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Safe Platooning in Automated Highway Systems Part II: Velocity Tracking Controller

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SUMMARY

This paper presents the design of a velocity tracking controller for safe vehicle maneuvering in Automated Highway Systems (AHS) in which traffic is organized into platoons of closely spaced vehicles. The notion of safety is related to the absence of collisions that exceed a given relative velocity threshold. In a companion paper, state dependent safety regions for the platoons are designed in such a way that, whenever the state of a platoon is inside these safety regions, it is guaranteed that platoon maneuvering will be safe and follow the behavior prescribed by the finite state machines that control vehicles maneuvers. Velocity profiles inside these safety regions are derived for all the single lane maneuvers and a nonlinear velocity tracking controller is designed to track these profiles. This controller attempts to complete the maneuvers with comfort in minimum time, whenever safety is not compromised. The control schemes presented in this paper were implemented and tested using AHS simulation software.

1. INTRODUCTION

This paper addresses one important control problem in the AHS hierarchical architecture of the California PATH program in [1]: the design of a velocity tracking controller for the safe execution of regulation layer maneuvers. The structure of this hierarchical architecture is described in a companion paper [2] and references therein and therefore it is not explained here.

One of the key issues of the architecture in [1] is that traffic is organized in platoons of closely spaced vehicles. As the behavior of a platoon is determined by the control law that is applied to its leader, the results presented in this paper focus on the control laws that are applied to platoon leaders. Under normal operation, there are five control laws for the leader of a platoon: leader law, join law, split law, decelerate to change lane law and change lane law. The leader law is used to

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keep a platoon traveling at a target velocity and at a safe distance of the platoon ahead. Join and split laws are used to perform longitudinal maneuvers. The decelerate to change lane law is used to execute an additional longitudinal maneuver that creates a safe distance from platoons in different lanes before a change in lane can occur. The change lane law controls the lateral motion of a vehicle when it goes from one lane to another. The analysis regarding the stability of the follower law, that applies to non-leader vehicles in a platoon, can be found in [3,4].

After the safety regions for platooning were established in the companion paper [2], it is necessary to determine the desired velocity profiles for each one of the single lane maneuvers that will always be inside these safety regions. In this paper, the desired velocity profiles are designed by considering that, in addition to safety, it is convenient to complete maneuvers in minimum time using comfort levels on jerk and acceleration, whenever safety is not compromised. A nonlinear velocity tracking controller is proposed to follow the desired velocity profiles. The controller presented here differs from the one in [5] in two important points. The first one is that it uses the full state of the platoons relative motion to calculate the jerk control signal; the controller in [5] employs only part of that state. The other difference is in the design of the observer to estimate the acceleration of the lead vehicle. In this paper the knowledge of the relative distance between vehicles and the velocity of the trail vehicle is exploited. The resultant observer has a smoother behavior than the one designed in [5].

The results obtained with the use of this controller are illustrated with simulation examples. Only cases in which no collision should occur between platoons in the AHS are included. These simulation results are different from the results reported in [5]; the design in [5] required to allow low relative velocity collisions in the AHS for the manuevers to be completed in all cases. The design presented in this paper extends the results in [5] by including the important case in which collisions in the AHS do not take place.

This paper is divided into four sections. Section 2 contains the design of the desired velocity profiles for the single lane maneuvers. Section 3 contains a description the non-linear velocity tracking controller, including the stability analysis. In Section 4, simulation results are presented for the case in which no collisions are supposed to happen during the normal mode of operation of the AHS. Finally, Section 5 contains the conclusions of the paper.

2. VELOCITY PROFILES

In this section the velocity for the relative motion of platoons during a maneuver is expressed as desired velocity profiles in the state space $(\Delta x,\ \Delta\dot{x},\ v_{lead})$. Fig. 1 defines the basic variables considered in the analysis. The rest of the notation corresponds to the one established in the companion paper [2].

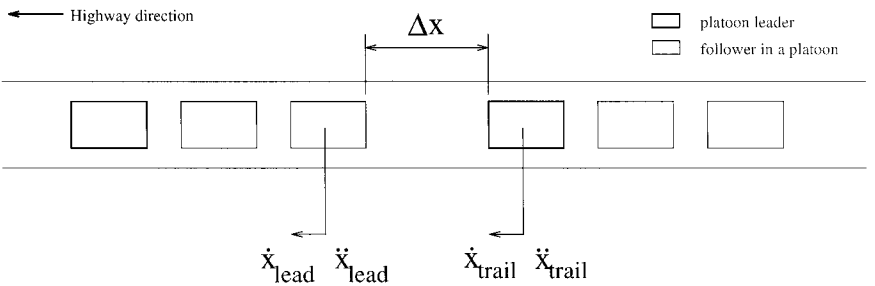


Fig. 1. Notation for two platoons on the highway.

The velocity profiles must allow the platoons to complete the maneuvers in minimum time while guaranteeing that the trajectories remain inside the safety region. The desired velocity profiles are established considering the following assumption.

Assumption 1

1. *Whenever safety is not compromised, platoons should keep the acceleration and jerk within comfort bounds.*
2. *Maneuvers are executed one at a time. No maneuver can begin before the previous one is completed.*

2.1. Join Law

The goal of the control law in a join maneuver is to decrease the initial relative displacement between the lead platoon and the trail platoon, $\Delta x(0)$, to a desired intraplatoon spacing Δx_{join}. The relative velocity, $\Delta\dot{x}$, should be null at the end of the join maneuver. The resulting trajectory of the state $(\Delta x, \Delta\dot{x}, v_{lead})$ during the join maneuver, according to Theorem 1 in [2], must be within the safety set X_{bound}.

In order to decrease the time the join maneuver takes to complete, the relative velocity between the trail and lead platoons, $\Delta\dot{x}$, should be minimized while observing the safety limits. This suggests that the state $(\Delta x, \Delta\dot{x}, v_{lead})$ of the join maneuver should be kept, as much as possible, in the boundary ∂X_{safe} of the safety set X_{safe} in Theorem 1 in [2]. This boundary consists of two smooth portions:

1. In the first portion, the trail platoon is far enough from the lead platoon so that maximum deceleration will stop the lead platoon before the trail platoon hits it at v_{allow}, if a collision occurs.
2. The other portion of the maximum safe velocity curve represents the case when full braking does not stop the lead platoon before the trail platoon hits it at v_{allow}, if a collision occurs.

According to [2], the maximum safe velocity v_{trail} of the trail platoon, for a given Δx and v_{lead} is

$$v_{trail}(v_{lead},\Delta x) = \begin{cases} -c_2 + \sqrt{2a_{min}^{trail}\Delta x + \alpha v_{lead}^2 + v_{allow}^2 + c_2 d}; & R_2(\Delta x, v_{lead}) > S(\Delta x, v_{lead}), \\ -c_2 + v_{lead} + \sqrt{v_{allow}^2 + \dfrac{\alpha - 1}{\alpha} a_{min}^{trail}(2\Delta x + c_2 d)}\ ; & R_2(\Delta x, v_{lead}) \le S(\Delta x, v_{lead}), \end{cases} \tag{1}$$

where:

$$R_1(\Delta x) = -c_2 + \sqrt{v_{allow}^2 + \frac{\alpha - 1}{\alpha} a_{min}^{trail}(2\Delta x + c_2 d)}\ ,$$

$$R_2(\Delta x, v_{lead}) = -c_2 - v_{lead} + \sqrt{2a_{min}^{trail}\Delta x + \alpha v_{lead}^2 + v_{allow}^2 + a_{min}^{trail} c_2 d}\ ,$$

$$R_3(v_{lead}) = (\alpha - 1) v_{max} - c_2 + v_{allow}\ ,$$

$$S(\Delta x, v_{lead}) = \max\big(R_1(\Delta x), R_3(v_{lead})\big),$$

$$c_2 = \big(a_{max}^{trail} + a_{min}^{trail}\big) d,$$

$$\alpha = a_{min}^{trail} / a_{min}^{lead} > 0.$$

Ideally, the maximum desired velocity for the trail platoon while performing a join maneuver should be the one indicated by Eq. (1). It should be noticed, however, that in Eq. (1) it is assumed that the velocity of the trail platoon, v_{trail}, is on the boundary of X_{safe}. Therefore, when the state $(\Delta x,\ \Delta\dot{x},\ v_{lead})$ is not in this boundary, the actual value of the other state, $\Delta\dot{x}$, affects the trajectory of the state as time passes. Consider, for example, the situation depicted in Fig. 2. The flow departing from point p in the figure will have an initial direction given by the resultant of $\Delta\dot{x}$ and the effective acceleration a at that particular instant of time.

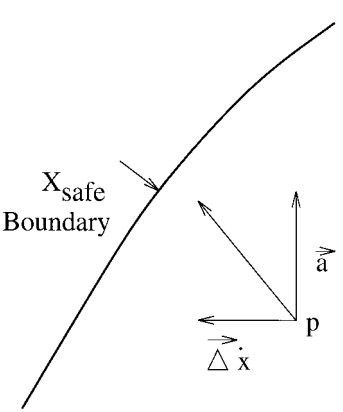


Fig. 2. Effect of the relative velocity $\Delta\dot{x}$ on the desired velocity for the trail platoon.

For this reason it is suggested to set the maximum desired velocity for the trail platoon, v_{safe} as a function of the full state $(\Delta x,\ \Delta\dot{x},\ v_{lead})$ as follows

$$v_{safe}(\Delta x, \Delta\dot{x}, v_{lead}) = \begin{cases} -c_2 + \sqrt{2a_{min}^{trail}(\Delta x + \eta\Delta\dot{x}) + \alpha v_{lead}^2 + v_{allow}^2 + a_{min}^{trail} c_2 d}\ ; & R_2(\Delta x, v_{lead}) > S(\Delta x, v_{lead}), \\ -c_2 + v_{lead} + \sqrt{v_{allow}^2 + \dfrac{\alpha - 1}{\alpha} a_{min}^{trail}(2(\Delta x + \eta\Delta\dot{x}) + c_2 d}\ ; & R_2(\Delta x, v_{lead}) \le S(\Delta x, v_{lead}), \end{cases} \tag{2}$$

where $\eta > 0$ is a gain.

To finish the join maneuver in minimum time, it is necessary to slow the trail platoon to v_{lead} at the end of the join. According to assumption 1 the trail platoon should decelerate at the maximum comfortable level. The velocity in the deceleration curve, v_{min}, written as a function of $(\Delta x,\ \Delta\dot{x},\ v_{lead})$ is

$$v_{min}(\Delta x, \Delta\dot{x}, v_{lead}) = \min \begin{cases} v_{lead} + \sqrt{2a_{com}(\Delta x + \eta\Delta\dot{x} - \Delta x_{join})}\ , \\ v_{fast}\ . \end{cases} \tag{3}$$

where a_{com} is the magnitude of the comfort acceleration and deceleration for vehicles in a highway, Δx_{join} is the desired intraplatoon distance and v_{fast} is the maximum recommend velocity for a platoon to travel on the highway.

To ensure the safety of the join control law and to allow the maneuver to be completed in minimum time, the velocity of the trail platoon should satisfy

$$v_d(\Delta x, \Delta\dot{x}, v_{lead}) = \min(v_{min}, v_{safe})\,.$$

Eqs. (2) – (3) define a desired velocity profile for the trail platoon during a safe join law. Fig. 3 shows an example of this desired velocity profile in the $\Delta x(\cdot)$ vs. $\Delta\dot{x}(\cdot)$ phase plane. For the profile in Fig. 3 it is assumed that the lead platoon is traveling at constant velocity and that the braking capabilities of the lead and trail platoons are equal. The acceleration portion in Fig. 3 will be produced by the velocity tracking controller to be described in the next section.

The desired phase-plane trajectory for the trail platoon velocity includes abrupt changes in acceleration at the points where different curve sections intersect. It is convenient to smooth these transitions so as not to violate jerk comfort constraints.

2.2. Split Law

In the split maneuver the goal is to increase the distance between the lead and trail platoon, Δx, to a desired value Δx_{split}. To accomplish this increment at relative distance, the relative speed between platoons, $\Delta\dot{x}$, must be positive. For this reason, in most cases, the velocity of the trail platoon will be lower than the velocity of the lead platoon, and thus the threat of high-speed collisions during a split maneuver will be inherently reduced.

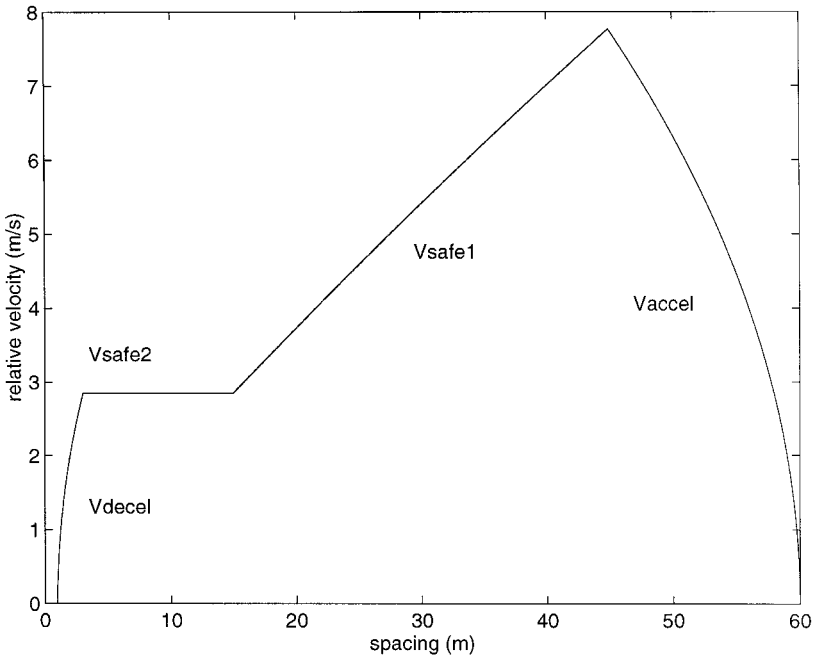


Fig. 3. Basic velocity profile for 60 m initial spacing. The maximum allowable velocity for collisions is 3 m/s and the lead platoon is moving at a constant velocity of 25 m/s.

To design the desired velocity profile for a split maneuver, a similar approach to the one used for the join maneuver can also be used. Two boundary curves are established for the velocity of the trail platoon, v_{trail}. The first one, related to safety, is derived from the results in [2] by assuming $v_{allow} = 0$. Thus, for a given state (Δx, $\Delta \dot{x}$, v_{lead}), the maximum velocity of the trail platoon for the split law to be safe is

$$v_{safe}(\Delta x, \Delta \dot{x}, v_{lead}) = \begin{cases} -c_2 + \sqrt{2a_{min}^{trail}(\Delta x + \eta \Delta \dot{x}) + \alpha v_{lead}^2 + a_{min}^{trail} c_2 d}\,; & R_2(\Delta x, v_{lead}) > S(\Delta x, v_{lead}), \\ -c_2 + v_{lead} + \sqrt{\dfrac{\alpha - 1}{\alpha} a_{min}^{trail}(2(\Delta x + \eta \Delta \dot{x}) + c_2 d}\,; & R_2(\Delta x, v_{lead}) \le S(\Delta x, v_{lead}), \end{cases} \tag{4}$$

where

$$R_1(\Delta x) = -c_2 + \sqrt{\frac{\alpha - 1}{\alpha} a_{min}^{trail}(2\Delta x + c_2 d)}\,,$$

$$R_2(\Delta x, v_{lead}) = -c_2 - v_{lead} + \sqrt{2a_{min}^{trail}\Delta x + \alpha v_{lead}^2 + + a_{min}^{trail} c_2 d}\,,$$

$$R_3(v_{lead}) = (\alpha - 1) v_{max} - c_2\,,$$

$$S(\Delta x, v_{lead}) = \max(R_1(\Delta x), R_3(v_{lead})),$$

$$c_2 = \left(a_{max}^{trail} + a_{min}^{trail}\right)d.$$

The other boundary curve is related to time-optimality. This curve establishes a lower bound on the velocity of the trail platoon. To determine this lower bound, it is assumed that, for a given state $(\Delta x, \Delta\dot{x}, v_{lead})$, if the trail platoon is traveling at minimum velocity, then it will reach the desired intraplatoon distance Δx_{split} with null relative velocity by applying maximum comfort acceleration. It is also assumed that a minimum velocity v_{slow} exists below which it is not recommended to travel on the highway under normal circumstances. The minimum velocity of the trail platoon is therefore given by

$$v_{min}(\Delta x, \Delta\dot{x}, v_{lead}) = \max\begin{cases} v_{lead} - \sqrt{2a_{com}\left(\Delta x_{split} - \Delta x - \eta\Delta\dot{x}\right)}, \\ v_{slow}. \end{cases} \tag{5}$$

At any particular state $(\Delta x, \Delta\dot{x}, v_{lead})$ of a split maneuver, the velocity of the trail platoon should satisfy the safety requirements, therefore from Eqs. (4) and (5)

$$v_d(\Delta x, \Delta\dot{x}, v_{lead}) = \min(v_{min}, v_{safe}).$$

2.3. Decelerate to Change Lane Law

The decelerate to change lane control law attempts to create a safe distance between platoons in different lanes before any actual change lane maneuver can take place. The decelerate to change lane law can be treated similarly to the split law. The only distinction is that, while safety is considered in terms of the lead platoon in the same lane, the time optimal part of the trajectory has to be calculated in terms of the lead platoon in the lane the trail platoon is changing into. Notice that, according to assumption 1, it is enough to calculate for safety only for the platoon in the same lane, because the change lane maneuver will not occur until the decelerate to change lane maneuver is completed. The maximum safe velocity for the trail platoon is therefore the same as in the split law in Eq. (4).

The minimum velocity of the trail platoon is established in the same way as in the split control law, but considering the target velocity and distance with respect to the platoon in the adjacent lane. Thus

$$v_{min}(\Delta x_{next}, \Delta\dot{x}_{next}, v_{next}) = \max\begin{cases} v_{next} - \sqrt{2a_{com}\left(\Delta x_{change} - \Delta x_{next} - \eta\Delta\dot{x}_{next}\right)}, \\ v_{slow}. \end{cases} \tag{6}$$

where $(\Delta x_{next}, \Delta\dot{x}_{next}, v_{next})$ is the state of the platoon performing the change lane maneuver, relative to the lead platoon that is in the lane the trail platoon is

changing into and Δx_{change} is the required spacing after the decelerate to change lane maneuver has been completed. At any particular stage of a decelerate to change lane maneuver, the velocity of the trail platoon should satisfy the safety requirements, therefore from Eqs. (4) and (6)

$$v_d(\Delta x, \Delta\dot{x}, v_{lead}, \Delta x_{next}, \Delta\dot{x}_{next}, v_{next}) = \min(v_{min}, v_{safe}).$$

2.4. Leader Law

The leader law is intended to keep a platoon traveling on a highway at a target velocity and at a safe distance of the platoon ahead. When no propagation of collisions is desired on the highway, the state $(\Delta x,\ \Delta\dot{x},\ v_{lead})$ of a platoon executing the leader law has to remain within the set X_{leader} defined in Theorem 2 in [2]. When safety is not critical, the target velocity for a platoon leader executing the leader law is no longer the velocity of the platoon ahead, but some desired velocity v_{link}. This velocity is given by a highway link layer traffic controller according to the section of the highway where the leader of the platoon is currently located [6].

The maximum safe velocity curve v_{safe} for a platoon in leader law, given $(\Delta x,\ \Delta\dot{x},\ v_{lead})$, according to Theorem 2 in [2] is

$$\begin{aligned} &v_{safe}(\Delta x, \Delta\dot{x}, v_{lead}) \\ &\quad = -c_2 - v_{allow} + \sqrt{2a_{min}^{trail}(\Delta x + \eta\Delta\dot{x}) + \alpha(v_{lead} - v_{allow})^2 + a_{min}^{trail}c_2 d}\,. \end{aligned} \tag{7}$$

The desired velocity for a platoon under the leader law is therefore

$$v_d(\Delta x, \Delta\dot{x}, v_{lead}) = \min(v_{link}, v_{safe}).$$

It is also important to remark that whenever $v_{link} > v_{safe}$ then the relative spacing Δx will decrease until it reaches

$$\Delta x_{leader} = \frac{(v_{lead} + v_{allow} + c_2)^2 - \alpha(v_{lead} - v_{allow})^2 - a_{min}^{trail}c_2 d}{2a_{min}^{trail}}. \tag{8}$$

Substituting the value of Δx_{leader} in Eq. (8) into Eq. (7), $v_d(\Delta x_{leader}, 0, v_{lead}) = v_{lead}$ and therefore the desired velocity of the trail platoon will be the velocity of the lead platoon.

3. VELOCITY PROFILE TRACKING CONTROLLER

In this section a velocity tracking controller is introduced. This controller commands the actual velocity of a platoon to follow the desired velocity profile

derived in the previous section. The design of this controller is based on the following assumptions.

Assumption 2

1. *Positions and velocities of both the lead and the trail platoons are measured quantities.*
2. *The acceleration of the trail platoon is known.*
3. *The acceleration of the lead platoon is estimated.*
4. *The jerk of the lead platoon is modeled as noise.*

The velocity tracking controller is designed using the backstepping procedure [7]. This controller design combines an observer for the lead platoon state with a nonlinear controller for the jerk of the trail platoon.

3.1. Backstepping Design

Let $v_d(\Delta x, \Delta\dot{x}, v_{lead})$ be the value of the desired velocity flow field for the trail platoon. Introduce, for convenience, the change of variables

$$(\Delta x, \Delta\dot{x}, v_{lead}) \Leftrightarrow (\Delta x, v_{lead}, v_{trail}),$$

that follows directly from $v_{trail} = v_{lead} - \Delta\dot{x}$. Define the velocity error by

$$e = v_{trail} - v_d(\Delta x, v_{lead}, v_{trail}).$$

The velocity error dynamics is given by

$$\dot{e} = a_{trail} - \begin{pmatrix} \dfrac{\partial v_d}{\partial \Delta x} & \dfrac{\partial v_d}{\partial v_{lead}} & \dfrac{\partial v_d}{\partial v_{trail}} \end{pmatrix} \begin{pmatrix} v_{lead} - v_{trail} \\ a_{lead} \\ a_{trail} \end{pmatrix}, \tag{9}$$

where a_{lead} and a_{trail} denote the second time derivative of x_{lead} and x_{trail}, respectively. According to assumption 2, let $\hat{a}_{lead}$ be the estimated acceleration of the lead car. Define the lead platoon acceleration estimation error, $\tilde{a}_{lead}$ as

$$\tilde{a}_{lead} = \dot{v}_{lead} - \hat{a}_{lead} = a_{lead} - \hat{a}_{lead}. \tag{10}$$

From Eq. (10) into Eq. (9)

$$\dot{e} = a_{trail} - \begin{pmatrix} \dfrac{\partial v_d}{\partial \Delta x} & \dfrac{\partial v_d}{\partial v_{lead}} & \dfrac{\partial v_d}{\partial v_{trail}} \end{pmatrix} \begin{pmatrix} v_{lead} - v_{trail} \\ \hat{a}_{lead} \\ a_{trail} \end{pmatrix} - \frac{\partial v_d}{\partial v_{lead}} \tilde{a}_{lead}. \tag{11}$$

Assume, for the moment, that there is no error in the estimation of the acceleration, i.e., $\tilde{a}_{lead} = 0$, then, if the dynamics of e is desired to be stable, it is possible to define a fictitious control for the acceleration of the trail platoon as

$$\Gamma(\Delta x, v_{lead}, v_{trail}, \hat{a}_{lead}, a_{trail}) = -\lambda_1 e + \begin{pmatrix} \frac{\partial v_d}{\partial \Delta x} & \frac{\partial v_d}{\partial v_{lead}} & \frac{\partial v_d}{\partial v_{trail}} \end{pmatrix} \begin{pmatrix} v_{lead} - v_{trail} \\ \hat{a}_{lead} \\ a_{trail} \end{pmatrix}, \tag{12}$$

Using Eq. (12) into Eq. (11)

$$\dot{e} = -\lambda_1 e + a_{trail} - \Gamma - \frac{\partial v_d}{\partial v_{lead}} \tilde{a}_{lead}. \tag{13}$$

Define $\tilde{\Gamma}$ to be the difference between a_{trail} and Γ, i.e.,

$$\tilde{\Gamma}(t) = a_{trail}(t) - \Gamma(\Delta x, v_{lead}, v_{trail}, \hat{a}_{lead}, a_{trail}). \tag{14}$$

From Eq. (14) in Eq. (13), the velocity error dynamics is

$$\dot{e} = -\lambda_1 e + \tilde{\Gamma} - \frac{\partial v_d}{\partial v_{lead}} \tilde{a}_{lead}. \tag{15}$$

Consider now the dynamics of $\tilde{\Gamma}$,

$$\dot{\tilde{\Gamma}} = j_{trail} - \begin{pmatrix} \frac{\partial \Gamma}{\partial \Delta x} & \frac{\partial \Gamma}{\partial v_{lead}} & \frac{\partial \Gamma}{\partial v_{trail}} & \frac{\partial \Gamma}{\partial \hat{a}_{lead}} & \frac{\partial \Gamma}{\partial a_{trail}} \end{pmatrix} \begin{pmatrix} v_{lead} - v_{trail} \\ a_{lead} \\ a_{trail} \\ \dot{\hat{a}}_{lead} \\ j_{trail} \end{pmatrix}, \tag{16}$$

where $j_{trail} = d^3 x_{trail}/dt^3$ is the control jerk of the trail platoon and $\dot{\hat{a}}_{lead}$ is the time derivative of the estimate of the lead platoon's acceleration. The expression for the latter depends on the implementation of the lead platoon state observer and will be defined later when this observer is presented.

The following control for j_{trail} is proposed

$$\left(1 - \frac{\partial \Gamma}{\partial a_{trail}}\right) j_{trail} = -\lambda_2 \tilde{\Gamma} - \beta e + \begin{pmatrix} \frac{\partial \Gamma}{\partial \Delta x} & \frac{\partial \Gamma}{\partial v_{lead}} & \frac{\partial \Gamma}{\partial \dot{x}_{trail}} & \frac{\partial \Gamma}{\partial \hat{a}_{lead}} \end{pmatrix} \begin{pmatrix} v_{lead} - \dot{x}_{trail} \\ \hat{a}_{lead} \\ \ddot{x}_{trail} \\ \dot{\tilde{\hat{a}}}_{lead} \end{pmatrix}, \tag{17}$$

where $\dot{\tilde{\hat{a}}}_{lead}$ is an estimate of the time derivative of the lead platoon acceleration. When $\hat{a}_{lead}$ is estimated using a full order observer, $\dot{\tilde{\hat{a}}}_{lead} = \dot{\hat{a}}_{lead}$, when $\hat{a}_{lead}$ is estimated using a reduced order observer, their difference is proportional to the error in the estimate of the lead platoon acceleration.

The dynamics of $\tilde{\Gamma}$ under Eq. (17) becomes

$$\dot{\tilde{\Gamma}} = -\beta e - \lambda_2 \tilde{\Gamma} - \left(\frac{\partial \Gamma}{\partial v_{lead}} + \frac{\partial \Gamma}{\partial \hat{a}_{lead}} d_1 \right) \tilde{a}_{lead}.$$

Define

$$\boldsymbol{g} = \begin{pmatrix} -\dfrac{\partial v_d}{\partial v_{lead}} \\ -\dfrac{\partial \Gamma}{\partial v_{lead}} - d_1 \dfrac{\partial \Gamma}{\partial \hat{a}_{lead}} \end{pmatrix}$$

$$= \begin{pmatrix} -\dfrac{\partial v_d}{\partial v_{lead}} \\ -(\lambda_1 + d_1)\dfrac{\partial v_d}{\partial v_{lead}} - \dfrac{\partial v_d}{\partial \Delta x} - \dfrac{\partial^2 v_d}{\partial \Delta x \partial v_{lead}}(v_{lead} - v_{trail}) - \dfrac{\partial^2 v_d}{\partial v_{lead^2}} \hat{a}_{lead} - \dfrac{\partial^2 v_d}{\partial v_{trail} \partial v_{lead}} a_{trail} \end{pmatrix},$$

the combined dynamics of e *and* $\tilde{\Gamma}$ and are given by

$$\frac{d}{dt}\begin{pmatrix} e \\ \tilde{\Gamma} \end{pmatrix} = \begin{pmatrix} -\lambda_1 & 1 \\ -\beta & -\lambda_2 \end{pmatrix} \begin{pmatrix} e \\ \tilde{\Gamma} \end{pmatrix} + \boldsymbol{g}\tilde{a}_{lead}. \tag{18}$$

Notice that the state evolution matrix in Eq. (18) is stable when λ_1, λ_2 and β are positive. The design values of these parameters can be obtained by minimizing the effect of $\tilde{a}_{lead}$ on e using linear methods and assuming constant values of $\boldsymbol{g}$.

3.2. Lead Platoon Acceleration Observer

The velocity profile tracking controller uses the estimate of the acceleration of the lead platoon that, by assumption 2, is not measured. An observer to estimate this lead platoon acceleration is presented. This observer estimates the relative position between platoons and the velocity and acceleration of the lead platoon. The formulation presented here is different from that in [5] in that the absolute position of the lead platoon is no longer used. This produces a more robust numerical calculation of the lead platoon acceleration. The effects on velocity tracking due to the saturation of the jerk of the lead platoon are also considered now.

According to assumption 2, the dynamics of the jerk of the lead platoon

$$\frac{d^3}{dt^3} x_{lead} = j_{lead}, \tag{19}$$

acts as a disturbance. Defining

$$\boldsymbol{x} = \begin{pmatrix} \Delta x \\ v_{lead} \\ a_{lead} \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} \Delta x \\ v_{lead} \end{pmatrix}, \quad \boldsymbol{u} = \begin{pmatrix} v_{trail} \\ j_{lead} \end{pmatrix}$$

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \boldsymbol{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

Eq. (19) is included in

$$\dot{\boldsymbol{x}} = \boldsymbol{A}x + \boldsymbol{B}\boldsymbol{u}, \qquad \boldsymbol{y} = \boldsymbol{C}x.$$

It is straightforward to check that $(\boldsymbol{A}, \boldsymbol{B})$ is controllable and $(\boldsymbol{A}, \boldsymbol{C})$ is observable.

An observer for the lead platoon acceleration is

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{x} - \boldsymbol{L}(y - \boldsymbol{C}\hat{x}) + \boldsymbol{q},$$
$$\hat{a}_{lead} = (0\,0\,1)\,\hat{x}, \tag{20}$$

where $\hat{\boldsymbol{x}} = (\Delta\hat{x},\ \hat{v}_{lead},\ \hat{a}_{lead})^T \in \mathscr{R}^3$ is the state estimate, the observer gain $\boldsymbol{L} \in \mathscr{R}^{3\times 2}$ is such that $\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C}$ is asymptotically stable, and $\boldsymbol{q} = \boldsymbol{q}(t)$ is a tuning function to be determined.

The dynamics of the acceleration estimation error $\tilde{a}_{lead}$ is given by

$$\dot{\tilde{\boldsymbol{x}}} = (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\,\tilde{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} - \boldsymbol{q},$$
$$\tilde{a}_{lead} = (0\,0\,1)\,\tilde{x}, \tag{21}$$

where $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$.

3.3. Stability Analysis

In the stability analysis for the velocity tracking controller, the designs for the trail platoon jerk control and the lead platoon acceleration observer are combined. The value of the observer function, $\boldsymbol{q}$, is related with the value of the nonlinear term in the jerk control, $\boldsymbol{g}$, in such a way that stable behavior can be obtained for both the error dynamics of $(e, \tilde{\Gamma})^T$ in Eq. (18) and the estimation error, $\tilde{a}_{lead}$, in Eq. (21).

Assumption 3

1. *The jerk of the lead platoon is bounded, i.e.,* $\|j_{lead}(\cdot)\|_\infty \le j_{max}$.
2. *The maximum braking jerk of the lead platoon can be sustained for at most d seconds.*

The following theorem establishes a bound on the velocity tracking error $e(t) = v_{trail}(t) - v_d(\Delta x(t), v_{lead}(t), v_{trail}(t))$, when the observer is used to estimate the acceleration of the lead platoon.

Theorem 1 *Let the dynamics of* $\boldsymbol{s} = (e, \tilde{\Gamma})^T$ *be given by Eq. (18) with* λ_1, λ_2, $\beta > 0$ *and the dynamics of lead platoon acceleration estimation error,* $\tilde{a}_{lead}$, *be given by Eq. (21). Choose the control law for the jerk of the trail platoon according with Eqs. (12) and (17). Then, under assumptions 2 and 3, there is a time* t_1 *such that for any* $t \geq t_1$ *and any* $\epsilon > 0$ *the velocity tracking error of the trail platoon* $e(t) = v_{trail}(t) - v_d(\Delta x(t), v_{lead}(t), v_{trail}(t))$ *satisfies*

$$|e(t)| \leq v_f(1 + \epsilon); \quad v_f > 0. \tag{22}$$

Moreover if assumption 3.2 holds, then

$$lim_{t \to \infty} |e(t)| = 0.$$

Proof: First notice that λ_1, λ_2, $\beta > 0$ implies that the matrix

$$\boldsymbol{F} = \begin{pmatrix} -\lambda_1 & 1 \\ -\lambda_2 & \beta \end{pmatrix},$$

is stable.

Define $\boldsymbol{A}_F = \boldsymbol{A} - \boldsymbol{LC}$ to be the stable evolution matrix of the full order observer in Eq. (21). Let $\boldsymbol{Q} \in \mathscr{R}^{2\times 2}$, and $\boldsymbol{P} \in \mathscr{R}^{3\times 3}$ be positive definite symmetric matrices that satisfy the Lyapunov equations

$$\boldsymbol{QF} + \boldsymbol{F}^T\boldsymbol{Q} = -2\boldsymbol{C}_1; \qquad \boldsymbol{PA}_F + \boldsymbol{A}_F^T\boldsymbol{P} = -2\boldsymbol{C}_2,$$

where $\boldsymbol{C}_1 \in \mathscr{R}^{3\times 3}$ and $\boldsymbol{C}_2 \in \mathscr{R}^{2\times 2}$ are positive definite matrices.

Consider the Lyapunov function

$$V = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{Qs} + \gamma\frac{1}{2}\tilde{\boldsymbol{x}}^T\boldsymbol{P}\tilde{\boldsymbol{x}}, \tag{23}$$

where $\gamma > 0$. The time derivative of Eq. (23) is

$$\begin{aligned}\dot{V} = &-\boldsymbol{s}^T\boldsymbol{C}_1\boldsymbol{s} - \gamma\tilde{\boldsymbol{x}}^T\boldsymbol{C}_2\tilde{\boldsymbol{x}} + \boldsymbol{s}^T\boldsymbol{Qg}(0\,0\,1)\tilde{\boldsymbol{x}} - \gamma\tilde{\boldsymbol{x}}^T\boldsymbol{Pq} + \gamma v_{trail}\boldsymbol{B}_{\cdot 1}^T\boldsymbol{P}\tilde{\boldsymbol{x}} \\ &+ \gamma j_{lead}\boldsymbol{B}_{\cdot 2}^T\boldsymbol{P}\tilde{\boldsymbol{x}},\end{aligned} \tag{24}$$

after substitution of Eqs. (18) and (21) into Eq. (24). $\boldsymbol{B}_{\cdot i}$ $i = 1,2$ denotes the i-th column of matrix $\boldsymbol{B}$.

Choose the tunning function $\boldsymbol{q}$ in Eq. (21) to be

$$\boldsymbol{q} = \frac{(\boldsymbol{g}^T\boldsymbol{Qs})}{\gamma}\boldsymbol{P}^{-1}\begin{pmatrix}0\\0\\1\end{pmatrix} + \frac{1}{\gamma}\boldsymbol{B}_{\cdot 1}v_{trail}. \tag{25}$$

Then

$$\dot{V} = -\boldsymbol{s}^T\boldsymbol{C}_1 s - \gamma\tilde{\boldsymbol{x}}^T\boldsymbol{C}_2\tilde{\boldsymbol{x}} + \gamma j_{lead}\boldsymbol{B}_{\cdot 2}^T\boldsymbol{P}\tilde{\boldsymbol{x}}. \tag{26}$$

Let $\boldsymbol{F} = \boldsymbol{T}_1\ \Lambda_1\ \boldsymbol{T}_1^{-1}$ and $\boldsymbol{A}_F = \boldsymbol{T}_2\ \Lambda_2\ \boldsymbol{T}_2^{-1}$ be the real Schur decomposition [8] of $\boldsymbol{F}$ and $\boldsymbol{A}_F$ respectively. Pick

$$-2\boldsymbol{C}_1 = \boldsymbol{T}_1^{-T}\left(\Lambda_1 + \Lambda_1^T\right)\boldsymbol{T}_1^{-1}, \quad -2\boldsymbol{C}_2 = \boldsymbol{T}_2^{-T}\left(\Lambda_2 + \Lambda_2^T\right)\boldsymbol{T}_2^{-1}. \tag{27}$$

It can be shown that

$$\boldsymbol{Q} = \boldsymbol{T}_1^{-T}\boldsymbol{T}_1^{-1}, \quad \boldsymbol{P} = \boldsymbol{T}_2^{-T}\boldsymbol{T}_2^{-1}. \tag{28}$$

Using Eqs. (27) and (28) in Eq. (26)

$$\dot{V} = \boldsymbol{s}^T\left(\boldsymbol{QF} + \boldsymbol{F}^T\boldsymbol{Q}\right)\boldsymbol{s} + \gamma\tilde{\boldsymbol{x}}^T\left(\boldsymbol{PA}_F + \boldsymbol{A}_F^T\boldsymbol{P}\right)\tilde{\boldsymbol{x}} + \gamma j_{lead}\boldsymbol{B}_{.2}^T\boldsymbol{P}\tilde{\boldsymbol{x}}. \tag{29}$$

Hence, it follows that

$$\dot{V} \le -2\zeta V + \gamma\tilde{\boldsymbol{x}}^T\boldsymbol{P}_{.3}j_{lead}, \tag{30}$$

where $-\zeta$ is minimum real part of the eigenvalues of $\boldsymbol{F}$ and $\boldsymbol{A}_F$ and $\boldsymbol{P}_{.3}$ is the third column of $\boldsymbol{P}$.

From (23) it follows that

$$e^2 \le \rho V, \quad \text{where} \quad \rho = \frac{2Q_{22}}{\det(\boldsymbol{Q})} \tag{31}$$

and

$$\left(\boldsymbol{P}_{.3}^T\tilde{\boldsymbol{x}}\right)^2 \le \frac{\delta}{\gamma}V, \quad \text{where } \delta = 2P_{33} \tag{32}$$

with Q_{ij} and P_{ij} the (i,j)th elements of the matrices $\boldsymbol{Q}$ and $\boldsymbol{P}$, respectively.

The time derivative of the square root of Eq. (23) is

$$\frac{d}{dt}\left(V^{\frac{1}{2}}\right) = \frac{1}{2}V^{-\frac{1}{2}}\dot{V} \tag{33}$$

Using Eqs. (30), (31) and (32) in Eq. (33) and by assumption 3

$$\frac{d}{dt}\left(V^{\frac{1}{2}}\right) \le -\zeta V^{\frac{1}{2}} + \sqrt{\frac{\delta}{\gamma}}\,j_{max}. \tag{34}$$

Hence, from Eqs. (31) and (34), it follows that for any initial conditions $(\mathbf{s}(0), \tilde{\boldsymbol{x}}_{lead}(0))$, and for any $\epsilon > 0$, there is a time t_1 s.t. if $t \ge t_1$,

$$\sqrt{\frac{1}{\rho}}\,|e(t)| \le V^{\frac{1}{2}}(t) \le \frac{j_{max}}{\zeta}\sqrt{\frac{\delta}{\gamma}}\,(1+\epsilon),$$

Therefore, after a long enough time

$$|e(t)| \le v_f(1+\epsilon),$$

where

$$v_f = \frac{j_{max}}{\zeta}\sqrt{\frac{\delta\rho}{\gamma}}\ .$$

Now consider assumption 3.2, $j_{lead}(t) = 0;\ \forall t \geq d$. Therefore from Eq. (30)

$$\dot{V}(t) \leq -2\zeta V(t) \leq 0; \quad \forall t \geq d.$$

4. REGULATION LAYER SIMULATION RESULTS

The control laws simulation results shown here are from a Matlab program that simulates just two adjacent platoons involved in a maneuver. The program was written to test the control laws for different behaviors of the platoon ahead. The control used was the velocity tracking controller presented in Section 3. Most of the control laws were also implemented in SmartPath [9]. The results of both Matlab and SmartPath simulations were the same concerning vehicle safety and performance. The values for most parameters values in the simulations are shown in Table 1.

Five plots are included for each simulation:

1. Relative distance $\Delta\dot{x}$ vs. time.
2. Relative velocity $\Delta\dot{x}$ vs. time.

Table 1. Parameters used for the simulations.

$a_{com} = \pm 2\ m/s^2$	This is the value used in the current merge [10]. It is commonly accepted in the literature. See [11,12].
$\mu = 1.12$	This value was derived graphically from the results for the follower law in [4].
$A_{MIN} = 5\ m/s^2$	This is the absolute value of the maximum deceleration. This value is used in the current merge.
$A_{MAX} = -2.5\ m/s^2$	This is a rough approximation based on data presented in [13]. The road is assumed to be flat. The vehicles are assumed to have automatic transmissions in third gear.
$j_{com} = \pm 2.5\ m/s^3$	Lygeros and Godbole [10] set the comfortable jerk limit at 5 m/s^3 in the current merge. Most examples in the literature suggest the limit is between 2 m/s^3 and 2.5 m/s^3. See [11,14,12].
$j_{max} = -50\ m/s^3$	This value was selected as a physical limit on jerk. It is less than the one given in [15].
$\Delta x_{join} = 1\ m$	This is the current intraplatoon spacing.
$\Delta x_{split} = 60\ m$	This is the current interplatoon distance.
$\Delta x_{range} = 91\ m$	This value corresponds to the maximum range of the sensor currently used in PATH.
$d = 30\ ms$	Simple brake models often include pure time delays of about 50 ms. It is shown in [16], however, that delays in the current braking system for PATH are greater than 150 ms. By redesigning the brake system, delays near 20 ms could be achieved [17]. Delays from sensing, filtering and differentiating are also possible, but they could be small at a high sample rate.

3. Acceleration of the trail platoon vs. time.
4. Jerk of the trail platoon vs. time.
5. Phase portrait in the $\Delta x - \Delta \dot{x}$ plane. The plot includes the two safety boundaries defined in Section 2 of [2], ∂X_{safe} and ∂X_{bound}. The controller reference is obtained by reducing ∂X_{safe} by a constant factor to account for discrete time and controller tunning effects.

4.1. Simulations with no collisions allowed ($v_{allow} = 0$)

The following set of results uses the approach presented in this paper to produce regulation layer maneuvers in which not even low speed collisions will occur. For these simulations the remaining parameters are shown in Table 2.

Fig. 4 shows results for a merge from 30 m initial spacing. The velocity of the platoon ahead was constant at 25 m/s. The maneuver was completed in 11.9 s. Jerk and acceleration comfort limits were not exceeded. The final relative velocity is not zero as the simulation only ran to the point where the follower law takes effect.

Fig. 5 shows results from a merge with an initial spacing of 60 m. The lead platoon maintained a constant velocity. The merge took 16.5 s in this case.

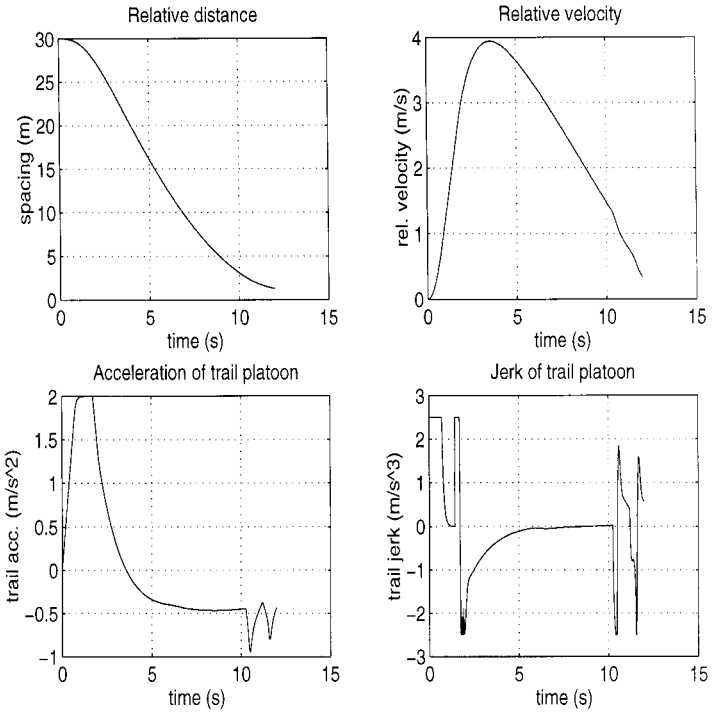
Fig. 6 show the case in which the lead platoon applies maximum braking when the trail platoon has maximum relative velocity. Note that the simulation shows no collisions, as expected. This figure includes a large spike in jerk. The controller is designed so that comfort limits are disregarded when safety becomes critical. In these cases, the comfort jerk was overridden once the large lead platoon deceleration was detected.

In the final merge simulation, the lead platoon braked at comfortable deceleration. No collision occurred. The results are shown in Fig. 7. It should be noticed that in the last part of the maneuver the acceleration of the trail platoon exceeded the comfort limit. The controller was designed to allow this behavior in order to avoid collisions.

The split law was also simulated. Figs. 8 and 9 show the results of split from 1 and 30 m to 60 m spacing respectively. The cases when the lead platoon applies comfort and full braking while the trail platoon is attempting a split are shown in Figs. 10 and 11 respectively.

Table 2. Additional parameters used for the simulations in the no collision case.

$v_{allow} = 0\ m/s$	Desired condition for normal mode of operation in AHS.
$\alpha_c = 1.15$	Value obtained from [2].
$a_{min}^{trail} = A_{MIN}/\mu = 4.46\ m/s^2$	For the join and split laws.
$a_{min}^{trail} = A_{MIN}/\mu^2\ \alpha_c = 3.4661$	For the leader law.
$a_{min}^{lead} = A_{MIN}/\mu\alpha_c = 3.88\ m/s^2$	For the join and split laws.
$a_{min}^{lead} = A_{MIN}$	For the leader law.
$a_{max}^{trail} = 2.0\ m/s^2 < A_{MAX}$	
$v_{max} = 25\ m/s \approx 55\ mi/hr$	



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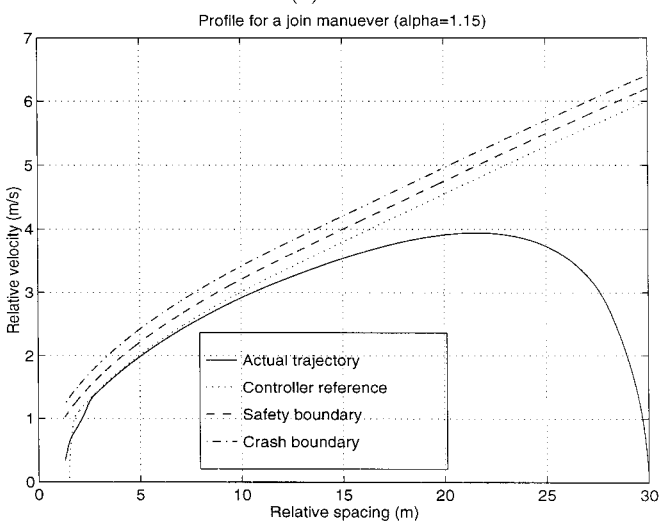
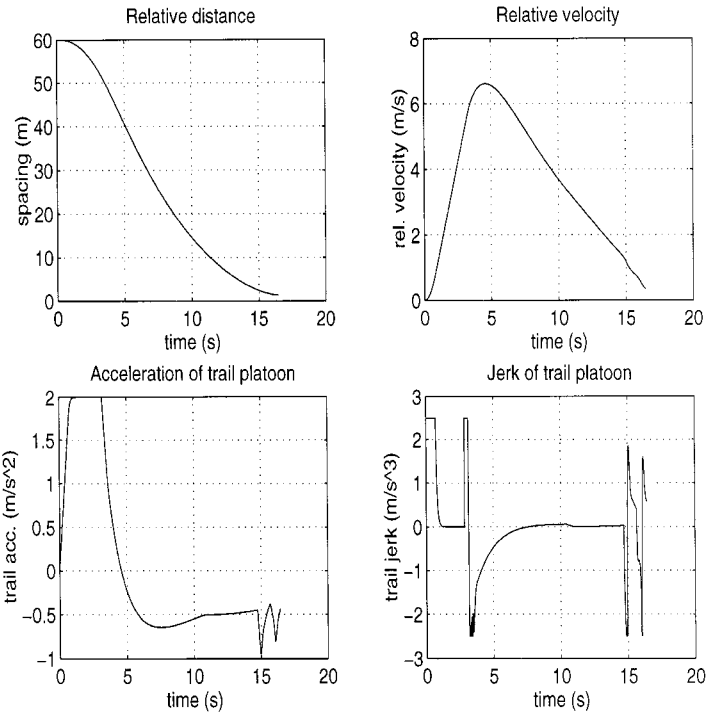


(b)

Fig. 4. Simulation results of merge from 30 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.



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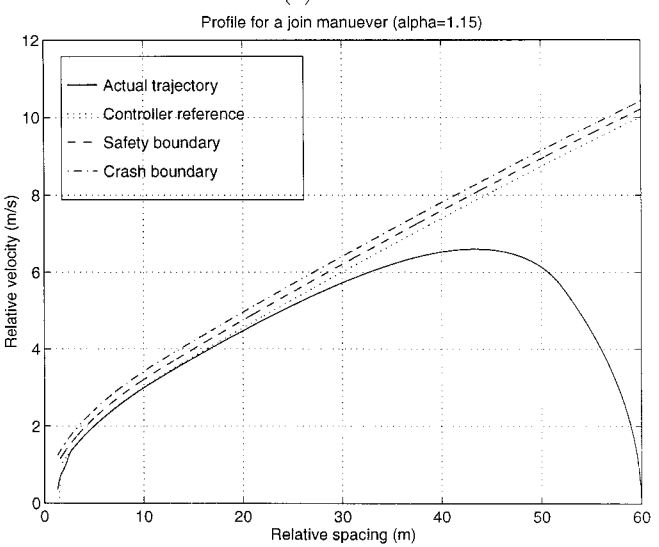
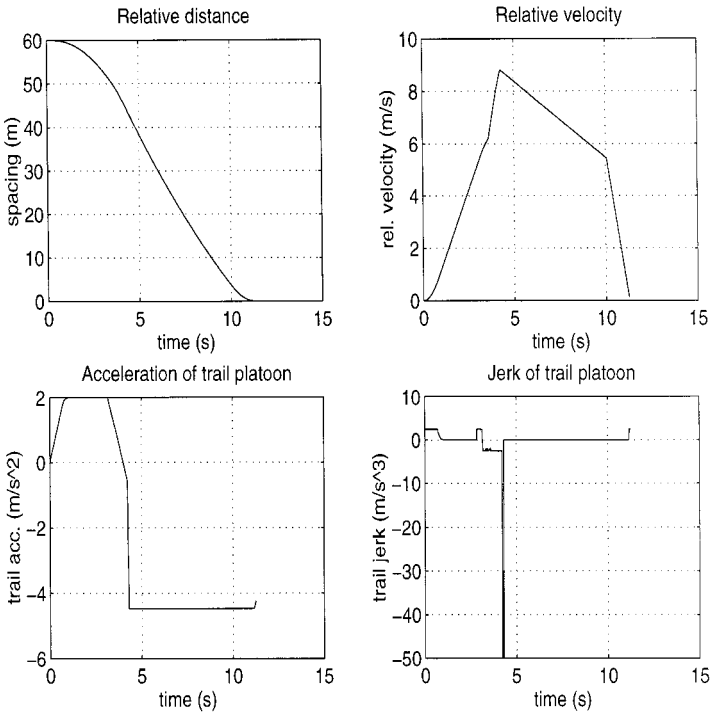


(b)

Fig. 5. Simulation results of merge from 60 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.



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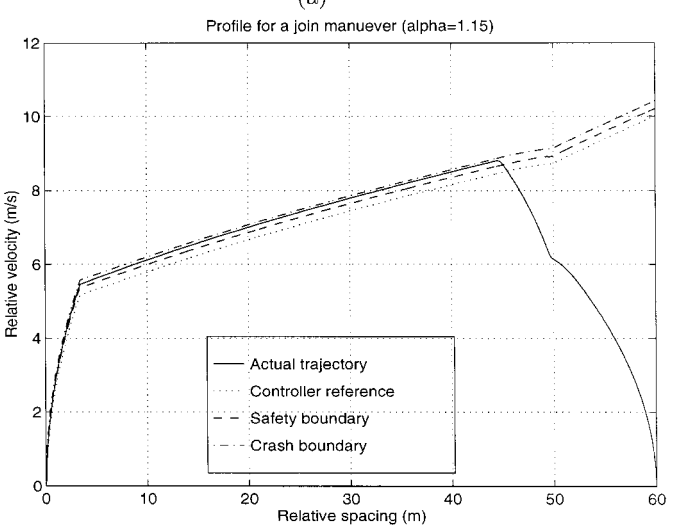

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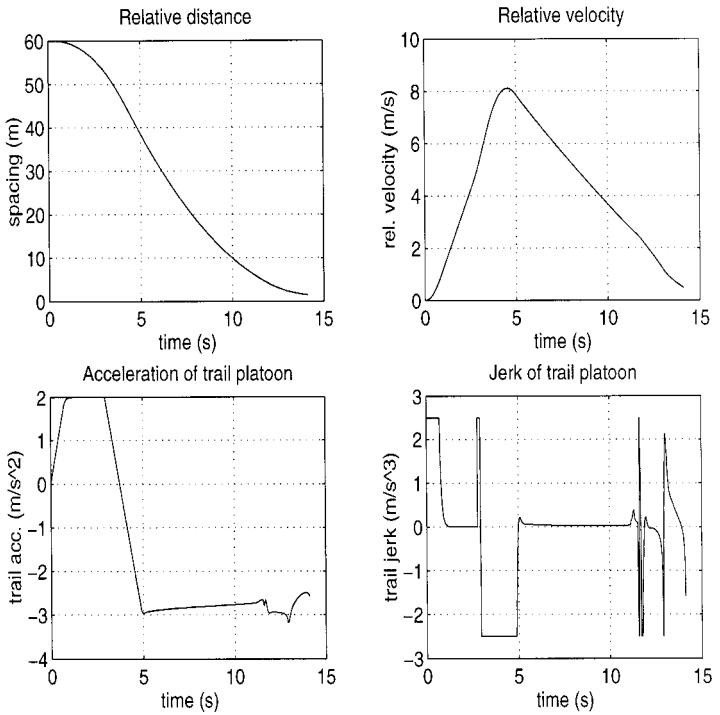
Fig. 6. Simulation results of merge from 60 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. The lead platoon applied maximum braking at 3.5 s. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.



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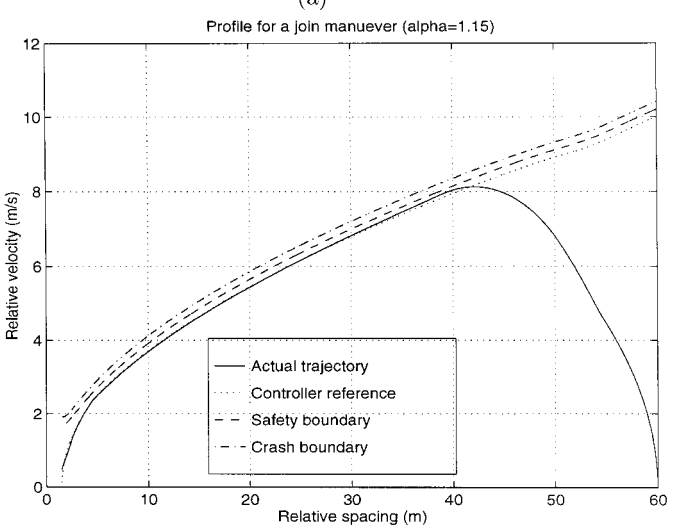


(b)

Fig. 7. Simulation results of merge from 60 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. The lead platoon applied comfort braking at 4.1 s. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.

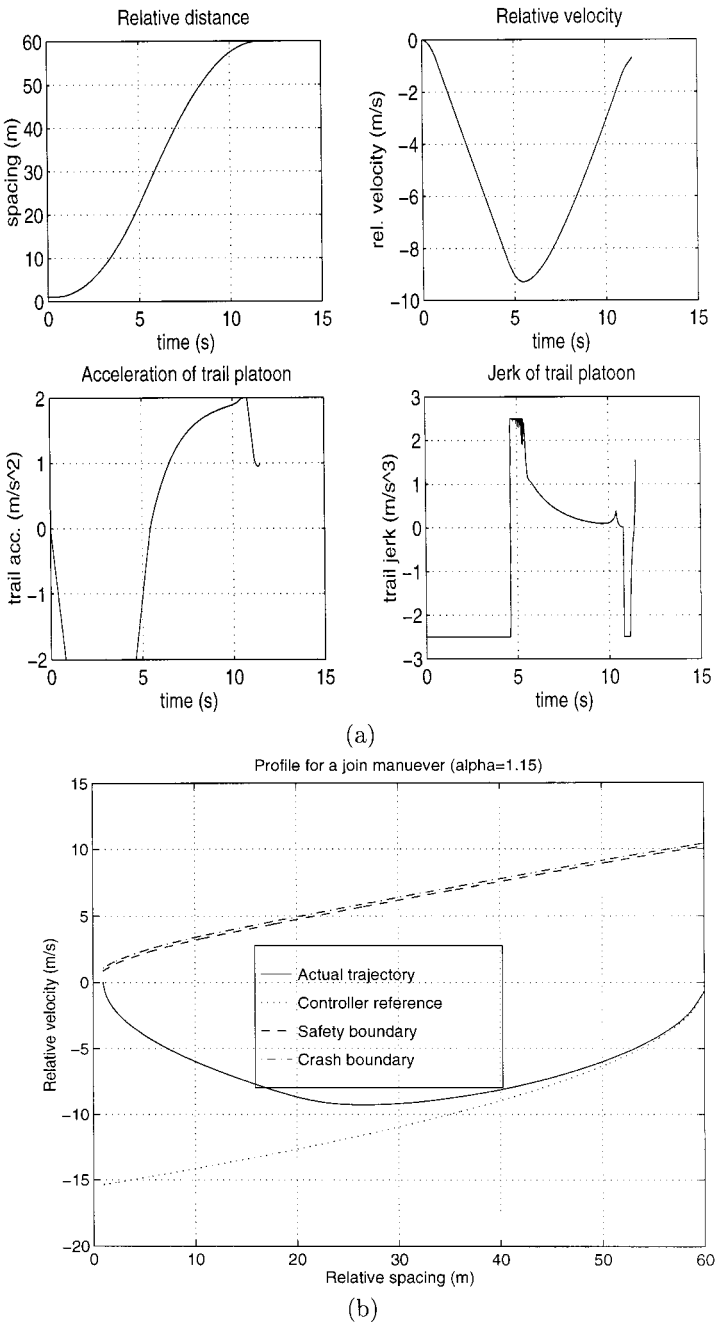
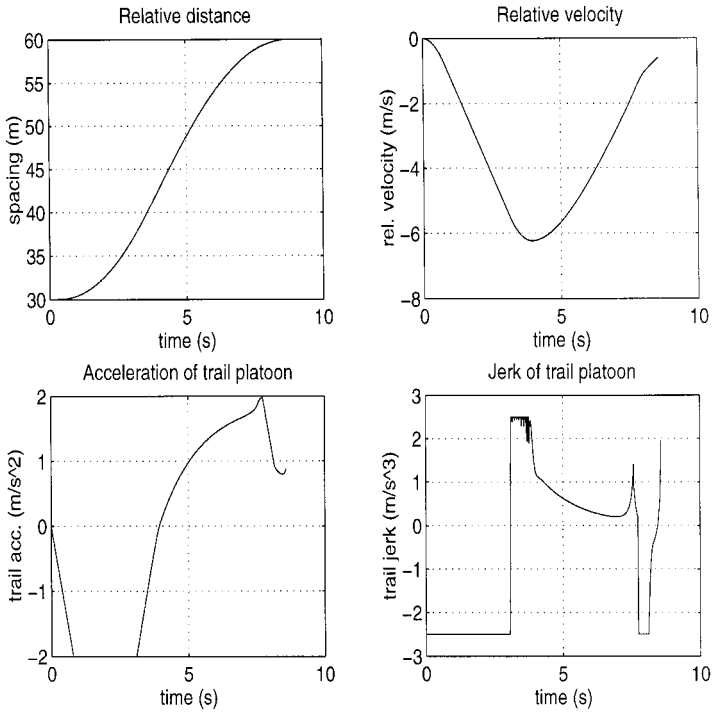


Fig. 8. Simulation results of split from 1 to 60 m spacing: The initial velocity of both platoons was 25 m/s. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.



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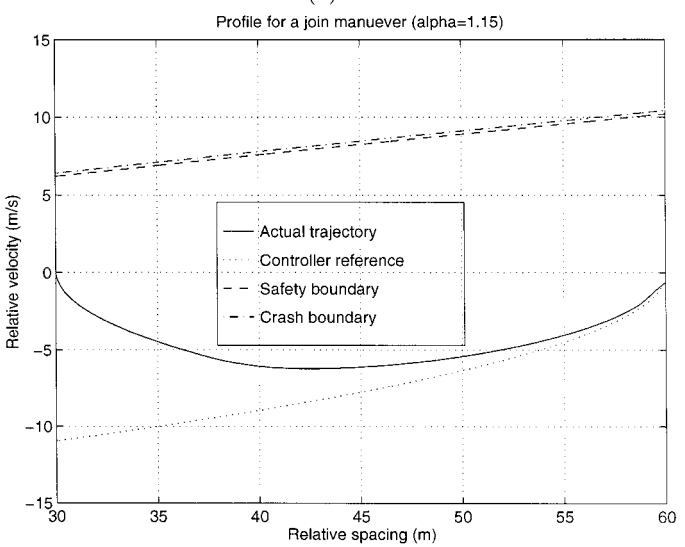

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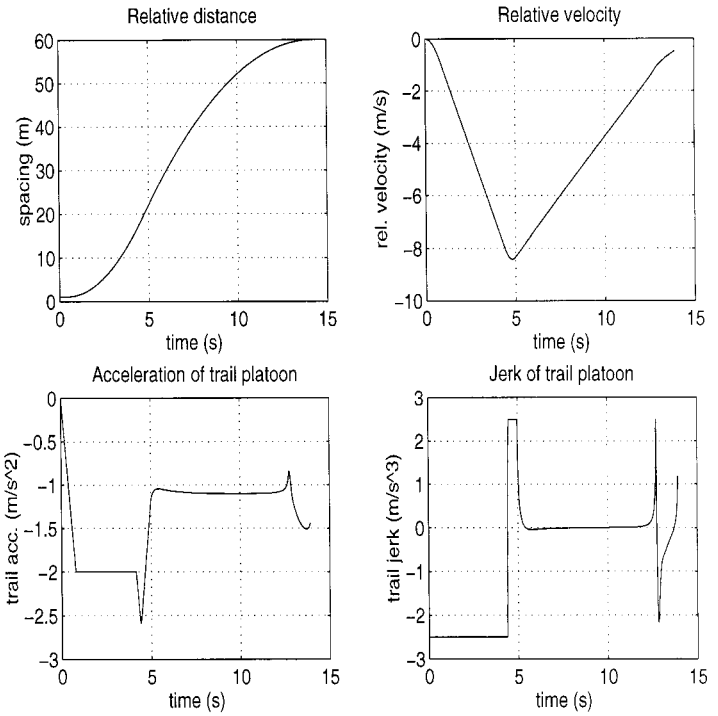
Fig. 9. Simulation results of split from 30 to 60 m spacing: The initial velocity of both platoons was 25 m/s. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.



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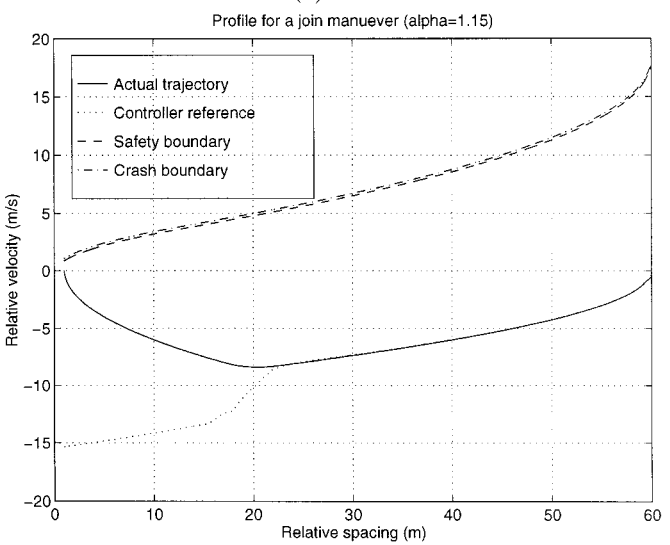
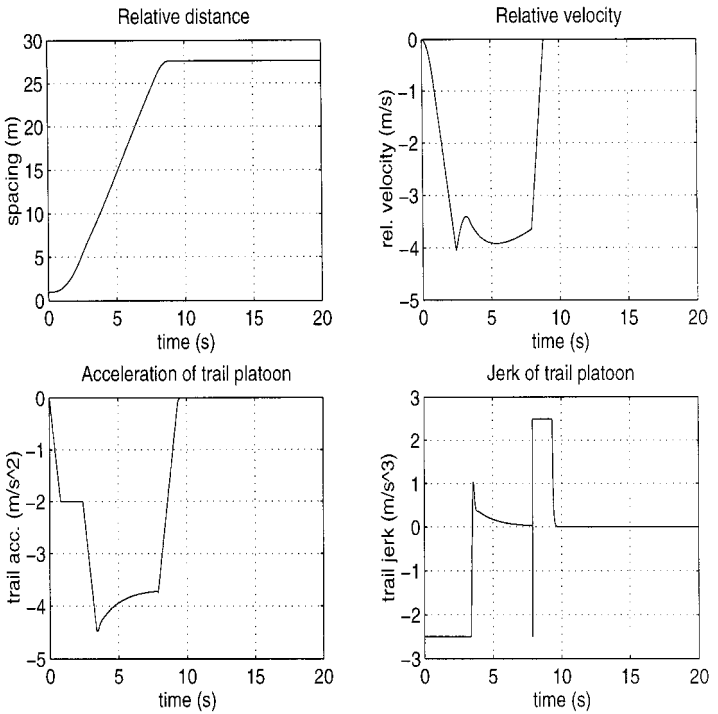


(b)

Fig. 10. Simulation results of split from 1 to 60 m spacing: The initial velocity of both platoons was 25 m/s. The lead platoon applies comfort braking at $\Delta x = 15$ m. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.



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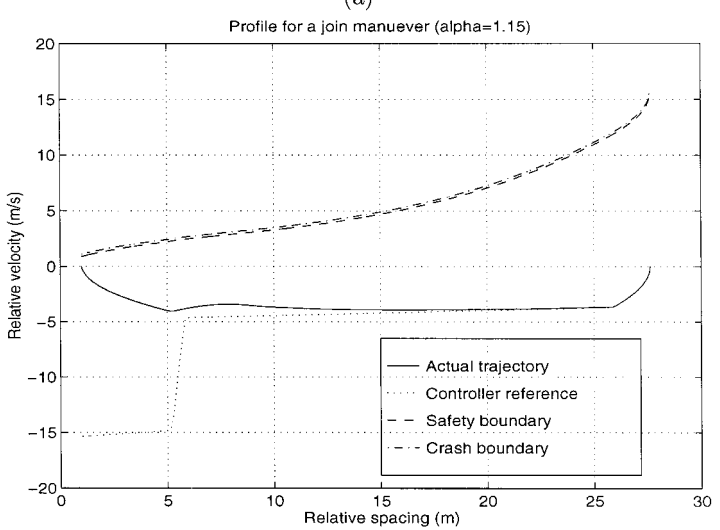
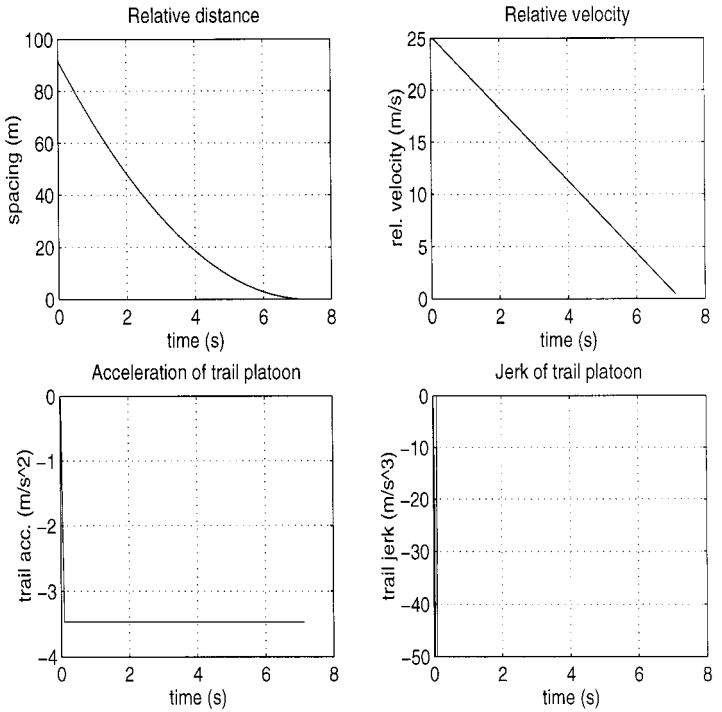


(b)

Fig. 11. Simulation results of split from 1 to 60 m spacing: The initial velocity of both platoons was 25 m/s. The lead platoon applies maximum braking at $\Delta x = 5$ m. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.



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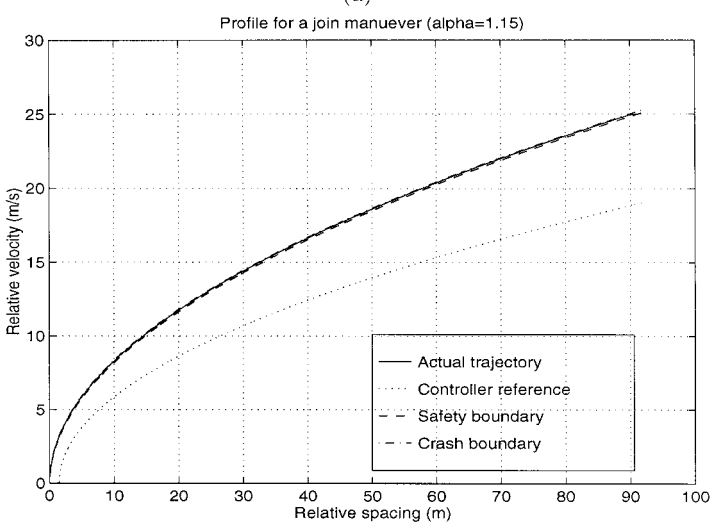

(b)

Fig. 12. Simulation results of the leader law. A platoon is traveling at 25 m/s and detects an stopped platoon 90 m in front of it. a) Results vs. time. b) Results in the phase plane Δx vs. $\Delta \dot{x}$.

Table 3. Effect of the time delay in the time for maneuvering completion.

Maneuver	Delay (s)			
	0.03	0.05	0.20	0.15
Join from 30 m	11.9	12.5	14.7	17.1
Join from 60 m	16.4	17.1	19.4	22.1

Table 4. α_M vs. relative distance sensor range.

Range	α_M
40	0.50
50	0.63
60	0.76
70	0.88
80	1.01
90	1.14
100	1.27
110	1.40

The last simulation results in Fig. 12 correspond to the leader law. An extreme case was simulated. The lead platoon detects a stopped platoon in front of it, while traveling at maximum speed. Notice that in this case the controller reference is calculated assuming comfort braking levels that are not possible in this situation.

Table 3 shows the effect of the delay d in the time for completion of a join maneuver and Table 4 the effect of the range of the relative distance sensor in the value of α_M.

The results for other control laws are not shown, since they are similar to the ones already presented.

5. CONCLUSIONS

The design of safe velocity tracking controller for the regulation layer control laws for the hierarchical architecture of [1] is presented. The notion of safety is that no platoon is allowed to collide with the platoon ahead of it at a relative velocity greater than a prescribed limit. The results show that for a safe normal mode operation of AHS, it is necessary to establish bounds on the parameters that determine the vehicle's behavior during the execution of the regulation layer maneuvers. These bounds allow to rule out out the cases reported in [18] in which safety can be compromised because of platoons' different braking capabilities.

Based on the safe platooning analysis presented in the companion paper [2], velocity profiles are derived for all the single lane maneuvers. These profiles are described in the state space of the platoons' relative motion and are therefore suited for feedback control implementation. A non-linear feedback velocity tracking controller is presented. This controller allows the maneuvers to be completed

in minimum time and with comfort values of jerk and acceleration, whenever safety is not compromised. The simulation results presented illustrate the effectiveness of the designed control laws.

The approach presented here to design control laws for maneuvers in the normal mode of operation of the regulation layer is also being applied to the degraded mode maneuvers. Simulation results are presented.

ACKNOWLEDGEMENTS

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