Parametric Uncertainty Identification and Model Reduction for Dual-Stage Robust H2 Track-following Control Synthesis

R. Conway, S. Felix, and R. Horowitz, *University of California at Berkeley*

***Abstract*—This paper presents a systematic method for identifying parameters and parametric uncertainty for a set of dual-stage hard disk drives. A modal analysis technique is selected to extract parameters from a batch of frequency response data. Next, an optimal model truncation methodology is presented to eliminate redundant modes from each multiple-input-multiple-output system model. Finally, principle component analysis is employed to obtain a low-order, minimally conservative approximation of uncertain parameters. The result is a reduced order state space model and parameter uncertainty set to be used in robust H2 control synthesis for a track-following servo.**

Index Terms—**Model reduction, parametric uncertainty, robust control, system identification**

I. INTRODUCTION

Increasing areal data densities in hard disk drives continue to motivate the development of high performance servo schemes for track-following control. Among the emerging configurations are dual-stage servos which incorporate a smaller-scale actuator onto the disk drive suspension in addition to the voice coil motor. It has also been shown that the use of optimal robust control synthesis techniques result in dual-stage controllers with superior tracking performance subject to realistic disturbance models and modeling uncertainties [1].

A significant challenge in the implementation of control synthesis techniques such as robust H2 is the identification of model parameters and uncertainties. Since robust H2 synthesis optimizes over all the worst case combinations of the uncertain parameters, the computation time increases exponentially as the number of uncertain parameters increases. Thus, obtaining a description of the plant uncertainty with a minimal number of parameters is essential for posing a computationally feasible control design problem. This can be achieved through proper selection and identification of the model form, appropriate model truncation, and reduced-order uncertainty approximation. The following presents a systematic methodology for each of these phases of system identification, originating from a set of experimental frequency response functions (FRFs). The methods are presented in the context of a specific example application.

II MODEL FORM AND PARAMETER IDENTIFICATION

Fig. 1 shows the experimental frequency response data for a batch of 36 dual-input, single output hard disk drive plants obtained from [2]. The secondary input is piezoelectric (PZT) actuation of the suspension.

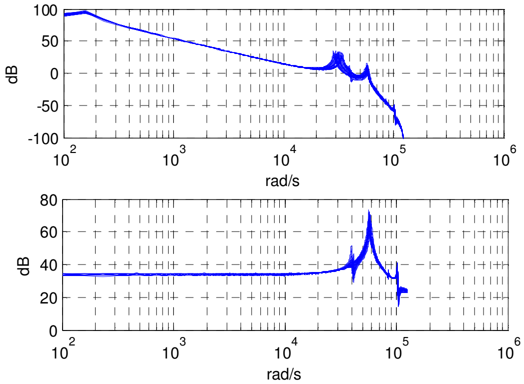

Fig. 1 Magnitude frequency response for 36 experimental plants. Top: output response from the VCM input, bottom: output response from the PZT input.

Several distinct resonance modes are apparent in the data. It is therefore appropriate to represent the transfer function between each input-output pair, $G_{ij}(s)$, as a summation of *m* modes, as in (1). This form requires only three parameters per mode: natural frequency, ω_n, damping coefficient, η, and modal constant, b_o. Further, these parameters relate to insight about physical plant variations, which is important later in the uncertainty analysis.

$$G_{ij}(s) = \sum_{i=1}^{m} \frac{b_{0m}}{s^2 + \eta_m \omega_{nm} s + \omega^2{}_{nm}} \tag{1}$$

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R. Conway (e-mail: rconway345@gmail.com), S. Felix (e-mail: sarfelix@me.berkeley.edu), and R. Horowitz (e-mail: horowitz@me.berkeley.edu) are with the Computer Mechanics Laboratory at the University of California at Berkeley, USA.

Parameters for each mode can be identified readily using single degree of freedom (SDOF) modal analysis. The SDOF assumption should be verified by examining the Nyquist plot of the data. The data points around each peak should form a distinct circle in the Nyquist plane. This property is exploited for parameter identification, since natural frequency, damping, and modal constants can be extracted from the geometry of this Nyquist circle. This technique, called the circle fit method, is used to obtain modal parameters from the two FRFs for each plant in the batch. A priori knowledge is required from the designer about which modes are to be identified. Finally, for each plant, the identified transfer functions are combined to form a multiple input multiple output (MIMO) state space realization such that

$$G(s) = B(sI - A)^{-1}C \, . \tag{2}$$

III OPTIMAL MODEL REDUCTION

As seen in Fig. 1, there are several modes that appear in both sets of FRFs. This leads to nearly redundant eigenvalue pairs in the *A* matrix of the state space system, with slight differences being the result of error in measurements and parameter estimation. The repeated pairs of modes are weakly observable, and balanced model truncation effectively eliminates the redundant eigenvalues in the A matrix. However, the modes are not necessarily weakly controllable, so after truncation, information is lost in the *B* and *C* matrices, resulting in error shown in Fig. 2.

An additional optimization step improves the accuracy of the reduced-order system. This is accomplished by minimizing the 2-norm between the actual system, *G(s)*, and the reduced-order system, $\hat{G}(s)$. The minimization problem

$$\min_{\hat{B}} \left\| G(s) - \hat{G}(s) \right\|_2 \tag{3}$$

can be solved using the solution of a Lyapunov equation. A dual minimization exists for the *C* matrix. Note that *B* and *C* cannot be optimized simultaneously using this methodology, but they can be optimized one at a time in an alternating, iterative fashion until the model reduction cost in (3) converges. The black solid curve in Fig. 2 shows the improved reduced order model. This process is performed on each plant. Again, knowledge is required from the designer to specify how many redundant modes exist.

IV UNCERTAINTY APPROXIMATION

After a suitable reduced order system is obtained for each plant, the uncertainty in each parameter can be characterized. It was observed that coupling exists in the plant variations such as frequency and damping. This can be seen in Fig. 3 where, in several modes, the natural frequency apprears to increase as damping increases. This coupling can be exploited using principle component analysis to identify a transformed set of uncertain parameters, ordered by “importance”. This acheives two goals. The first is that the number of uncertain parameters can be reduced by choosing to neglect weaker directions. Second, the coordinate transformation yeilds a less conservative envelope around the spread of parameter values. A transformation matrix is computed to approximate the original parameters from the reduced set of uncertain parameters. Cost function, *J*, is defined to quantify the accuracy of the approximation and aid the designer in selecting an appropriate number of uncertain parameters. Let v be a vector of actual parameter values and $\hat{v}$ be the approximated set of parameters. Then,

$$J = \sum_i \left\| v_i - \hat{v}_i \right\|_2^2 \, . \tag{4}$$

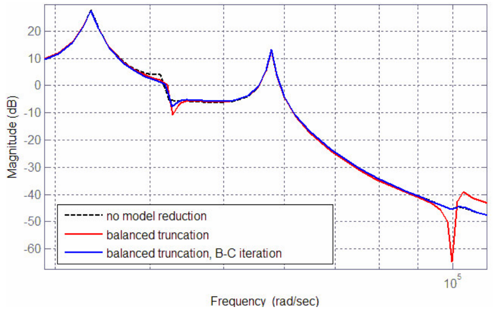

Fig. 2 Magnitude response of full-order model and reduced-order approximations. Plot is zoomed on modes where error is significant.

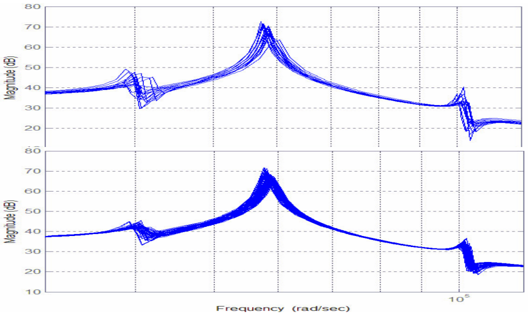

Fig. 3 Actual and simulated plant variations shown for response from PZT input. Top: experimental data, bottom: simulated variations using only three uncertain parameters.

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