OMETAN Model Improvement for Traffic Control

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Abstract—The METANET model, deduced based on equilibrium state assumption, provides a candidate model for freeway traffic control design since it has both speed and density dynamics, but the previous work parameterized the speed control variable to be highly nonlinear, which caused difficulty in control design and implementation. Besides, the model could not catch quick and significant changes in traffic dynamics. This paper suggests improvement on the dynamics model in two aspects: (a) to drop the nonlinear parameterization in the speed control variable for simplicity; and (b) to propose several alternatives for the convection term of the speed dynamics. Model calibration using Berkeley Highway Lab data and simulation for comparison are presented to show the effectiveness of the improvements.

I. INTRODUCTION

As recognized in recent years, ramp metering can only control the demand from onramps and the average density immediately downstream. It only indirectly affects the traffic upstream by congestion back propagation, for example. Ramp metering does not influence driver behavior on the mainline. To control mainline flow, VSL is one possible strategy, which is complementary to RM. The goal of the model improvements in this paper is to support design of freeway traffic control strategies combining Variable Speed Limits (VSL) and Coordinated Ramp Metering (CRM). To achieve this, it is necessary to represent both speed and density (occupancy) dynamics accurately. The main functions of the traffic dynamics model in traffic control design are (a) for simulation, (b) for traffic state estimation and prediction, and (c) for control design and synthesis.

To design a model-based controller to achieve good performance, there are two requirements on the model: (a) to capture the traffic dynamics in the relevant traffic situations, including free flow and all the congested equilibrium states and non-equilibrium transitions between them; and (b) to be simple enough to incorporate in the real-time control implementation. There is often a subtle trade-off between model complication and faithfulness subjected to control performance requirement. This paper will show that simplifications in the METANET model will not degrade the model match with the traffic dynamics, which is desirable.

The paper is organized as follows: Section 2 is for brief literature review; Section 3 presents some details about the METANET model; Section 4 reports the main results for model improvement; Section 5 uses field data for model calibration; and future work is presented in Section 6.

II. LITERATURE REVIEW

The second order METANET model by Payne [1] provided a candidate model for combined speed and ramp metering control design since it contains both speed and density dynamics. It was used for traffic state parameter estimation, and for ramp metering control in [2, 3]. The model was later improved in [4]. It is essentially a 2nd order model with coupled density and speed dynamics. The model has been further improved in [5, 6] by introducing the weaving effect due to lane changing. Since the authors intended for ramp meter control and different ramp meter strategies evaluation only, there was essentially only one control variable – ramp meter rate. The speed dynamics is not intended to be independent. Instead, it generates a reference speed based on a static density-speed relationship – the Fundamental Diagram -- and went back to the loop to affect the density dynamics indirectly through some coupling. In this sense, the second order model essentially represents the dynamics of density with some driver behavior added through the speed dynamics. This model was used in [7] for combined VSL and Ramp Metering for reducing shockwave. A good reference is referred to [8] for understanding several second order traffic model based on fluid dynamics. The explanation is very interesting, particularly the car following model and the Payne-Papageorgiou model.

It is generally accepted that crashes and incidents can be reduced between 25–40 percent by using VSL [9]. In recent years, VSL has been investigated in both theory and practice for mobility improvement [7, 10, 11]. Speed control needs its dynamics. Therefore, most works in coupled VSL and CRM design used the METANET model.
III. THE METANET MODEL

The following assumptions are made though the rest of the discussion. A freeway corridor is divided into links; a link contains exactly one onramp meter; a link may have any number of off-ramps; a link could be divided into cells if necessary; for simplicity, each link is considered as a cell; and each link contains at least one traffic sensor.

\( m \) – link (or cell) index

\( T \) – time step for model update

\( L_m \) – length of link \( m \)

\( \lambda_m \) – number of lanes in link \( m \)

\( \rho_m (k), v_m, q_m (k) \) - density, distance mean speed and flow for link (cell) \( m \) during time interval \( k \)

\( s_m (k) \) - flow at off-ramp \( m \), measured

\( r_m (k) \) - on-ramp flow (metering rate) to be determined

\( u_m (k) \) - speed control variable, to be determined

Payne [1] deduced the speed dynamics based on the certain assumptions and the following equilibrium state assumption:

\[ v(x, t + \tau) = V(\rho(x + \Delta x, t)) \] (3.1)

which could be interpreted as: the density in the \( \nu - \rho \) relationship of the Fundamental Diagram (FD) has been predicted ahead over distance \( \Delta x \), but the average driver’s response has been delayed by \( \tau \) in time. Using Taylor series expansion to both sides of the equation with respect to \( t \) and \( x \) respectively and discretizing it, the following speed dynamics equation is obtained:

\[ v_m (k + 1) = v_m (k) + \frac{T}{\tau} V(\rho_{m+1}(k) - v_m (k)) - \frac{T}{L_m} v_m (k)(v_{m+1}(k) - v_m (k)) - \frac{\nu T}{\tau L_m} \frac{\rho_{m+1}(k) - \rho_m (k)}{\rho_m (k) + \kappa} \] (3.2)

where \( T \) – time step length

\( L_m \) – cell length

\( \tau, \nu, \kappa \) are parameters to be calibrated from field data.

Each term of the right hand side of the model (3.2) could be interpreted as follows:

1. The first – relaxation term: It is a high gain filter \( \left( \frac{1}{\tau} \right) \) with small \( \tau \) from a dynamic systems viewpoint [12]. The collective of drivers is to achieve the desired speed \( V(\rho_m (k)) \) - the control variable. The selection of the desired speed is critical to reflect the driver behavior.

2. The second – convection term: the effect of the traffic into the downstream cell from upstream cell, i.e., the speed increase/decrease caused by in-flow and out-flow vehicle speeds. It can be modified by adding a factor \( \text{sat}(\rho_{m+1} / \rho_m) \) where \( \text{sat}(\cdot) \) is the saturation function to address the driver speed change with respect to density variation between the two consecutive cells [13]. This term is the most sensitive one in speed dynamics and will be discussed further later.

3. The third – density gradient term: when downstream density increases/decreases, the speed in the current cell will decrease/increase:

\[ \frac{\nu T}{\tau L_m} \frac{\rho_{m+1}(k) - \rho_m (k)}{\rho_m (k) + \kappa} = \frac{1}{\tau} \frac{\nu T}{L_m} \frac{\rho_{m+1}(k) - \rho_m (k)}{\rho_m (k) + \kappa} \] (3.3)

where \( \tau \) is the time delay for the response of a collective of drivers to the perception of the traffic density (basically, what each driver could observe is the inter-vehicle distance in the immediate vicinity - which could be interpreted as the driver version of local density); \( V \) is a sensitivity factor. The part in the bracket expresses the effect of downstream cell density: the higher the downstream density, the lower the speed for the current cell. \( \rho_m (k) \) in the denominator is for normalization.

The parameter \( \kappa > 0 \) is added for two purposes:

- To force the model to only work for medium to high density;
- To avoid the singularity or the sensitivity of the term to the model in low density situations.

The physical meaning of the three terms including the parameters was also explained in detail related to driver behavior by [13]. Those explanations could be used as the basis for parameter identification.

Since \( V(\rho_m (k)) \) is basically the speed control parameter to be designed, it could be parameterized with any other value instead of density \( \rho_m (k) \) or even without parameterization at all. Doing this is just a matter of coordinate transformation. However, parameterization with density directly links the control design to the shape of the FD. Putting together the density dynamics and speed dynamics, the METANET model is obtained as follows [1, 5, 6, 14]:

\[ \rho_m (k + 1) = \rho_m (k) + \frac{T}{L_m \lambda_m} (\rho_{m+1}(k)v_{m+1}(k) - \rho_m (k)v_m (k)) + r_m (k) - s_m (k) \] (3.4)

\[ v_m (k + 1) = v_m (k) + \frac{T}{\tau} (V(\rho_m (k), b_m (k)) - v_m (k)) \]

\[ \frac{T}{L_m} v_m (k)(v_{m+1}(k) - v_m (k)) - \frac{1}{\tau} \frac{\nu T}{\tau L_m} \frac{\rho_{m+1}(k) - \rho_m (k)}{\rho_m (k) + \kappa} \]

Parameters \( (\tau, \nu, \kappa) \) are to be determined in model calibration using field data. In the equation (3.4), the control variable \( V(\rho_m (k), b_m (k)) \) was re-parameterized in [4, 5, 7, 14, 15] as:

\[ V(\rho_m (k), b_m (k)) = v_j [b_m (k)] \exp \left[ - \frac{1}{\alpha [b_m (k)]} \left( \frac{\rho_m (k)}{b_m (k)} \right)^{\alpha[b_m (k)]} \right] \]

\[ v_j [b_m (k)] = v_j \cdot b_m (k) \Rightarrow b_m (k) = \frac{v_j [b_m (k)]}{v_j} \]

\[ \rho_\alpha [b_m (k)] = \rho_m \left[ 1 - 2\alpha (1 - b_m (k)) \right] \]

\[ \alpha [b_m (k)] = \alpha \left[ E - (E - 1)b_m (k) \right] \]

(3.5)
where \((v'_j, \rho'_j, \alpha', E, A)\) are to be determined from field data in the model calibration process.

- \(v'_j\) – the free-flow speed
- \(\rho'_c\) – critical density for traffic to break down from free flow or from a homogeneous flow
- \((\alpha', E, A)\) – model parameters for FD in the form of speed-density relationship.

The purpose of the parameterization could be explained as:

- Desired speed and density are restricted to follow the FD curve;
- Control parameter \(b_m(k)\) determines the shape of the FD; therefore, selection of control strategy determines the shape of desired FD;
- The objective of VSL control is to make the speed and density follow the desired FD pattern.

The authors believe that the model needs improvement in the following aspects:

- The control variable \(b_m(k)\) appears highly nonlinear in the dynamics (3.4); if the constraints of the control variable are not set properly, the feasible set (decision parameter domain) could be very small, which is likely to cause problems for the numerical optimization process;
- Density and thus ramp meter rate through the density dynamics (3.4) are tightly and nonlinearly coupled with the speed control variable \(b_m(k)\) (3.5), which further complicates the control design;
- Most importantly, the model is deduced from the FD, which is intrinsically a static relationship. Therefore, the model may not be able to capture fast transition phases of the traffic dynamics very well, which can be observed in the simulation later.

Those points are the main motivation of this paper.

IV. MODEL IMPROVEMENT

Model improvement in this paper includes both simplification and modification.

A. Constraint Simplification

In (3.1), set \(V = u_m(k)\) as the speed control variable. The following constraints are proposed to address the system dynamics property and traffic characteristics:

\[
\begin{align*}
    v_m(k) & \leq Q_m, \\
    0 & \leq \rho_m(k) \leq \rho, \\
    0 & \leq v_m(k) \leq V_f, \\
    \rho_m(k) & \leq \varphi_m(u_m(k)), \quad \text{or} \quad u_m(k) \leq \psi_m(\rho_m(k)) \\
    0 & \leq u_m(k) \leq V_f, \\
    r_m(k) & \leq s_m(k) + v_{m-1}(k) \rho, \\
    r_{m,\min} & \leq r_m(k) \leq r_{m,\max}
\end{align*}
\]

where \(\rho_j\) - jammed density; \(V_j\) - flow speed. The first three constraints in (4.1) are constraints on state variables; and the fourth and fifth constraints are for VSL; and the sixth and seventh are on ramp metering rate.

The following discussion justifies the constraints. It is clear that the second, third, and fifth are just simple physical limits. In the Cell Transmission Model (CTM) [16], the following equality is based on physical limits of the traffic flow:

\[
q_m(k) = \min \{v_{m-1}(k)\rho_{m-1}(k), Q_m, v_{m-1}(k)(\rho_j - \rho_{m-1}(k))\} \quad (4.2)
\]

If we take into account the flow from on-ramp and off-ramp, (4.2) becomes

\[
q_m(k) = \min \{v_{m-1}(k)\rho_{m-1}(k) + r_m(k) - s_m(k), Q_m, v_{m-1}(k)(\rho_j - \rho_{m-1}(k))\} \quad (4.3)
\]

\(q_m(k)\) assumes one of the three possible values in braces. If it assumes the first value, then

\[
q_m(k) = v_{m-1}(k)\rho_{m-1}(k) + r_m(k) - s_m(k) \quad (4.4)
\]

It is the conservation of the flow, or the first dynamics in (3.1). If \(q_m(k) = Q_m\), it is the first inequality of (4.2). If

\[
q_m(k) = v_{m-1}(k)(\rho_j - \rho_{m-1}(k))
\]

then

\[
\rho_m(k) = v_{m-1}(k)(\rho_j - \rho_{m-1}(k)) \Rightarrow r_m(k) - s_m(k) = v_{m-1}(k)\rho_j \Rightarrow r_m(k) = s_m(k) + v_{m-1}(k)\rho_j
\]

in which (4.4) is used. This justifies the sixth constraint in (4.1). Here the \(v_{m-1}(k)\) can be predicted based on the speed dynamics of the last time step. If \(s_m(k)\) is measured or predicted, then the constraint is again linear with respect to the control variable \(r_m(k)\) – the inflow rate.

The last three constraints are for control variables – VSL and ramp meter rate added for traffic operation. \(\{r_{m,\min}, r_{m,\max}\}\) are minimum/maximum ramp meter rates. \(\rho_m(k) = \varphi(u_m(k))\) is the curve of a specified traffic speed drop probability contour as indicated in. For a given acceptable traffic drop probability, the contour gives an upper bound for the feasible region [17].

The advantages of dropping the FD constraint are as follows:

- Speed control variable appears linearly, which significantly simplifies the control design problem;
- The model has two degrees of freedom for control design: both VSL and CRM rate are free in the feasible region bounded by the constraint (4.1) in the speed-density plane;
- Model mismatch caused by discrepancies between field data and the FD curve could be avoided.
B. Alternative Convection Term of Speed Dynamics

Analyzing the convection term in speed dynamics:

\[
\frac{T}{L_m} v_m(k) \left[ (1 - \alpha) v_{m-1}(k) - v_m(k) \right]
\]

(4.5)

it indicates that if the upstream (link \( m-1 \)) speed in the last time step is greater than the speed of link \( m \), then the speed of link \( m \) will likely increase, otherwise, it will decrease. This may be true if the downstream (link \( m+1 \)) is free flow and the link \( m \) is not saturated yet. However, if the downstream (link \( m+1 \)) is congested and/or the link \( m \) is saturated, this may not be correct. This motivates us to consider other possible convection terms in the speed dynamics of the METANET model. The following alternative (a) is proposed:

\[
\frac{T}{L_m} v_m(k) \left[ \alpha \cdot v_{m-2}(k) + (1 - \alpha) \cdot v_{m-1}(k) - v_m(k) \right]
\]

(4.6)

0 ≤ \( \alpha \) ≤ 1

or,

\[
\frac{T}{L_m} v_{m-1}(k) \left[ \alpha \cdot v_{m-2}(k) + (1 - \alpha) \cdot v_{m-1}(k) - v_m(k) \right]
\]

(4.7)

0 ≤ \( \alpha \) ≤ 1

which means that speed further upstream is used to predict the current cell speed. This is expected to lead to less time delay in speed dynamics. Alternative (b) is suggested as:

\[
\frac{T}{L_m} \left( v_m^2(k) - v_m(k)^2 \right) \cdot v_m(k)
\]

(4.8)

The idea here is to use the geometric mean of current and upstream speed to replace upstream speed. It reduces the significance of the upstream speed effect on the speed of the current link, and also the sensitivity of the term. Alternative (c) is:

\[
\frac{T}{L_m} \left( \frac{v_m^2(k) + v_m^2(k)}{2} - v_m(k)^2 \right) \cdot v_m(k)
\]

(4.9)

This also reduces the effect of upstream link speed on the speed of the current link. Alternative (d) is suggested as:

\[
\frac{T}{L_m} \left( \frac{v_{m-1}(k)^2 + v_m^2(k)}{2} - v_m(k)^2 \right) \cdot v_{m-1}(k)
\]

(4.10)

This term reduces the effect of upstream link speed due to the first factor, but increases the significance of upstream speed due to the second factor \( v_{m-1}(k) \). Model calibration and simulation results using this term will show later that it improves model matching.

C. Model Application

The model has been applied to control design in several ways:

- Assume known ramp metering rate and then conduct VSL design [18];
- Design combined CRM and VSL with coupled density and speed dynamics (3.1, 4.1);
- Design VSL at each step before CRM – the former linearized the model which leads to a linear model for CRM design [19]; the control problem can be formulated as an Linear Programming (LP) at each time step with MPC.

From those works, the benefits of model simplification and improvement can be observed. However, further model simplification and improvement are still necessary.

V. MODEL VALIDATION AND SIMULATION

The original and modified METANET models are validated and simulated for comparison in model matching using field data, which include the following cases: with and without FD, with convection term (4.5) and (4.10). It is noted that the model has the following common parameters to be identified: \((\tau, \nu, \kappa)\). Other parameters are assumed to have the values suggested by [15, 13, 5, 20].

A. Field Data

The Berkeley Highway Lab (BHL) [21,22] dual loop data with 1 second update rate and 60 Hz loop on/off information are used for the calibration. 20 days of data have been processed by cleaning, imputation and correction, and then averaged across all lanes and aggregated into 20 s and 60 s traffic state variables in distance mean speed, density and flow and then are used for model calibration. Both East and West in the BHL are used; Onramp and off-ramp flows are ignored since the measurements are not available.

In the simplified METANET (4.2) the relaxation (equilibrium speed) term is removed in model calibration and simulation. This implicitly assumes that drivers follow the desired speed reasonably well. This can also be justified from a dynamical systems identification viewpoint [12]; i.e., the speed tracks the desired speed quickly with high gain \( \frac{1}{\tau} \) (small enough \( \tau \)). This is another reason why the parameter searching region later in the model calibration has a limit 0 ≤ \( \tau \) ≤ 1.8.

B. Model Calibration Method

Objective Function: The following objective function is selected for minimizing both speed and flow errors.

\[
Z = \sum_m \sum_k \left[ (\hat{v}_m(k) - v_m(k))^2 + \sigma (\hat{q}_m(k) - q_m(k))^2 \right]
\]

Here \( m \) is over all the links, and \( k \) is time steps in the calibration time period (e.g. 24 hours). \( v_m(k) \) and \( q_m(k) \) are estimated by the model. \( \hat{v}_m(k) \) and \( \hat{q}_m(k) \) are the corresponding values from field data. The weighting parameter \( \sigma = 0.15 \) is used.

Inserting the estimated state from (3.1, 4.1), it can be observed that model calibration is a nonlinear optimization problem. For a given region, there may be multiple solutions for a given convergence threshold. Therefore, the search region for the optimization is very important. For this reason, several parameter search ranges have been conducted. The one that is believed to be more suitable for control design is presented here.
Parameter searching range: Let $\mathbf{b} = [\tau, \kappa, \nu]$ be the vector of parameters. The search interval $[\beta_{\text{min}}, \beta_{\text{max}}]$ is suggested as follows: $\beta_{\text{min}} = [0, 30.7], \beta_{\text{max}} = [1.8, 38, 9]$. Such setup is based on the following considerations: $0 \leq \tau \leq 1.8$ is the average driver response delay. Small $\tau$ will force the speed dynamics to track the desired speed fast, which represents the practical control situation. It is noted that larger $\tau$ might lead to better model matching for some datasets, which explains the driver behavior without control, but that is not suitable for control since the speed will no track the desired speed and therefore will not be adopted. The rest two are based on simulation and on previous work [15, 13, 5, 20].

The performance measure is the averaged Root Mean Relative Square Error (RMRSE) which is defined for time sequence $x$ and its estimation $\hat{x}$ as follows:

$$RMRSE(x) = \frac{1}{M} \frac{\sum_{m} \sum_{k} |x_m(k) - \hat{x}_m(k)|}{\sum_{m} \sum_{k} x_m(k)}$$

where $M$ is the total number of links, and $k$ is the overall number of time steps in the calibration time period.

The optimization algorithm is the 2-D Golden Section Search in Matlab to find the minimum overall points in the region. The error tolerance is set as $\varepsilon = 0.1$. The maximum number of iterations, $N_{\text{max}}$, is set to be 10.

**C. Simulation Results**

After model calibration, the obtained model parameters were inserted into the model for simulation of the dynamical behavior of the model. The following setups for initial and boundary conditions are adopted:

- **Initial conditions:** Flow, density, and speed at the first time step for all the links assume the field measured values, but after that, they were determined by the model dynamics.
- **Boundary conditions:** Flow, density, and speed at Cells 1 and 8 are assumed measured value all the time. Therefore, only the 6 links in between the two terminal links are simulated.

The following tables collect the modeling and simulation results, with and without FD, and using the default convection term Table 1. It can be observed from the tables that:

(a) model matching with FD and without FD does not make a significant difference on average. Therefore, since a linear control variable is highly desirable in control design it is better to omit the FD;

(b) Using convection term (4.10) in speed dynamics improves the model matching for both cases, with and without FD. This is obvious by looking at the last line for the average error over the 5 datasets.

**Table 1.** Comparison of modeling and simulation results with field data with FD and without FD; using different convection terms; average over 5 work days

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Using Convection Term (4.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau, \kappa, \nu$</td>
</tr>
<tr>
<td><strong>Without FD</strong></td>
<td>0.1308</td>
</tr>
<tr>
<td><strong>With FD</strong></td>
<td>0.1409</td>
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<table>
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<tr>
<th>Parameters</th>
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<tr>
<td></td>
<td>$\nu$</td>
</tr>
<tr>
<td><strong>Without FD</strong></td>
<td>0.0279</td>
</tr>
<tr>
<td><strong>With FD</strong></td>
<td>0.2580</td>
</tr>
</tbody>
</table>

Fig. 1. Speed and density of Cell 2 and 3; Red: measured data; Blue: simulated

In Fig. 1, speed and density have a reasonable match with measure data over a whole day for Cell 2 and 3. In general, speed matching is better than density for other day simulations as well.

**VI. CONCLUDING REMARKS**

This paper suggests two improvements on the METANET model: (a) dropping the nonlinear parameterization in the speed control variable and directly adopting the linear desired speed as the control variable; and (b) suggesting several alternatives for the convection term of the speed dynamics for better model matching. Model calibrations using BHL field data have been conducted, with results presented for comparison. As the results showed, the original METANET model and its simplified counterpart are comparable in model matching, but the suggested new convection term in speed
dynamics improves the model matching. However, the density model still could not always capture the traffic dynamics accurately based on comparison of simulations with field data. This may be due to several reasons including: (1) data quality – whether the data accurately represents the traffic dynamics for the freeway section; (2) the model needs further improvement. For example, during simulation, we recognized that the density gradient term in the speed dynamics is rather sluggish during the time.

The computational aspects will be addressed in future work for the comparison of using the original METANET model and the simplified model in control design. It would be difficult for such a comparison without control design.

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REFERENCES