i(k) = S_{i+1}(k)$ and $\frac{r_i(k)}{S_{i+1}(k)} > \frac{\min(C_i, l_i(k))}{\min(n_i(k)V_i, F_i)(1 - \beta_i(k)) + \min(C_i, l_i(k))}$:

$$\begin{aligned} d_i(k) &= \min(C_i, l_i(k)), \\ v_i(k) &= \frac{\min(C_i, l_i(k))}{n_i(k)(1 - \beta_i(k))} \times \left(\frac{S_{i+1}(k)}{r_i(k)} - 1 \right) \\ v_i(k) &\leq \frac{\min(C_i, l_i(k))}{n_i(k)(1 - \beta_i(k))} \\ &\times \left(\frac{\min(n_i(k)V_i, F_i)(1 - \beta_i(k)) + \min(C_i, l_i(k))}{\min(C_i, l_i(k))} - 1 \right) \\ &\leq \frac{\min(n_i(k)V_i, F_i)(1 - \beta_i(k))}{n_i(k)(1 - \beta_i(k))} \leq V_i \end{aligned}$$

$$\begin{aligned} \frac{n_i(k)v_i(k)(1 - \beta_i(k))}{\min(C_i, l_i(k))} &= \left(\frac{S_{i+1}(k)}{r_i(k)} - 1 \right) = \frac{f_i(k)}{r_i(k)} \\ \Rightarrow f_i(k) &= n_i(k)v_i(k) \times \frac{S_{i+1}(k)}{n_i(k)v_i(k)(1 - \beta_i(k)) + d_i(k)} \\ \text{and } r_i(k) &= d_i(k) \times \frac{S_{i+1}(k)}{n_i(k)v_i(k)(1 - \beta_i(k)) + d_i(k)} \end{aligned}$$

Thus, all the cases highlighted above satisfy the flow conditions of **Problem A** ■

Lemma 3.2: Let $A = n_i(k), f_i(k), l_i(k), r_i(k), d_i(k), v_i(k)$ be a feasible solution of **Problem A**, then $B = n_i(k), f_i(k), l_i(k), r_i(k)$ is a feasible solution for **Problem B**.

Proof: Clearly, B satisfies the conservation equations and the constraints of **Problem B**. Also, by direct substitution and manipulation, we can prove that B satisfies the flow constraints of **Problem B**. ■

Theorem 3.1: Let $B = \bar{n}_i^*(k), \bar{f}_i^*(k), \bar{l}_i^*(k), \bar{r}_i^*(k)$ be an optimal solution of **Problem B** and $A = n_i^*(k), f_i^*(k), l_i^*(k), r_i^*(k), v_i^*(k), d_i^*(k)$ be the solution derived using **Algorithm A**. Then A is an optimal solution for **Problem A**.

Proof: The feasible sets of **Problem A** and **Problem B** are equivalent, as shown in the previous lemmas. Also, **Algorithm A** provides a mapping of a solution of **Problem B** to a solution of **Problem A**. Thus A , derived from B is an optimal solution of **Problem A**. ■

Remarks In this section, we have used a relaxation technique to map the non-linear optimization problem to a linear optimization problem. The relaxation technique works only when variable speed limits are applied to all links, and all ramps are metered. In the problem formulation, we have allowed the split ratio $\beta_i(k)$ to be possibly time-varying. However, one needs to be careful while searching for an optimal speed control profile in case of time-varying split ratios. For example, consider the case that split ratios for the first cell increases with time. In this case, an optimal speed control law might hold back vehicles initially (by decreasing the speed limit), so that the vehicles catch a higher split ratio, and exit the freeway. This effect is exacerbated when J_a is considered as the objective, since vehicles that exit do not contribute to the Total Travel Time in the downstream links. In contrast, augmenting the objective with $-f_i(k)$ serves to alleviate this effect in this case, as vehicles exiting the freeway do not contribute to the flow downstream. In the case of decreasing split ratios, the roles of these terms are reversed. Hence we argue that J_b or J_c is a better objective function to consider for the problem. In the case of constant split ratios, this problem does not arise. This problem is not unique to an optimal control formulation using the LN-CTM, but arises due to the use of a split-ratio based routing scheme adopted by this model. This observation was hinted by the authors in [4].

IV. MODEL PREDICTIVE RAMP METERING AND SPEED CONTROL

In this section we present a model predictive controller based on the optimal control formulation presented in the previous section. The Model Predictive Controller solves an open loop optimal control problem online based on a plant model at each sampling time, using the state information measured at the current sampling time. The controller implements the control steps of the obtained optimal control profile till the next sampling time, and then the process is repeated.

Let T and N_p denote the model time step and prediction horizon used in the optimization problem respectively. We execute the MPC every $T_c = N_c \times T$ time instants ($N_p, N_c \in \mathbb{N}$). In the model predictive controller, the split ratio is assumed to be constant, equal to the 'smoothed' split ratio observed around the instant the controller is initiated. This averts the problem related to time varying split ratios detailed in the previous section, and does not usually lead to any appreciable decrease in the controller performance within the MPC framework. We make one modification to the optimal

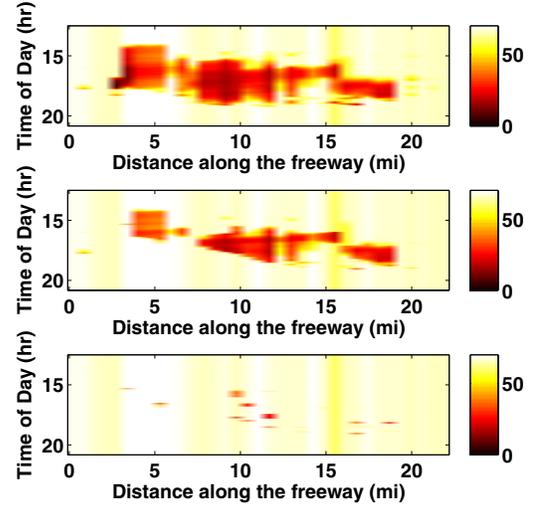


Fig. 3. Top : Speed contours in the uncontrolled case. Middle : Speed contours with ramp metering and VSL. Bottom : VSL specified by the MPC.

control formulation presented in the previous section. We convert the hard constraints on queues to a soft constraint and add a term to the cost function. Let $\zeta_i(k)$ be the new variable that captures the queue violation. Then, we modify the cost function as $\bar{J} = J + C \sum_{i,k} \zeta_i(k)$, and add $\bar{l}_i(k) - L_i \leq \zeta_i(k)$ $i = 1..N, k = 1..K$, and $0 \leq \zeta_i(k)$ $i = 1..N, k = 1..K$ to the constraints. It can be easily seen that this does not affect any of the results presented in the previous section. Since it is impossible to exactly predict the on-ramp demands, queue constraints are expected to be frequently violated even when an MPC incorporating hard constraints is executed. The presence of these new soft constraints eliminates problems related to infeasibility.

For the simulation experiments presented in this paper, we use a calibrated model of the I-80E freeway in the Bay area between the Bay Bridge and the Carquinez Bridge. The procedure for model calibration is similar to the one explained in [9]. Figure 3 (Top) shows the speed contours produced by the model without any control measures activated. We apply a Model predictive controller with $T = 10s$, $N_p = 200$ and $N_c = 9$ to specify the ramp metering rates and variable speed profile for this freeway. A queue limit of $L_i = 50 \forall i$ was imposed for this simulation. Figure 3 (middle) represents the speed contour observed when the MPC is used, and Figure 3 (bottom) shows the speed limit profile generated by the

MPC. Given the limited queue size constraint imposed on the controller, the controller did not completely eliminate the congestion present in the freeway. However, the MPC succeeds in delaying the onset of congestion on the freeway. In this scenario, the controller resulted in a delay reduction of 17.44%. From Figure 4, we see that the queue constraints are not adversely violated in this case, where $C = 5$ was chosen. We also explore the effect of various parameters on

TABLE II
MPC PARAMETER STUDY

N_c ($N_p = 120, L_i = 50$)	6	12	18	24	30
Delay reduction	17.77%	16.83%	15.224%	12.72%	7.81%
N_p ($N_c = 9, L_i = 50$)	30	60	90	120	150
Delay reduction	17.39%	17.41%	17.41%	17.42%	17.43%
L_i ($N_p = 120, N_c = 9$)	10	20	50	100	∞
Delay reduction	6.9%	10.6%	17.42%	23.4%	25.34%

the performance of the Model Predictive controller. Table II lists the performance of the MPC when critical parameters - control horizon, prediction horizon and the maximum queue limit, are varied. We see that the control horizon is more critical than the prediction horizon. In particular, we note that prediction horizons can be as short as 10 minutes, while the control horizon should be sufficiently small i.e 1 or 2 mins. Longer control horizons lead to considerable decrease in controller performance (this was found to be the case irrespective of the prediction horizon chosen). Short control horizons necessitates the use of a fast optimization routine in the MPC. Finally, we also see that ramp queue limits have a major effect on the efficiency gains that can be expected out of the controlled system.

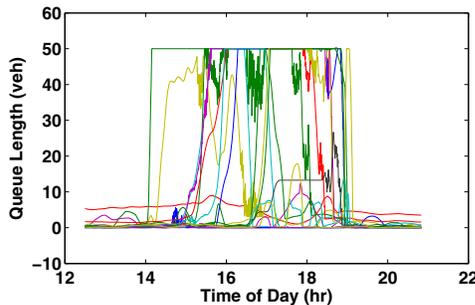


Fig. 4. Queue lengths in the controlled case.

V. CONCLUSION AND COMMENTS

In this paper, we presented the optimal congestion control problem for freeway networks, using ramp metering and variable speed limits. The optimization problem based on the LN-CTM had non-linear and non-convex constraints. We proposed a relaxed optimization problem, with only linear equalities and inequalities, and provided a procedure to map the solution of the relaxed problem to a ramp metering and speed limit profile for the original problem. We proved that the resulting control is also optimal for the original problem.

Given the large-scale nature of a typical centralized freeway optimal control problem, our solution enables us to

solve the optimization problem in real-time to incorporate it in a model predictive framework. Typically, a problem with $N_p = 200$, $T = 10s$ and $n = 33$ had around 55000 and 12000 inequality and equality constraints. This problem can be efficiently solved within 10 seconds using the MOSEK® solver, which is a fraction of the controller time horizon. The approach taken in this paper has advantages over the approaches presented in [3], [5] that use second order models; particularly with respect to global optimality and computation speed. The global optimality guaranteed by this approach ensures that optimal controller (executed using a MPC) can be used to compare and evaluate other control methodologies applied to the same setup.

The present framework also has some drawbacks. In the current formulation, we do not constrain speed limit variations. However, under constant split ratios, the optimal controller did not lead to sudden speed drops along the freeway, unless the section downstream was also congested. In fact, the speeds observed using the control were usually higher than the speeds observed without control. Also, when the MPC controller is implemented as a higher level controller, the profiles provided by the controller can be fed into a lower level speed limit specification module which will enforce the optimal control taking safety aspects into consideration. Finally, the model does not include the notion of capacity drop. The authors are currently pursuing active research in this topic.

VI. ACKNOWLEDGMENTS

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