Fundamental Diagram Modelling from NGSIM Data

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Abstract: This paper is to investigate lane-wise flow-density (or equivalently speed-density) relationship which is generally called Fundamental Diagram (FD) over a stretch of homogeneous freeway section using the microscopic NGSIM data (Alexiadis, 2004). Particularly, it investigates how a homogenous traffic breakdown through data analysis and modeling. The breakdown of a homogenous traffic is understood as the significant flow drop and density increase with noticeable shock-wave back-propagation. The corresponding density is a generalization of the critical density for free-flow. Variable structure models with two limbs are proposed to model the homogenous flow and its further breakdown. A special Generalized Polynomial Model (with fraction coefficients) is also proposed for the right limb. Properly aggregated NGSIM data are used to fit the model with results compared with some other models over time at fixed location using Root Mean Square Errors (RMSE) as measure.

Keywords: Fundamental Diagram, variable structure modeling, speed-density relationship, equilibriums state, homogenous flow, NGSIM data

1. INTRODUCTION

In the nominal work of Greenshields (1934), the Fundament Diagram (FD) was defined and used as the relationship between traffic flow q and density ρ for an equilibrium traffic state. Sine then, several works have been conducted to establish a static relationship between flow and density in theory and in empirical modelling with filed data fitting. It is generally recognized that FD is location dependent due to road geometry and traffic characteristics. FD may have several equivalent forms: flow-density (occupancy) which is concave, speed-density (occupancy) which is monotone decreasing, and speed-flow with two foliations: upper limb and lower limb. It is noted that the speed-u/c (volume/capacity ratio) relationship in HCM (Highway Capacity Manual) is equivalent to a speed-flow relationship. Data aggregation level in time for modelling determines the application of the FD model: short time aggregated data leads to model for traffic operation, and long time aggregated data lead to model for planning (Skabardonis, 1977). Although some models are for planning purpose, some are for operation, and several models could be used for both purpose depending on the time aggregation level. In fact, several models calibrated on the same set of field data could produce the similar outputs for the same input data with similar level of model mismatch error such as Mean Square Error (MSE). A good model should be flexible enough to capture the intrinsic functional relationship for a range of field data.

Lighthill and Whitman (1995) investigated the FD in $q-\rho$ (flow-density) plane and suggested that the FD should have a flat top as depicted by Greenshields (1934). Del Castillo et al (1995) considered the functional form of the speed-density relationship. The FD can be obtained from $q = v \cdot \rho$ which is true if v is distance means speed and if the data aggregation level for the three are the same.

The FD is the basis of several traffic flow modelling approaches. A generally accepted model is the macroscopic LWR model (Lighthill and Whitham, 1955) which has been used for traffic simulation and control. For practical calculation, the model is usually simplified based on the assumption of the existence of an FD. Thus the speed is eliminated from the LWR model with the density left as the only traffic state variable for the dynamic system, as in the Cell Transmission Model (CTM) (Daganzo, 1994, 1995). A primary result of that paper is that the CTM is equivalent to the LWR hydrodynamic model based the existence of the FD and the assumption of homogeneous highway traffic. For non-homogeneous highway traffic (phase transition), the equivalence would be difficult to prove.

Beside the importance for model simplification, FD is also used to estimate some critical traffic parameters such as capacity/critical flow, critical/jammed density, etc.

provided that the FD truly reflects the intrinsic traffic characteristics.

Controversial opinions regarding the existence and forms of the FD are well-known. Different forms of FD have been suggested: (a) the simplest form - a triangle; (b) reversed parabolic and reversed Gambell; and (c) a reversed λ etc. However, it is generally agreed that $q = q(\rho)$, if it exists, is a concave function of density ρ defined in $[0, \rho_J]$ (ρ_J – *jam density*) and boundary conditions $q(0) = q(\rho_J) = 0$; and the corresponding speed-density relationship is monotone decreasing.

Critical density ρ_c is originally defined as the density at which free-flow traffic is to breakdown. It is well-known that traffic may have infinite number of equilibrium state which can be described as homogeneous flow sustainable for certain period of time with mean speed within the interval $(0, v_f]$ (v_f – free-flow speed), in which speed and density (thus flow) are close to some constants. However, the traffic could further breakdown (or transition) from one equilibrium state to another with *lower flow*. An strong evidence for this is the shockwave observed through NGSIM data which are collected in peak hours (Lu and Skabardonis, 2007**)**. Traffic breakdown from free-flow is to transit from free-flow (a special homogenous flow) to a congested flow. The objective of *active traffic control* is to achieve the following in the order of priority and feasibility: (i) to smoothly transit to equilibrium state with higher flow; (ii) to keep it a homogenous flow (without shockwave); (iii) to smoothly transit to a homogeneous traffic with lower flow if it is unavoidable (without shockwave). For this purpose, it is not only necessary to understand how the traffic transit (breakdown) to a equilibrium state with lower flow, but also necessary to understand the mechanism and characteristics how to transit smoothly from one equilibrium state to another. The *break-down density* is denoted as ρ_c^h for *homogenous traffic state h, which* is understood as the density with abrupt flow drop and density increase, such as, with shock-wave back-propagation.

The most accepted FD model from a practical traffic data viewpoint is a reversed λ shape with the two limbs, which has also been observed in NGSIM data analysis. The left limb of the FD corresponds to the homogenous-flow (free-flow as a special case) which is close to a straight line and it matches pretty well with practical data. However, different ideas arise as to what shape the right limb corresponding to congested traffic should look like. Notice the following facts: (a) practical data are scattered for the congested flow as shown by many studies; (b) jammed density is fixed as generally accepted, for example, $\rho_J = 230$ vehicle per lane per mile (vplpm), at which $v(\rho_J) = 0$ and $q(\rho_J) = 0$ must hold; and (c) the other end of the right limb has to intersect with the left limb at critical density and maximum flow. Usually, the scattered data do not fit the right limb well with those constraints if the model is not properly chosen. To solve the problem, this paper proposes several choices for modelling the right limb.

The main contributions of the paper are: (a) A Generalized Polynomial Model with some non-integer coefficients is proposed for $v - \rho$ model, which produce concave $q - \rho$ relationship; Variable structure model is proposed for the reversed λ shape FD for homogenous traffic flow. The left limb includes linear and parabolic curves, and the right limb includes linear, Generalized Polynomial, and Edie models; (b) Those variable structure models are compared using the properly aggregated NGSIM data for both $v - \rho$ and $q - \rho$ relationships using Root-Means- Square Error (RMSE) as the performance measure; (c) The density range for further breakdown of the homogenous traffic recorded in NGSIM ρ_c^H is investigated with the available data sets. It shows that the range is $[90, 120]$ per-veh-per-mile for the recorded saturated traffic; this is might be significantly higher than the generally believed critical density range for the free-flow to breakdown;; and (d) The impact of the value for ρ_c^h in the range on modeling is investigated with RMSE for comparison of different model combinations. It shows that this value is different, though within the range, in times as well as in locations for the same model.

It is noted that Edie (1961) also suggested using a variable structure model to represent the traffic breakdown near maxim density ρ_m, where from field data, $\rho_m \in [75, 100]$ which was close to what is observed with NGSIM data.

The paper is structured as follows: Section 2 review previous work on Fundamental Diagram modeling and application; Section 3 proposes a Generalized Polynomial Model and investigate the temporal behaviour of several FD models for the same location using properly aggregated NGSIM data; Section 4 establishes some variable structure models which are calibrated using properly aggregated NGSIN data. Section 5 is for concluding remarks.

2. LITERATURE REVEW

Many models exist for modelling the static $v - \rho$ or $q - \rho$ relationship. Although the function expressions are different, they are more or less similar in the domain $\rho \in [0, \rho_J]$. However, some of them do not satisfy the two boundary conditions $v(0) = v_f, (\rho_J) = 0$ simultaneously.

2.1 Models for Speed-Density ($v - \rho$) relationship

(1) Edie Model

Edie (1961) Showed that the Greenberg model (1959)

$$v = V_m \ln\left(\frac{\rho}{\rho_J}\right) \quad (2.1)$$

can be obtained by integration of the following car-following model

$$M\ddot{x}_{n+1}(t) = \lambda_1 \frac{\left(\dot{x}_n(t+\Delta t) - \dot{x}_{n+1}(t+\Delta t)\right)}{\left(x_n(t+\Delta t) - x_{n+1}(t+\Delta t)\right)}$$

M – vehicle mass
λ_1 – driver sensitivity character coefficient
Δt – average time lag, a constant, for driver-car system
$x_n(t), x_{n+1}(t)$ – the coordinate of front vehicle and the subjective vehicle with respect to the an inertia coordinate system at time *t*. $x_n(t) - x_{n+1}(t)$ is the distance headway. It is noted that a constant Δt would not affect the integration. Using the relationship between average headway and density $\rho = 1/y = \frac{1}{x_n - x_{n+1}}$, the boundary condition $v(\rho_J) = 0$ and $V_m = \lambda_1 / M$, (2.1) is obtained. General Greenberg model is obtained by adding parameters for data fitting flexibility:

$$v(\rho) = g_1 + g_2 \ln\left(\frac{\rho}{\rho_J}\right) \quad (2.2)$$

It is pointed out the flaw of the Greenberg model is

$$\lim_{\rho \to 0} V_m \ln\left(\frac{\rho_J}{\rho}\right) = \infty$$

which means the model is not suitable for sparse traffic. Edie suggested a further improvement by starting the following car following model for the uncongested traffic

$$M\ddot{x}_{n+1}(t) = \lambda_1 \dot{x}_{n+1}(t) \frac{\left(\dot{x}_n(t+\Delta t) - \dot{x}_{n+1}(t+\Delta t)\right)}{\left(x_n(t+\Delta t) - x_{n+1}(t+\Delta t)\right)^2}$$

with the boundary condition

$v(0) = v_f$ (free-flow) if $\frac{1}{x_n - x_{n+1}} = \frac{1}{y} = 0$ or infinite spacing

One can reach the following model noticing the density and headway relationship $\rho = 1/y$:

$$\rho = \rho_m \ln\left(\frac{v_f}{v}\right)$$

$$\rho_m = 1/y_m$$

Or equivalently

$$v = v_f \exp\left(\frac{-\rho}{\rho_m}\right)$$

which is exactly the Underwood model (1961).
y_m – the spacing of maximum flow which is deduced by minimizing the $q(v)$; ρ_m – the density of maximum flow. Parameters w_1 and w_2 are added for flexibility in data fitting:

$$v = V(\rho) = \exp\left(-w_1 \frac{\rho}{\rho_J} + w_2\right) \quad (2.3)$$

It is strictly concave for $\rho \in [0, 2\rho_m)$.

(2) The following polynomial model is cited in (Zhang, 1999) as the one-parameter polynomial model:

$$v = v_f\left(1 - \left(\frac{\rho}{\rho_J}\right)^n\right) \quad (2.4)$$

where v_f - the free-flow speed; ρ_J - the jammed density. $n = 1$ is the Geenshields model (Geenshields , 1934).

(3) An exponential model used in (Hegyi et al, 2002):

$$V(\rho) = v_f \exp\left(-\frac{1}{a}\left(\frac{\rho}{\rho_c}\right)^a\right) \quad (2.5)$$

v_f – free-flow speed;
a – model parameter;
ρ_c – critical density, the same as the ρ_m used above.
It generalizes somehow the Underwood model.

(4) Some other models such as BPR model and Van Aerde model used in planning are referred to Skabardonis and Dowling (1997) and Van Aerde (1995).

3. GENERALIZED POLYNOMIAL MODEL FOR $v(\rho)$

Since the NGSIM data is microscopic from video camera instead of inductive loops, vehicle count and distance mean speed can be estimated directly. The $v - \rho$ relationship $v(\rho)$ is investigated. A new Generalized Polynomial model with non-integer power for FD is proposed for $v(\rho)$ with unit-sum and non-negative constraints on the coefficients.

3.1 A Generalized Polynomial FD Model

It can be shown that the most of the previous models can be approximated by or generalized to the following polynomial with non-negative coefficients and non-integer power:

$$v(\rho) = V_m \cdot \left[\sum_{i=1}^{P} a_i \left(\frac{\rho}{\rho_J}\right)^{b_i}\right] \quad (3.1)$$

$$b_i > 0, a_i \geq 0, \; i = 1, ..., P$$

To avoid any ambiguity, b_i, $i = 1, ..., P$ are assumed to be known real numbers and $b_i \neq b_j$, for $i \neq j$. (3.1) is called *Generalized Polynomial FD Model*. The nonnegative coefficients $(a_0, a_1, ..., a_P)$ are to be determined by fitting from practical data. In practice, one could choose lower or higher order instead of 6.

3.2 Concavity and Boundary Conditions

The concavity is also true for the Generalized Polynomial FD (3.1) in $q-\rho$ relationship for $\rho \in (0, \rho_J]$. In fact, it is easy to calculate that

$$\frac{d^2 q(\rho)}{d\rho^2} < 0 \;\text{ for }\; \rho > 0$$

which means that $q(\rho) = \rho \cdot V(\rho)$ is strictly concave for $\rho>0$ since the coefficients $(a_0, a_1, ..., a_P)$ are all nonnegative. It is clear that $v(0) = v_f$ the free-flow speed. $v(\rho_J) = 0$ leads to the constraint that $\sum_{i=0}^{P} a_i = 1.$ It is thus called Generalized Polynomial Model with Unit Sum Coefficients (GPMUSC).

Theorem 1. The generalized polynomial model (3.1) for the flow-density ($q-\rho$) relationship is strictly concave for $\rho>0$.

3.3 A Specific Model for $v(\rho)$

In practice, (3.1) may be too general. A special case of the Generalized Polynomial FD Model is proposed. It is noted that $\frac{\rho}{\rho_J} < 1$ for most interested cases, the power β plays a significant role in $\left(\frac{\rho}{\rho_J}\right)^{\beta}, \beta > 0$ since

$$\left(\frac{\rho}{\rho_J}\right)^{\beta} > \frac{\rho}{\rho_J}, \;\; \beta < 1$$

$$\left(\frac{\rho}{\rho_J}\right)^{\beta} < \frac{\rho}{\rho_J}, \;\; \beta > 1$$

To exploit such characteristics for modelling the traffic variation in transition phases, it might be necessary to include terms with non-integer power. Based on this consideration, the following model is proposed for $v(\rho)$:

$$V(\rho) = v_m \cdot \left(1 - P\left(\frac{\rho}{\rho_J}\right)\right) \qquad (3.2)$$

$$P\left(\frac{\rho}{\rho_J}\right) = a_{0.3}\left(\frac{\rho}{\rho_J}\right)^{0.3} - a_{0.6}\left(\frac{\rho}{\rho_J}\right)^{0.6} - a_1\left(\frac{\rho}{\rho_J}\right) - a_2\left(\frac{\rho}{\rho_J}\right)^{2} - a_3\left(\frac{\rho}{\rho_J}\right)^{3} - a_4\left(\frac{\rho}{\rho_J}\right)^{4}$$

$$a_{0.3} + a_{0.6} + a_1 + a_2 + a_3 + a_4 = 1$$

$$a_{0.3} \geq 0, a_{0.6} \geq 0, a_1 \geq 0, a_2 \geq 0, a_3 \geq 0, a_4 \geq 0$$

Linear Least Squares Method with non-negativity constraints (Lawson and Hanson, 1974) can be used for model fitting. For model calibration, one parameter is to be eliminated using the unit sum constraint, say, a_1 with

$$a_1 = 1 - (a_{0.3} + a_{0.6} + a_2 + a_3 + a_4)$$

$$1 - \frac{V}{v_m} = a_{0.3}\left[\left(\frac{\rho}{\rho_J}\right)^{0.3} - \left(\frac{\rho}{\rho_J}\right)\right] + a_{0.6}\left[\left(\frac{\rho}{\rho_J}\right)^{0.6} - \left(\frac{\rho}{\rho_J}\right)\right] + a_2\left[\left(\frac{\rho}{\rho_J}\right)^{2} - \left(\frac{\rho}{\rho_J}\right)\right] +$$
$$a_3\left[\left(\frac{\rho}{\rho_J}\right)^{3} - \left(\frac{\rho}{\rho_J}\right)\right] + a_4\left[\left(\frac{\rho}{\rho_J}\right)^{4} - \left(\frac{\rho}{\rho_J}\right)\right]$$
$$a_{0.3} \geq 0, a_{0.6} \geq 0, a_2 \geq 0, a_3 \geq 0, a_4 \geq 0 \qquad (3.3)$$

which can be estimated using Least Square Method. It is noted that the proposed model generalizes several existing models based on Taylor expansion. It opens a new class of possible models for FD while keeping the concavity of $q-\rho$ relationship.

4. VARIABLE STRUCTURE MODELING

This section presents some variable structure models for the reversed λ shape FD. The traffic drops from homogeneous flow happens between the two limbs. Edie model, and the GPMUSC are used to fit the left and right limbs of the reversed λ shape FD.

4.1 Reversed λ Shape FD

The simplest reversed λ shape FD is to adopt straight line segment for the two limbs as in Figure 1.

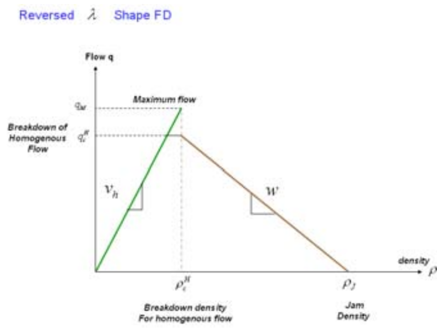

Figure 1. Reversed λ Shape FD with both limbs as straight line (constant slope)

With this shape, the slope of the left limb is the homogenous-flow speed v_h. Several possible models for the left and right limbs are listed bellow.

<u>*Left Limb Models:*</u>

(1) Linear model:

$$q = a_1 + b_1\rho, \;\; \rho \leq \rho_c^h$$

(2) Parabolic model:

$$q = \alpha + \beta\rho + \gamma\rho^2, \;\; \rho \leq \rho_c^h$$

<u>*Right Limb Models:*</u>

(1) Linear model:

$$q = a_2 + b_2\rho, \;\; \rho_c^h < \rho \leq \rho_J$$

(2) GPMUSC Model for $\rho_c^h < \rho \leq \rho_J$ as in (3.2);

(3) Edie model for $\rho_c^h < \rho \leq \rho_J$ as in (2.3);

The combinations of the left and right limb with the above alternatives provides 6 models.

4.2 Data for Model Fitting

NGSIM data are used to fit them. Since current NGSIM data were collected in peak periods for congested traffic, it was not for a free-flow breakdown directly. The model calibration will focus on saturated traffic due to data availability. Since the data need to contain homogenous-flow (not necessarily free-flow) traffic as well as shockwave, the data set for US101 Lane 1 in 3 time intervals are used including that for Figure 2. The distance aggregation is *170m* and time aggregation is *10s* which determines the size of the box A (aggregation level in time and space) in Figure 2.

Our experiences in aggregating the vehicle-by-vehicle NGSIM data indicate that the following aggregation seems reasonable for this purpose:

- time aggregation level: 10~20s
- distance aggregation level: 150~ 200m.

In those levels, distance mean speed, density and thus flow are well-defined and the noise reduction through aggregation is acceptable. It is noted that the original NGSIM data update rate is 10Hz. The basic parameters to be estimated are distance means speed v and density ρ for each lane. The lane-flow q is calculated from the former.

(1) Data Source: NGSIM I-80 5:00-5:15pm; and US-101 7:50-8:05am, with speed-time and distance-time plotted in Figure 2;

(2) Data Aggregation: Data aggregation over distance ΔD and time Δt is equivalent to shrink all the trajectory points within the rectangular box A with sides ΔD and Δt into one point. The size of ΔD and Δt need to be appropriate for traffic characteristics to show up.

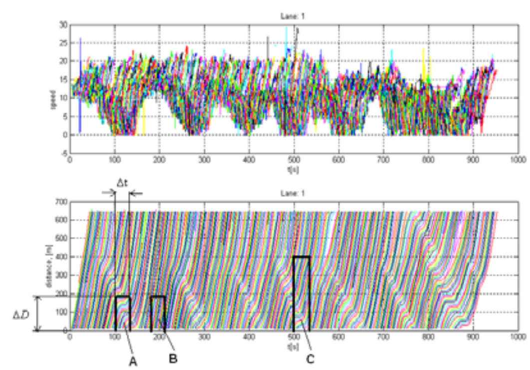

Figure 2. Data Source: NGSIM, US-101, 07:50-08:05am; Lane 1

The data are aggregated within the rectangular box as follows: vehicle count in that box divided by distance is the density ρ; the average of speed trajectories corresponding to those distance trajectories within the box is the distance mean speed v; and the flow q is then calculated as $q = \rho \cdot v$. It is noted that data in box **A** should reflect shockwave, and data in box **B** homogenous flow.

(3) Add the boundary condition under the assumption that $\rho_J = 190$ per/lane/per/mile:

$$v(0) = v(190) = 0$$

4.3 Model Fitting and Observations

The following facts are observed from model fitting using aggregated data:

(1) Break down density ρ_c^h is in a range $\left[\rho_{c,1}^h, \rho_{c,2}^h\right] = [90,120]$. The value of ρ_c^h which are different from lanes to lane;

(2) Selection of ρ_c^h is important for two reasons:

a. if it is not selected properly, traffic breakdown at ρ_c^h will not show up;

b. ρ_c^h selection affects estimation error for all the models with different magnitudes;

(3) In figure 3 and Figure 4, ρ_c^h is chosen such that the RMSE is minimum. Such a selection is possible because density has discrete value in the interval of interests;

(4) $\rho_c^h = 95$ veh/lane-mile seems to be more reasonable than other choices;

Data: US101 7:50~8:05am Ln1				
		Right Limb Model Coefficients		
Left Limb Model Coefficients	**Error type**	GPMUSC $v_m = 38.6$	Edie $g_1 = 131.7$ $g_2 = -25.8$	LIN $a_2 = 2853.7$ $b_2 = -16.0$
Linear: $a_1 = 763.5$ $b_1 = 12.6$	v_err	0.3448	0.3434	0.6010
	q_err	11.7049	11.2750	11.2121
Parabolic: $\alpha = -608.5$ $\beta = 58.9$ $\gamma = -0.4$	v_err	0.4821	0.4812	0.6890
	q_err	11.3106	10.8651	10.7997

It can be seen that for speed error, linear model for right limb is worse. However, for flow error, there is no significant different between the four models.

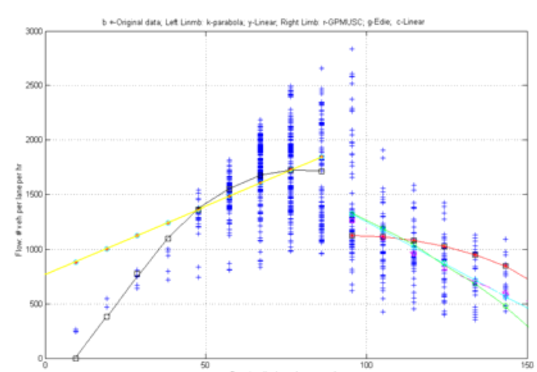

Figure 3. Variable Structure Model fitting from data set US101 7:50~8:05am Lane 1 in the first section of 170m; $\rho_c^h = 95$ with time arrogation interval 10 steps (1s).

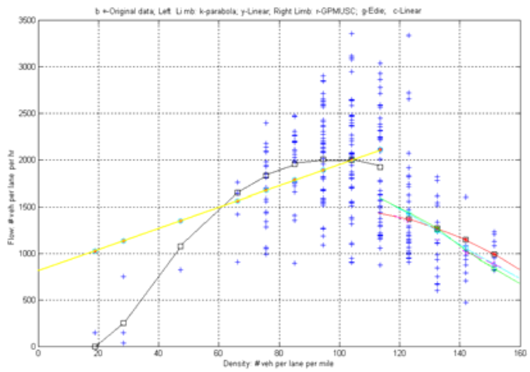

Figure 4. Variable Structure Model fitting from data set US101 Lane 1 and combined three time period: 0750-0805am, 0805-0820am, 0820-0835am; in the first section of 170m; $\rho_c^h = 115$ with time aggregation interval 10s;

Data: US101 7:50~8:05am Ln1					
		Right Limb Model Coefficients			
Left Limb Model Coefficients	**Error type**	GPMUSC $v_m = 38.6$	Edie $g_1 = 131.7$ $g_2 = -25.8$	LIN $a_2 = 2853.7$ $b_2 = -16.0$	
Linear: $a_1 = 763.5$ $b_1 = 12.6$	v_err	0.3448	0.3434	0.6010	
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	q_err	11.3106	10.8651	10.7997	

Similarly, for speed estimation error, linear model for right limb is worse than other three. However, for flow estimation error, there is no significant difference between the four models. It can also be observed that using data from different time periods for the same location leads to different estimation error. This, together with the difference in model coefficients, suggests that FD might not be static in time.

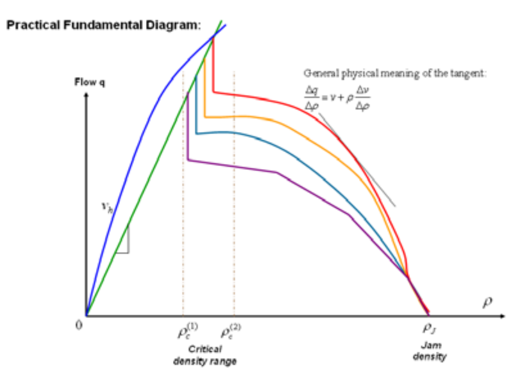

Figure 5. FD for homogenous flow; v_h is the speed of the homogenous flow; breakdown density for a homogenous flow falls into a range $\left[\rho_c^{h_1}, \rho_c^{h_2}\right]$ which is different in time and locations.

4.4 Practical Shape of FD

Based on the model variable structure model fitting for the reversed λ shape model, a general shape of a practical FD is proposed in Figure 5. With density in the range$\left[\rho_{c,1}^h, \rho_{c,2}^h\right]$, the homogenous traffic would have higher probability to breakdown.

It would be interesting to analyze analytically the physical meaning of the tangent for the right limb of the FD curve (Figure 5). Starting from $q = v \cdot \rho$ and using the difference rule in mathematics, it is obtained that

$$q = v \cdot \rho$$
$$\Delta q = \Delta v \cdot \rho + v \cdot \Delta \rho$$
$$\frac{\Delta q}{\Delta \rho} = v + \frac{\Delta v}{\Delta \rho} \cdot \rho$$

It is clear to have the following:

<u>*General Physical Meaning for the Tangent of Right Limb in FD*</u>: the tangent in the right limb is the distance means speed v plus an increment term $\frac{\Delta v}{\Delta \rho} \cdot \rho$ which must be negative since $\frac{\Delta v}{\Delta \rho} < 0$.

5. CONCLUDING REMARKS

FD describes the flow-density $q - \rho$ (speed-density $v - \rho$) relationship. FD is very important in understanding macroscopic traffic model and critical for traffic control such as ramp metering and VSL in model simplification, capacity estimation, and prediction of traffic drops from a homogenous flow, of which a special case is traffic direct breakdown from free-flow. This paper proposed a generalized polynomial model for the $v - \rho$ (or $q - \rho$) function which naturally satisfy the boundary conditions. Properly aggregated NGSIM data for saturated traffic are used for model calibration and comparison. Linear and parabolic lines are used for left limb; Edie, linear and GPMUSC are used for right limb. RMSE indicates that they are not significantly different for $q - \rho$ relationship although the GPMUSC is slightly better for $v - \rho$ relationship. The linear mode is not as good for the right limb of $v - \rho$. It thus opens a new class of models for further investigation.

Proper length of the distance and time intervals used for data aggregation from microscopic data to macroscopic data are very recognized important. Too short distance interval will not generate meaningful traffic parameters such as density, distance mean speed and flow(ρ, v, q); too long distance would smooth out traffic characteristics such as shockwaves and flow-drops. For NGSIM vehicle-by-vehicle tracking data with 10Hz update rate, the distance interval should be between 150~200m and time interval be 10~20s.

Different model coefficients determined through Least Squares fitting from data at the same location but different time periods and the same time periods but different locations indicate that the FD is neither static in time nor homogenous in distance. This is also enhanced by the fact that the estimation errors are different for the same model but data from different locations of the same time period, or the same location but different time periods. How those results would mean for traffic modelling and control needs further consideration.

6. ACKNOWLEDGEMENT

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