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# Automatic Calibration of the Fundamental Diagram and Empirical Observations on Capacity


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**ABSTRACT**

We present a method for automated, empirical calibration of freeway traffic flow characteristics. The method uses 5-min flow and density values for a section of freeway and rapidly and reliably estimates key parameters such as free flow speed, capacity, critical density, congestion wave speed and jam density, which are key inputs to many macroscopic traffic simulation models. The method consists of data filtering, capacity identification, and approximate quantile regression steps. The method was used to calibrate a cell transmission model of Interstate-880 in San Francisco Bay Area, California, a 40-mile long urban freeway with lots of recurrent and non-recurrent congestion and with dozens of loop detector stations. The calibrated model reproduced the observed traffic congestion behavior within 9% error for performance measures VMT (vehicle miles traveled), VHT (vehicle hours traveled) and total flow. Also, the empirical results suggest that capacity, defined as the maximum observed 5-minute flow rate over several days, differs from breakdown flow, defined as the flow that is observed just before the freeway section becomes congested.

*Keywords: fundamental diagram, capacity drop, cell transmission model, PeMS.*

## 1. INTRODUCTION

Most macroscopic models of vehicular traffic make use of the *fundamental diagram*, an empirical curve relating observed densities to observed flows at a particular point on the road. In the case of first order models derived from the LWR theory (1), such as the cell transmission model (CTM) (2), the fundamental diagram provides a direct mapping from density to flow. For higher order models, such as Payne's model (3) and its derivatives, it provides an equilibrium surface. In both cases, the selection of the parameters defining the curve is an important part of the model calibration procedure.

In general terms, the construction of a macroscopic model of a freeway involves certain well-defined steps. The first step is to define the geometrical characteristics of the site. These include the location of onramps and offramps, number of lanes, existence of HOV lanes, sensors, ramp meters, etc. The next step is data collection. A representative set of traffic measurements must be compiled in order to calibrate and construct realistic traffic models. A third step is to divide the length of the freeway into homogeneous segments, each represented in the model by a single cell or link. The method for dividing the freeway may depend on the requirements of the model. The collected traffic data must then be used to tune the model parameters for each of the segments of the freeway. In the case of the cell transmission model, all of these parameters are related to the fundamental diagram. Higher order models such as Metanet (4) involve additional parameters. Prior efforts in the estimation of the fundamental diagram include Van Aerde and Rakha (21), where a continuously differentiable function with 4 parameters was fitted to traffic data. The approach presented here differs from that previous efforts in that we assume that the calibrated model is ultimately used for simulation purposes, and not necessarily planning or highway design. Thus we impose a triangular form, which is compatible with the CTM. We also prioritize the capacity parameter over all others. Capacity is selected independently (as the largest observed flow), while the congestion parameter and jam density are constrained. We emphasize capacity because of its strong influence on congestion patterns and travel times.

This paper presents a model construction and calibration procedure applicable to macroscopic freeway models. The main focus is on the fitting of a triangular fundamental diagram to 5-minute flow and density data obtained from the PeMS database (5). PeMS is an online repository for traffic data gathered throughout the state of California. It allows the user to query raw data sets, but also provides illustrative reports and plots. The triangular form is convenient because the slopes of its two sides and its apex are explicit parameters in the cell transmission model. However, the procedure is not limited to the CTM, it may be applied to any macroscopic model that makes use of the fundamental diagram.

The second part of the paper provides empirical observations concerning the variation of capacity and its relation to breakdown and the capacity drop phenomenon. There are various definitions and estimation methods for the capacity of a roadway segment in the literature. The Highway Capacity Manual defines capacity as the maximum rate of hourly flow that can traverse the uniform cross-section of a road segment under prevailing road, traffic and control conditions (6). Lately, this deterministic approach has been challenged by the notion of *breakdown*, which describes the operation of a freeway near a bottleneck at a time instance where there is a change from free-flow to congestion (7). Numerous studies on the stochastic nature of capacity (8, 14, 15, 16, 20) suggest that the breakdown occurs randomly, affected by various external factors

such as driver behavior, road and weather conditions, incidents, etc. and capacity can be defined as a random variable with a specific probability distribution depending on the probability of breakdown.

Another important issue regarding the definition of capacity is its suggested dual nature. The discharge rate of a freeway section has been shown to drop below the pre-queue value once the section gets congested (9, 10, 11, 12). Therefore, for a deterministic model, the analyst should decide whether or not to adopt dual capacities (or which one to choose).

In the calibration algorithm presented in this paper, the fundamental diagram approach to capacity estimation has been adopted because of its suitability to simulation and compatibility to available data. Minderhoud et al. (13) name this method to be the best deterministic method available and the second-best method overall, next to the probabilistic product limit method (14). The criteria they base their subjective judgement on are data needs, required traffic state, viability of theory, expected uncertainties of outcomes and choice of capacity type (pre-queue vs. queue discharge). The empirical results presented in this paper agree with the capacity drop phenomenon and reveal significant disparity between breakdown and capacity, as defined in the fundamental diagram framework.

## 2. GEOMETRY AND TRAFFIC DATA

### 2.1. Freeway Representation

The first step in the calibration is the representation of the network in the form of successive cells. The freeway network is divided into cells each with at most one on- and/or off-ramp and one mainline vehicle detector station. The cells must be longer than the free-flow travel distance, i.e. $v_i T_s \leq l_i$ , where $v_i$ is the free-flow speed across cell i, $T_s$ is the simulation time step and $l_i$ is the length of the i'th cell, so that no vehicle is "lost" during simulation. Each cell is assumed to be homogeneous in terms of number of lanes, grade and geometrical features so that each cell can be represented by a single fundamental diagram. These fundamental diagrams are subsequently calibrated on the data provided by the individual mainline vehicle detector station of each cell. The 28 mile stretch of I-880S (extending between the 29$^{th}$ street onramp and the Auto Mall Parkway onramp) analyzed in this study consists of 52 such cells.

### 2.2. Data Acquisition, Selection and Correction

PeMS provides flow, speed and density data across the vehicle detector stations (vds) in the form of time series data over days of operation. The data are aggregated over an interval of 5 minutes such that each day contains 288 data points for each measured quantity. Detector and data health has been a major concern during the study. PeMS archives report detector performance for each day of operation. The data for calibration were chosen from days for which PeMS reported over 80% functionality for all detectors in the 28 mile stretch being analyzed. Further, for each detector station, only days on which that particular freeway section became congested were chosen, in an effort to observe capacity and congested flow characteristics. In this study, a freeway section is deemed congested when the speed across the detector falls below 40 mph for at least 5 minutes. This is in accordance with the PeMS definition of congestion. During the period between February 2007 and March 2008, 98 days were identified as functioning above

80% performance by PeMS and for the example vds presented in the following section, the cell corresponding to this vds became congested for 92 out of these 98 days. Thus, the calibration was performed on the data of these 92 "healthy and congested" days for this particular vds.

Since the fundamental diagrams are based on flow vs. density scatterplots, the measurement of these two quantities gains importance. The flows are measured directly from induction loop counts, whereas the densities and speeds are determined indirectly by the following formulae:

$$\text{Density} = \frac{\text{Flow}}{\text{Speed}} \qquad \text{Speed} = \frac{\text{Flow}}{\text{Occupancy}} \cdot \text{g-factor}$$

This "g-factor" or effective vehicle length is itself adaptively estimated (see 18 for details).

## 3. CALIBRATION OF THE FUNDAMENTAL DIAGRAM

### 3.1. Calibration of Free-flow Parameter, v

The free-flow speed, v, is estimated by performing a least-squares fit on the flow vs. density data at the time instants where the speed was reported to be above 55 mph. The scatterplot accumulated over 92 days' data for vds 400669 on I-880S is shown in Figure 1. Each dot on the plot corresponds to one observed flow – density pair and each day has 288 such data points. The regression line for the free-flow speed is shown in Figure 2.

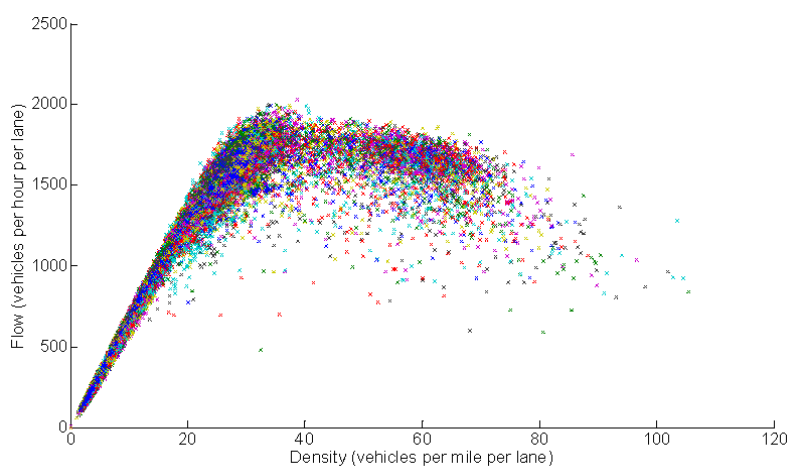


**FIGURE 1. Flow vs Density scatter plot**

### 3.2. Estimation of Capacity

The next step in the calibration is the estimation of capacity. Here, the Highway Capacity Manual (1) definition of capacity is adopted, i.e. capacity is the maximum amount of flow that can reasonably be expected to traverse the cross-section of a road segment. A discussion on the various definitions of capacity, its stochastic nature and its observed variation is presented in the next section; but for the purposes of model calibration, a deterministic and rather conservative

capacity estimate is implemented. This is done by assigning the maximum value of flow across the section among all observed days as the capacity of that section. This value is then horizontally projected to the free-flow line to establish the tip of the triangular fundamental diagram (Figure 2). The intersection is defined as the critical density for the section, above which the flow is congested.

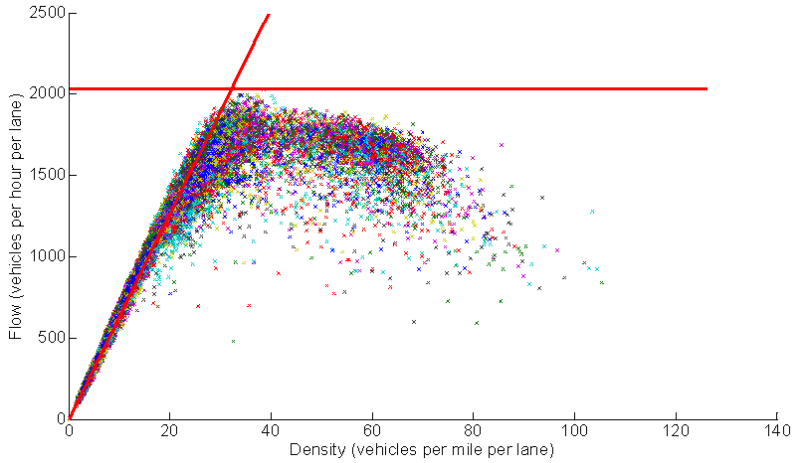


**FIGURE 2. Free-flow speed and capacity.**

The choice of this largest observed flow as the estimate of capacity is based on the assumption that external factors such as driver behavior, incidents, weather and road conditions always affect the capacity adversely and the actual capacities of freeway sections are rarely observed, if ever. In other words, capacity is assumed to be composed of a deterministic maximum value, plus a negative random factor. The calibration procedure estimates only the deterministic maximum part since for macroscopic modeling purposes, it is desirable that the model can reflect the ideal performance of the freeway. This way, there is room for reduction of the capacity in case, for instance, the effects of an incident on capacity are to be investigated in the simulation environment. Furthermore, for the testing of hypothetical control strategies, such as ramp metering, the ideal service quality needs to be reachable by the model for a healthy assessment of the performance of the applied strategy. This appraoch constrasts with others, for example that of [Elefteriadou et al], who suggest the use of either a median or a 15th percentile value of maximum pre-breakdown flow as the capacity.

### 3.3. Calibration of the Congestion Speed Parameter, w

The last parameter to be calibrated is the congestion speed parameter, w, which also defines the jam density for the section. Similar to capacity, this parameter shows significant diversity (11). Based on the same assumption as for capacity, an approximate quantile regression (17) was adopted to estimate this parameter at the higher end of its distribution. A detailed analysis on the statistics of this parameter is beyond the scope of this work.

After the critical density (the tip of the fundamental diagram) is determined, the flow-density points with density values higher than the critical density (the data points to the right of the tip) are partitioned along the horizontal axis (density axis) into non-overlapping bins of 10 data points each. Horizontally, each bin is summarized by "BinDensity," the mean of the 10 density

values in the bin. Vertically, each bin is summarized by "BinFlow," the largest non-outlier flow values among the the 10 flow values in the bin. Formally, this largest non-outlier is determined as follows:

$$\text{Bin} = \{ f_1 , f_2 , \ldots , f_{10} \}$$

$$\text{BinFlow} = \max_{f_i} \; ( f_i \mid f_i \; \varepsilon \; \text{Bin}, f_i < Q3 + 1.5IQR )$$

where, $f_1$ through $f_{10}$ and $f_i$ are the flow values inside one such bin, Q3 is the 75th percentile of the data points in the bin and IQR is defined as the difference between the 25th percentile and the 75th percentile of the data.

A constrained least-squares regression is performed on these BinDensity – BinFlow pairs to obtain the congested flow line and complete the fundamental diagram picture (Figure 3). It is required that the regression line passes through the tip of the fundamental diagram, so the regression is constrained to fulfill this requirement. The point where the regression line crosses zero flow is assigned as the jam density of the section.

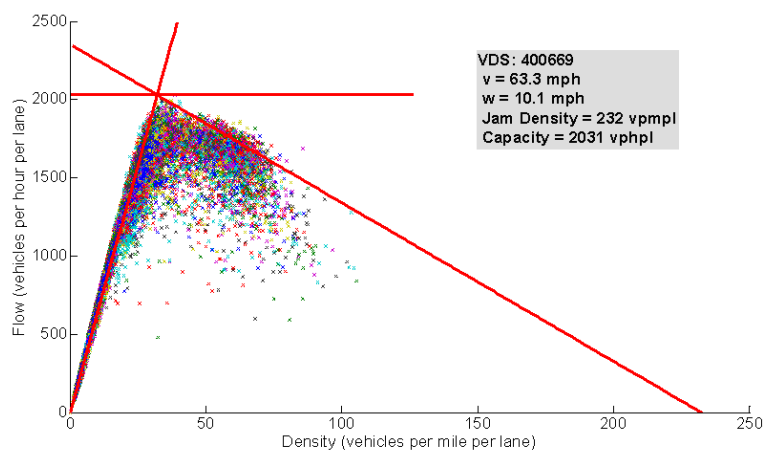


**FIGURE 3. The last result of the calibration**

## 4. EMPIRICAL OBSERVATIONS ON VARIATION OF CAPACITY, AND ITS COMPARISON TO BREAKDOWN AND CAPACITY DROP PHENOMENA

### 4.1. Daily Variation of Capacity

A preliminary study on the variation of capacity (defined as the maximum observed flow during a congested day) over different days yields the box plots shown in Figure 4. In the figure, the horizontal axis is the detector id's placed on the freeway in ascending order from upstream to downstream (left to right). The vertical axis reflects the normalized flow values across the cross-sections of the freeway. The number of data points used in the construction of each box plot is placed in the upper horizontal axis. They reflect the number of congested days among the total 98 days for each section. In this respect, the first section (the most upstream) became congested in 94 of those days and the last section had 86 congested days available for analysis. The sections

which only have few days available are mainly due to a lack of healthy data specific to those sections rather than a lack of congestion. The red lines inside the box plots correspond to the median of the observed capacities among days. The lower and upper box boundaries represent 1st and 3rd quartiles, or 25th and 75th percentiles, respectively. The whiskers span from either end of the box to the smallest and largest data points that are non-outliers, where "outliers" are defined as any points that are more than 1.5 interquartile range away from box boundaries. Individual outliers are marked as points.

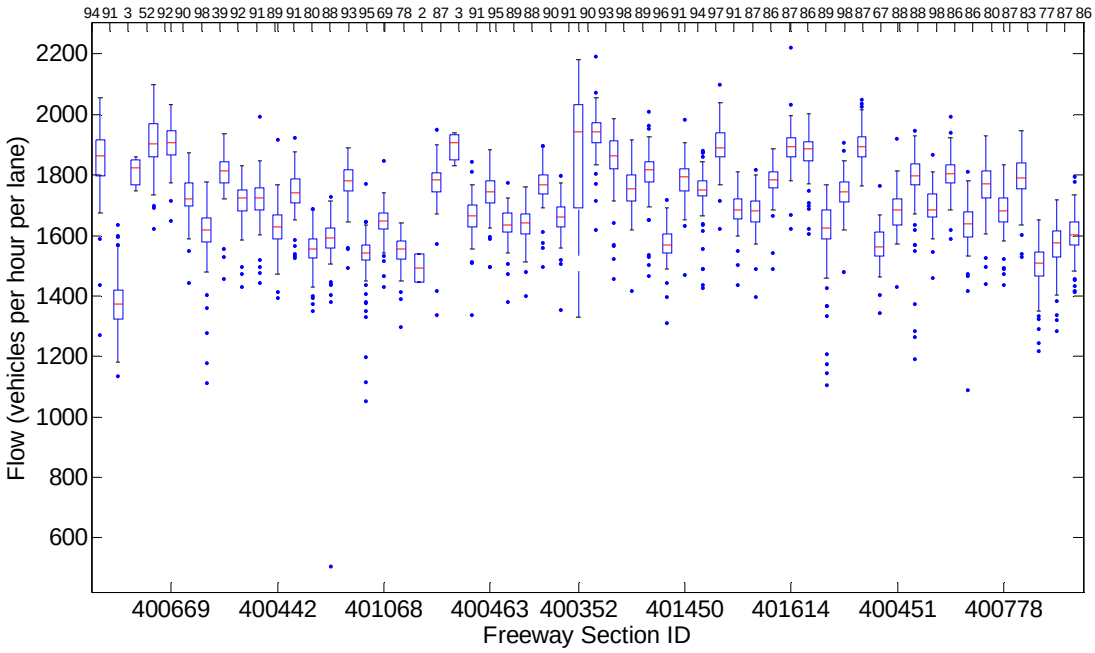


**FIGURE 4. Variation in capacity of successive freeway sections.**

The first striking feature of Figure 4 is that the maximum flow across a freeway section varies substantially among days of operation. In the extreme case of detector 400352, the capacity varies between 2186 and 1324 vehicles per hour per lane. Note that both these values reflect the highest observed flow on a single day of operation across this detector. Another observation on the box plots is that most of the outliers fall below the lower whisker, indicating that external random factors affect the capacity rather adversely. These observations justify the choice of capacity adopted in the calibration procedure.

### 4.2. Capacity vs. Breakdown

The next point of interest is to compare the capacity to flows at breakdown. Three sample sections were investigated for this comparison. These sections were chosen so that they do not contain any on-ramps or off-ramps to avoid the effects of any weaving inside the section. The chosen sections and some of their features are as follows:

- Section 1: vds ID: 400192, length: 792 ft, capacity: 1687
- Section 2: vds ID: 400427, length: 1901 ft, capacity: 1797
- Section 3: vds ID: 401614, length: 1320 ft, capacity: 2220

The analysis was based on the speed plots constructed from PeMS data. Three types of flow are defined in PeMS as follows:

- Flow Regime 1: Free Flow ( Speed > 55 mph )
- Flow Regime 2: Dense Flow ( 40 mph < Speed < 55 mph )
- Flow Regime 3: Congested Flow ( Speed < 40 mph )

To observe the flows at breakdown, speed plots of the three chosen sections were investigated on various days and the flows at breakdown were identified. Breakdown is defined as follows: There is a switch in the flow regime from 1 to 2, 2 to 3 or 1 to 3; and the downstream sections is in flow regime 1, so the upstream section is operating at active bottleneck conditions.

Figures 5, 6 and 7 reflect speed plots for each section on the dates 22-Oct-2007, 30-Nov-2007 and 09-Oct-2007, respectively. In these figures, the horizontal axis is the time of day and the vertical axis is the speed in mph. The solid line belongs to the upstream section that is being analyzed and the dashed line belongs to the section right downstream of it. The time when the daily maximum flow was observed is marked by the vertical dashed line, where the horizontal dashed lines mark the two speed thresholds for the identification of breakdown. The enumerated points on the speed plots correspond to the breakdown instances during the day and the corresponding flows before and after these breakdowns are listed to the lower left corner of the figure. The list also reflects the daily observed maximum flow for that day and the estimated capacity over all the days that were evaluated.

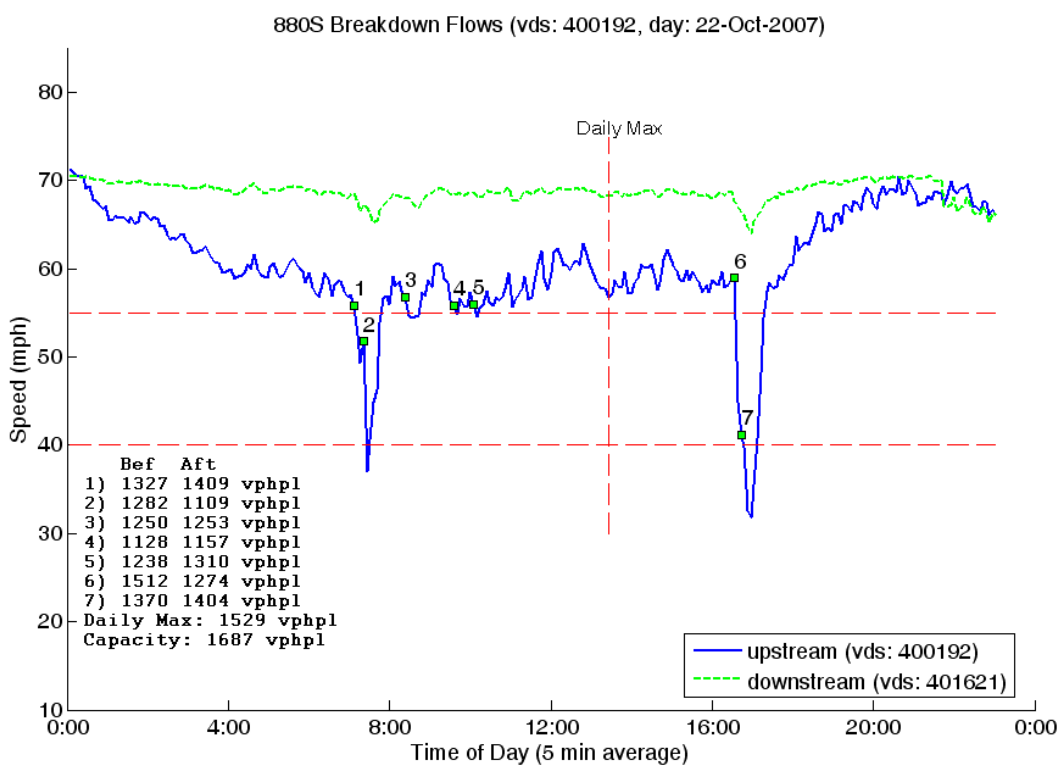


**FIGURE 5. Daily Speed Plot for vds 400192 on Monday, 22-Oct-2007**

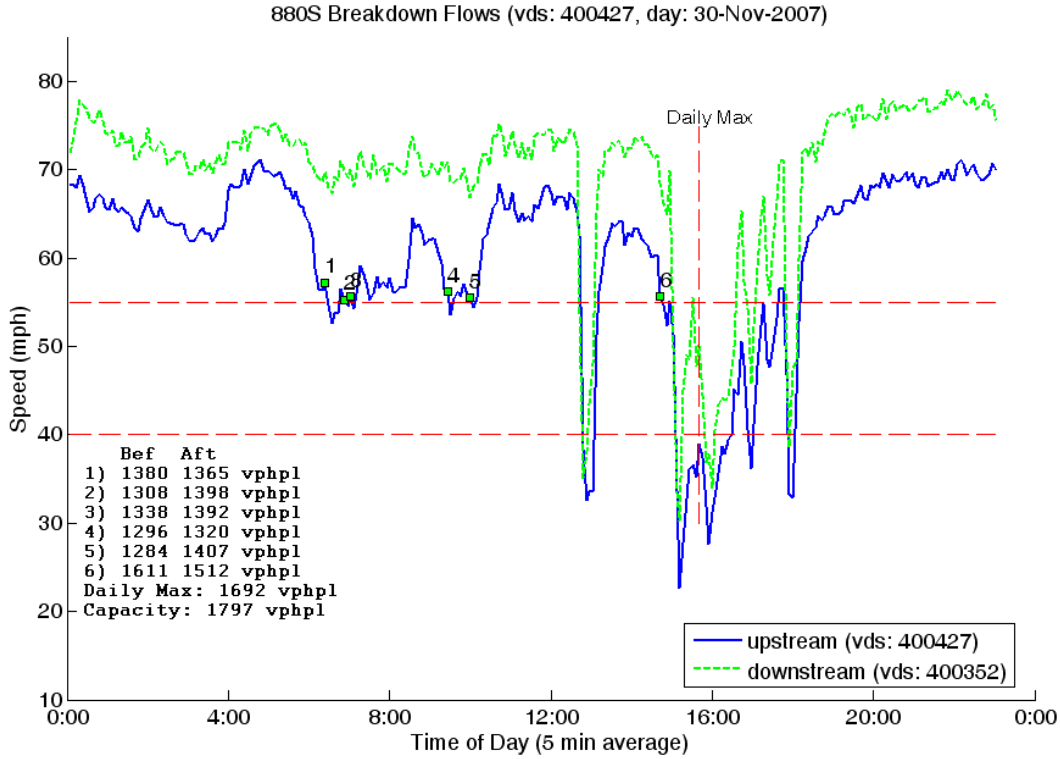


**FIGURE 6. Daily Speed Plot for vds 400427 on Friday, 30-Nov-2007**

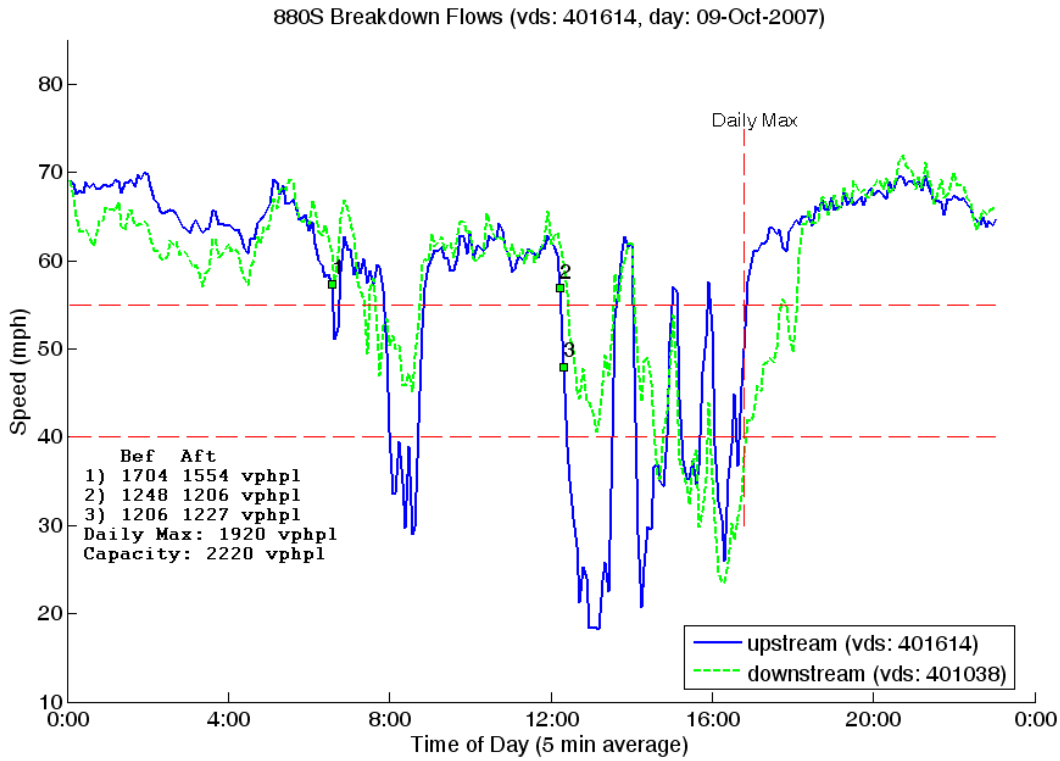


**FIGURE 7. Daily Speed Plot for vds 401614 on Tuesday, 09-Oct-2007**

The first plot reflects a daily maximum of 1529 vphpl (vehicles per hour per lane) whereas the capacity was found to be 1687 vphpl. This highest value (1529 vphpl) is observed at around 2:00pm at which time the freeway section is in free-flow and is away from any congested period of the day. Note that 7 breakdowns were observed on this day, none of which coincide with the highest observed flow. Furthermore, the pre-breakdown flow is not necessarily higher than the post-breakdown flow, as can be seen in several of the breakdowns. In the second plot, the highest flow of the day is observed during a congested period, again not at breakdown but during established congestion. The last plot shows highest flow at a time period where the downstream section is also heavily congested, meaning that the section is not operating as a bottleneck. Moreover, the breakdown flows show significant diversity ranging from 1206 to 1704 vphpl where the daily maximum is 1920 vphpl (observed during the evening rush hour at about 5:30pm) and the capacity was found to be 2220 vphpl (observed on another day). These results suggest that capacity, defined deterministically in the fundamental diagram framework, is usually observed apart from any breakdown instance.

### 4.3. Capacity Drop

Capacity drop at freeway bottlenecks is another important phenomenon related to capacity. This phenomenon was observable in the fundamental diagrams of all freeway sections, once the triangular constraint was removed. The quantile regression was also omitted and a pure least-squares regression was performed on the congested side of the flow vs. density scatterplot, in order to observe an overall measure for the capacity drop, rather than an upper quartile one. Under these conditions, the congestion line undercuts the free-flow line and the capacity drop among 52 freeway sections under consideration were observed to range between 10.1% and 25.1%. When the quantile regression scheme is applied to congested data as in the previous section, the range of capacity drops reduces to 3% - 15%. Examples are shown for the section with ID 400192 in Figure 8.

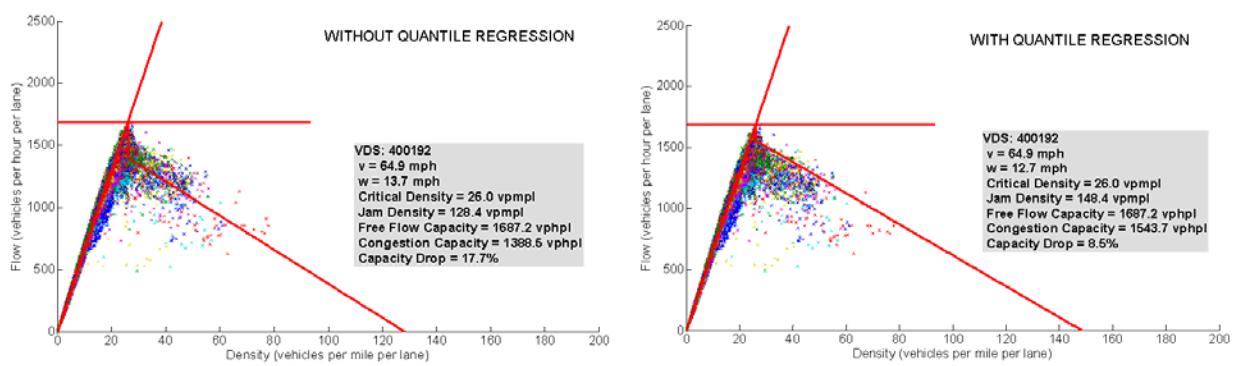


**FIGURE 8. Capacity drop at section 400192 without and with quantile regression in the congested side.**

Figure 8 suggests that there are two capacity values associated with the critical density, pre-queue and queue discharge. These results agree with the capacity drop observed previously in works of Cassidy et al. (9, 10) and the capacity drop percentages estimated using the quantile regression method are closer to the 10% Cassidy et al. observed.

## 5. RESULTS

Table 1 shows the calibrated parameters for 12 sections of the freeway. These results are based on the calibration procedure outlined in section 3, thus the choice of capacity is the pre-queue value in accordance with the CTM, on which the simulation is based. Incidentally, PeMS archives do not report ramp flows for I-880. For this reason, the missing ramp flows were imputed using the algorithm of Muralidharan and Horowitz (19).

| VDS ID | Postmile | v (mph) | w (mph) | Capacity (vphpl) | Critical Density (vpmpl) | Jam Density (vpmpl) |
|---|---|---|---|---|---|---|
| 400669 | 38.877 | 63.25 | 10.15 | 2031 | 32.111 | 232.21 |
| 400190 | 38.297 | 64.1 | 15.806 | 1872 | 29.204 | 147.64 |
| 400956 | 36.557 | 63.25 | 12.29 | 1935 | 30.593 | 188.04 |
| 400740 | 35.507 | 65.2 | 8.5095 | 1830 | 28.067 | 243.12 |
| 400134 | 34.977 | 64.825 | 14.054 | 1992 | 30.729 | 172.47 |
| 400228 | 33.757 | 62.175 | 13.695 | 1923 | 30.929 | 171.35 |
| 400192 | 33.577 | 64.86 | 20.313 | 1687.2 | 26.013 | 109.07 |
| 401621 | 33.427 | 64.92 | 14.596 | 1728 | 26.617 | 145.01 |
| 400942 | 32.947 | 63.8 | 13.74 | 1890 | 29.624 | 167.18 |
| 401068 | 32.357 | 63.9 | 16.015 | 1848 | 28.92 | 144.31 |
| 400225 | 30.877 | 66.6 | 16.482 | 1641 | 24.64 | 124.21 |
| 400312 | 29.597 | 68.6 | 14.05 | 1951.2 | 28.443 | 167.32 |

**TABLE 1. Calibrated parameters for 12 of the 52 freeway sections.**

To see how CTM reproduces observed congestion patterns and travel times using the calibrated parameters, the calibrated freeway model was simulated for a particular day, Jan 23$^{rd}$ 2008. The choice of this day was based on the maximum daily flows observed on this day and the health of the detectors. The maximum daily flows for this particular day were the closest to the capacities determined by the calibration, so it was assumed that this was an ideal day with no significant incidents and good road and weather conditions. Of course, it is not expected that each freeway segment reaches its capacity on any given day. Further, the CTM boundary conditions require that downstream of the simulated network is in free flow, meaning that the downstream flow will be overestimated whenever there is congestion in the last section, which is always the case for freeways in high demand such as I-880. For these reasons, the simulation results predict less congestion and less delay than the actual values. Nevertheless, the overestimation of capacity is rather advantageous because it enables the simulation of capacity drops caused by external factors, which are assumed to occur randomly. For example, all section capacities can be reduced by a certain proportion to simulate a rainy day or the capacity of a section can be cut in half if, say, half of its lanes are closed due to roadwork on a particular day. It is also important that the model implements these high capacities observed over many days to facilitate the simulation of hypothetical control scenarios, such as ramp metering.

The results of the simulation are given in terms speed contours and performance measures Vehicle Miles Traveled (VMT) and Vehicle Hours Traveled (VHT) in Figures 9 and 10.

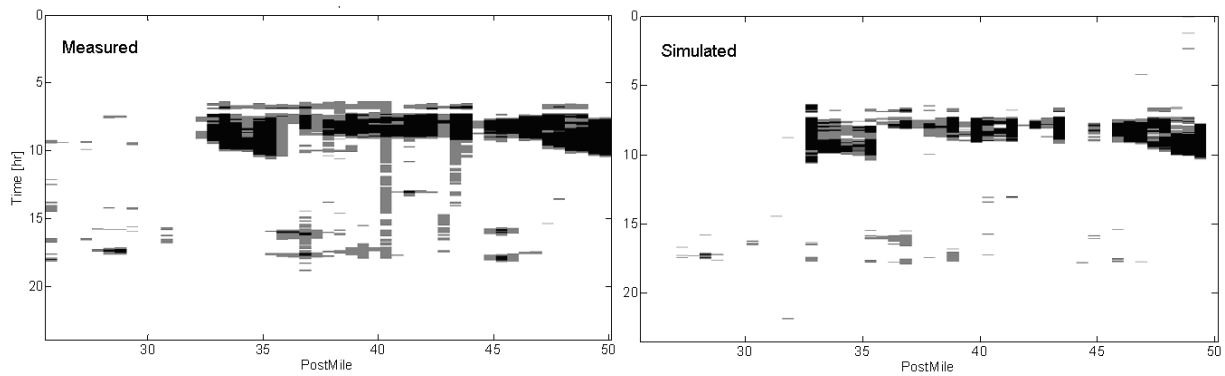


**FIGURE 9. Measured and simulated speed contour plot for Wednesday, 23-Jan-2008**

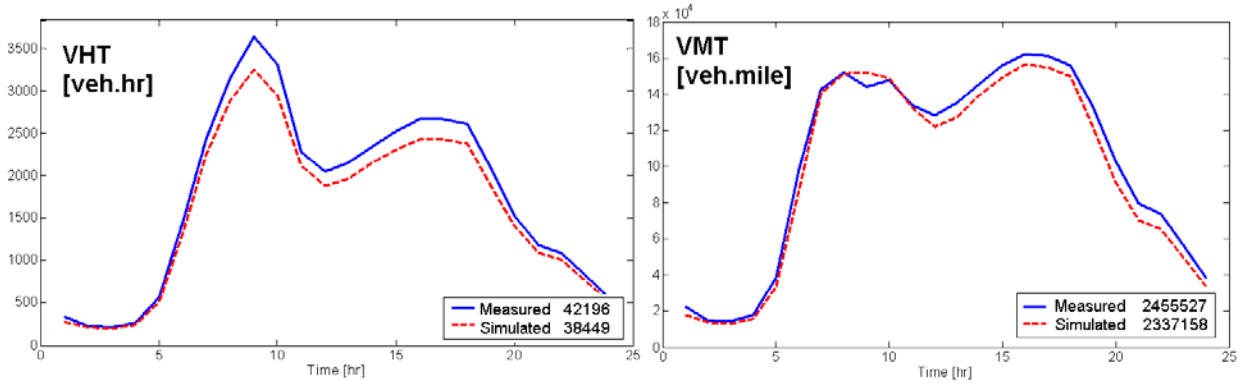


**FIGURE 10. VHT and VMT vs. time of day (measured and simulated)**

## 6. CONCLUSIONS AND FUTURE WORK

A procedure for the automatic calibration of freeway flow characteristics in terms of the fundamental diagram has been presented in this paper. The fundamental diagram, which relates densities to flows, features all parameters required by most macroscopic models. The Cell Transmission Model (CTM) was implemented for the simulation of the calibrated model for Interstate 880 case study in the San Francisco Bay Area. The calibrated model reproduced the measured speed contour pattern with significant accuracy (by visual inspection). The 4.8% error in the Vehicle Miles Traveled suggests that the demand data provided by PeMS is not very accurate. Nevertheless, the calibrated CTM was able to reproduce the Vehicle Hours Traveled with reasonable accuracy (8.8% error). In addition, the simulated total flow differed by 2.5 % from the measured total flow. The simulated delay was significantly lower than the measured delay because of the not-satisfied boundary condition of the model and the deliberate overestimation of freeway section capacities.

The estimation of capacity has been an important part of the study presented in this paper. The empirical results on the variation of capacity and its relation to breakdown and capacity drop phenomena have been investigated from the fundamental diagram perspective. Using the data that is normalized per lane and aggregated over 5 minute intervals, it has been shown that capacity, defined as the maximum observed flow, does not necessarily appear during breakdown.

It was also observed that the capacity of freeway sections exhibit significant drop when the section gets congested.

The main benefit of the presented calibration method is that it is completely automated, except for the data collection step, and provides a well-defined procedure for the estimation of free flow speed, capacity, critical density, congestion wave speed and jam density, which are key inputs to most macroscopic models. This estimation, however, is deterministic. The next stage of the research will be to extend the calibration to stochastic models. Preliminary results suggest that the capacities and congestion wave parameters of roadway segments show spatial and temporal correlation with each other, in addition to their relation to incidents and road conditions. Once these relations are unveiled, the probability distributions of these parameters can be estimated more rigorously.

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