Full Paper Sheet Control Using Hybrid Automata

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Abstract. Some high speed color printers require that the sheets be accurately controlled in order to achieve a precise alignment of colors. To accomplish this goal a steerable nips mechanism has been proposed as the actuator. This steerable nips mechanism allows the sheet to be precisely controlled in longitudinal, lateral and skew directions. In this paper we develop a control strategy based on hybrid automata that precisely controls the position of the sheet. This hybrid control law has four finite states among which the system switches during the trajectories tracking process. Switching from one state to another is necessary since the normal control mode cannot be used when the trajectory being tracked requires the wheels to have zero angular velocity. The proposed controller is able to move the sheet from an initial position at rest to an arbitrary final position also at rest. The system model is nonlinear and subject to four nonholonomic constraints. Two of these constraints come from the fact that the velocities perpendicular to the wheels must be zero, and the other two constraints are due to the no-slip condition.

1 Introduction

Some high speed color printers require that sheets be accurately positioned so that colors can be accurately placed on the sheet. This is a challenge especially at high speeds. In this paper we propose a solution to this problem using hybrid automata.

Hybrid automata are dynamical systems which involve the interaction of continuous and discrete dynamics. Systems of this type naturally arise in a number of engineering applications. For example, they have been successfully used in air traffic control [1], automotive control [2], bioengineering [3], process control [4,5], highway systems [6,7], and manufacturing [8]. The particular needs of these applications have sparked the development of theoretical and computational tools for modeling, simulation, analysis, verification, and controller synthesis for hybrid systems.

To accomplish our goal of accurately positioning sheets in a color printer, a steerable nips mechanism has been proposed as the actuator. It is schematically depicted in Figure 1. The problem of controlling paper trajectories with steerable nips is similar to a two wheel robot, such as the one studied in [9]. However, the proposed control law of [9] fails to account for singularities that arise when the steering angle of the wheels approach zero. Also, in the case of the two-wheel

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robot, three inputs are needed to follow the reference trajectories. Similar to the two-wheel robot, the steerable nips mechanism is a nonholonomic system. These systems have been extensively studied. Analytic work related to this subject can be found at [10], [11].

The control objective considered here is to move the sheet on the plane from an initial position $(x(0), y(0), \phi(0), \delta(0))$ at rest, to a final position $(0, 0, 0, 0)$. The generalized coordinates x, y, ϕ are the position of paper in longitudinal $\underline{i}_u$, lateral $\underline{j}_u$, and skew $\underline{k}_u$ directions. The generalize coordinate δ represents the change in length of the line that connects point 1 and 2 along the sheet, i.e. the amount of buckling or stretching of the sheet ($\delta = 0$ for a flat sheet). Hybrid automata are used to control the position of a sheet. This control strategy uses both steerable and non-steerable nips to track the paper trajectories. Steerable nips permit a more swift correction of lateral errors. Non-steerable nips can only indirectly correct lateral errors through the steering of the media.

Also, the paper can neither be stretched nor compressed, for this reason the proposed controller tracks to zero the velocity, $\dot{\delta}$, and the change in length of the line that connects point 1 and 2 along the sheet, δ. This can be seen in Figure 3. The system model has four inputs, inputs one and two rotate wheels one and two respectively. Inputs three and four steer wheels one and two respectively.

To move the sheet from rest to any other position, the proposed hybrid controller has four discrete control modes. During normal control operation both wheels are driven and steered to move the paper toward its final position while enforcing the constraints. The second control mode is needed because of the fact that the normal control fails when either velocity of the wheel approaches zero. For this case, an alternative control law is derived. In this control mode the longitudinal position, x, and the change in length of the line that connects point 1 and 2 along the sheet, i.e. the amount of buckling or stretching of the sheet, δ, are the only outputs being tracked. The third and fourth control modes are used at the beginning and at the end of the controlled motion. During these times the angles of the wheels are zero.

Results obtained in this paper show that by using hybrid automata it is possible to drive the paper from rest to any other position. This was accomplished while satisfying the nonholonomic constraints at all times.

The remainder of this paper is organized as follows. In §2 will derive that nonholonomic constraints, kinematic model, and dynamic model of the steerable nips mechanism. The control modes of the system will be derived in §3. The proposed hybrid automaton will be presented in §4. Results will be shown in §5. Finally, we draw some conclusions in §6.

2 Kinematic and Dynamic Model of the Steerable Nips Mechanism

The steerable nips and a sheet are shown in Figs. 2-3. The sheet moves on a flat surface. Figure 2 represents a sheet position while it is being tracked. The left corner of the sheet, point C is tracked to a desired trajectory. The angular

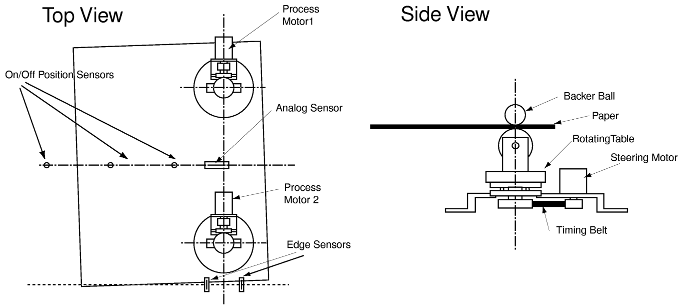

Fig. 1. Steerable Nips Shematic

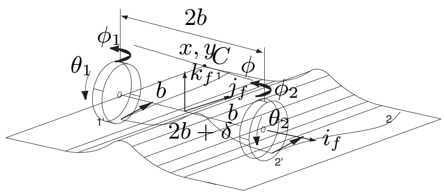

Fig. 2. Steerable Nips with Paper Buckled

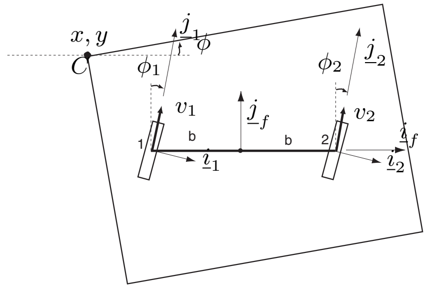

Fig. 3. Coordinate System of Steerable Nips

orientation of the sheet is also tracked. It is know that a sheet can be easily buckled but can not stretched.

2.1 Notation

Figure 3 shows a schematic representation of the modeling variables of the steerable nips system. This system has two independently steering wheels, located at points 1 and 2 respectively. These steerable wheels are separated by a distance $2b$.

Three coordinate frames are defined to describe the position and orientation of the paper. The coordinate frame with $(\underline{i}_f, \underline{j}_f, \underline{k}_f)$ represents the global coordinates. The other two sets of coordinates, $(\underline{i}_1, \underline{j}_1, \underline{k}_1)$, and $(\underline{i}_2, \underline{j}_2, \underline{k}_2)$ are local and are attached to wheel 1 and 2 respectively. The generalized coordinates of the system are $(x, y, \phi, \delta, \theta_1, \theta_2, \phi_1, \phi_2)$. Generalized coordinates x, y will be used to respectably represent the lateral and longitudinal position of the left corner of the sheet, the generalized coordinate ϕ represents the angular position of the sheet and the generalized coordinate δ represents the change in length of the line that connects point 1 and 2 along the sheet, i.e. the amount of buckling or stretching of the sheet ($\delta = 0$ for a flat sheet). Generalized coordinates θ_1 and ϕ_1 will be used to respectably describe the angular position of wheel 1 in the directions parallel and perpendicular to the sheet. Likewise, θ_2 and ϕ_2 will respectably describe the angular position of wheel 2 in the directions parallel and perpendicular to the sheet.

2.2 Velocities

The velocities of the paper at points 1 and 2 in global coordinates are

$$\underline{v}_1 = (\dot{x} + \dot{\phi}y)\underline{i}_f + (\dot{y} - \dot{\phi}(x + b))\underline{j}_f \tag{1}$$

$$\underline{v}_2 = (\dot{x} + \dot{\phi}y)\underline{i}_f + (\dot{y} + \dot{\phi}(-x + b + \delta))\underline{j}_f \tag{2}$$

Invoking the non-slip condition, they can also be written in terms of the angular speed of the wheels in its local coordinates

$$\underline{v}_1 = r\dot{\theta}_1\underline{i}_1 \tag{3}$$

$$\underline{v}_2 = r\dot{\theta}_2\underline{i}_2 \tag{4}$$

where r is the radius of the wheels.

2.3 Constraint Equations

Four constraint equations can be obtained by writing Eq. (1) and Eq. (2) in terms of the local coordinates. The velocities at 1 and 2 in local coordinates are:

$$\begin{aligned} \underline{v}_1 = &((\dot{x} + \dot{\phi}y)\cos\phi_1 - (\dot{y} - \dot{\phi}(x + b)\sin\phi_1))\underline{i}_1 \\ &+((\dot{x} + \dot{\phi}y)\sin\phi_1 + (\dot{y} - \dot{\phi}(x + b)\cos\phi_1))\underline{j}_1 \end{aligned} \tag{5}$$

$$\begin{aligned} \underline{v}_2 = &((\dot{x} + \dot{\phi}y)\cos\phi_2 - (\dot{y} + \dot{\phi}(-x + b + \delta)\sin\phi_2))\underline{i}_2 \\ &+((\dot{x} + \dot{\phi}y)\sin\phi_2 + (\dot{y} + \dot{\phi}(-x + b + \delta)\cos\phi_2))\underline{j}_2 \end{aligned} \tag{6}$$

This gives us four nonholonomic constraints. Two come from the fact that the velocities perpendicular to the wheels at point 1 and 2 are zero. This means that the velocity at 1 in the direction $\underline{i}_1$ must be zero. The same must be true at 2. Thus, its velocity in the direction $\underline{i}_2$ must also be zero.

$$(\dot{x} + \dot{\phi}y)\cos\phi_1 - (\dot{y} - \dot{\phi}(x + b)\sin\phi_1) = 0 \tag{7}$$

$$(\dot{x} + \dot{\phi}y)\cos\phi_2 - (\dot{y} + \dot{\phi}(-x + b + \delta)\sin\phi_2) = 0 \tag{8}$$

The other two constraints are due to the non-slip condition.

$$(\dot{x} + \dot{\phi}y)\sin\phi_1 + (\dot{y} - \dot{\phi}(x + b)\cos\phi_1) = r\dot{\theta}_1 \tag{9}$$

$$(\dot{x} + \dot{\phi}y)\sin\phi_2 + (\dot{y} + \dot{\phi}(-x + b + \delta)\cos\phi_2) = r\dot{\theta}_2 \tag{10}$$

Using the previously defined generalized coordinate $p = \begin{bmatrix} x \; y \; \phi \; \delta \; \theta_1 \; \theta_2 \; \phi_1 \; \phi_2 \end{bmatrix}^T$, each constraint can be written as

$$a_i(p)\dot{p} = 0 \quad i = 1, ..., 4 \quad p \in \mathbb{R}^8$$

For our system the constraints can be written in matrix form as

$$A(p)\dot{p} = 0 \tag{11}$$

where $A(p)$ is the 4×8 matrix defined below.

$$A(p) = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & y\cos\phi_1 + (x+b)\sin\phi_1 & 0 & 0 & 0 & 0 & 0 \\ \sin\phi_1 & \cos\phi_1 & y\sin\phi_1 - (x+b)\cos\phi_1 & 0 & -r & 0 & 0 & 0 \\ \cos\phi_2 & -\sin\phi_2 & y\cos\phi_2 - (-x+b+\delta)\sin\phi_2 & \cos\phi_2 & 0 & 0 & 0 & 0 \\ \sin\phi_2 & \cos\phi_2 & y\sin\phi_2 + (-x+b+\delta)\cos\phi_2 & \sin\phi_2 & 0 & -r & 0 & 0 \end{bmatrix}$$

Equation (11) is refered to as the Pfaffian constraint [12]. The constraint Eq.(11) is nonholonomic and therefore it cannot be integrated.

2.4 Kinematic Model

The kinematic model represents the kinematic relation of the system in the direction that it can move. These are the directions of motion where the nonholonomic constraints are satisfied at all times. As detailed in [12] this is given by a basis of the right null space of the constraints, which will be denoted by $g_j(p) \in \mathbb{R}^n$, $j = 1, ..., n - k = m$. By construction, this basis satisfies

$$a_i(p)g_j(p) = 0 \quad i = 1, ..., k \qquad j = 1, ..., n-k \quad p \in \mathbb{R}^n$$

and all allowable trajectories of the system can thus be written as the possible solutions of the system

$$\dot{p} = g_1(p)u_1 + ... + g_m(p)u_m. \tag{12}$$

That is, $p(t)$ is a feasible trajectory of the system if and only if $p(t)$ satisfies Eq. (12) for a choice of control $u(t) \in \mathbb{R}^m$.

For our system this basis can be obtained by casting equations (3) and (4) in terms of the global coordinates and equating them to (1) and (2) respectively. The velocities at 1 and 2 are respectively given by

$$r\dot{\theta}_1(\sin\phi_1\underline{i}_f + \cos\phi_1\underline{j}_f) = (\dot{x} + \dot{\phi}y)\underline{i}_f + (\dot{y} - \dot{\phi}(x+b))\underline{j}_f \tag{13}$$

$$r\dot{\theta}_2(\sin\phi_2\underline{i}_f + \cos\phi_2\underline{j}_f) = (\dot{x} + \dot{\phi}y)\underline{i}_f + (\dot{y} + \dot{\phi}(-x+b+\delta))\underline{j}_f \tag{14}$$

From Eq. (13) and Eq. (14) the following relations are obtained

$$\dot{x} = (r \sin\phi_1 + \frac{yr}{2b+\delta}\cos\phi_1)\dot{\theta}_1 - \frac{yr}{2b+\delta}\cos\phi_2\dot{\theta}_2 \tag{15}$$

$$\dot{y} = (r \cos\phi_1 - \frac{(x+b)r}{2b+\delta}\cos\phi_1)\dot{\theta}_1 + \frac{(x+b)r}{2b+\delta}\cos\phi_2\dot{\theta}_2 \tag{16}$$

$$\dot{\phi} = \frac{r}{2b+\delta}(\cos\phi_2\dot{\theta}_2 - \cos\phi_1\dot{\theta}_1) \tag{17}$$

$$\dot{\delta} = r\sin\phi_2\dot{\theta}_2 - r\sin\phi_1\dot{\theta}_1 \tag{18}$$

The above equations are the kinematic equations of our system. They can be written in the following form

$$\dot{\underline{p}} = G(\underline{p})\dot{\underline{\eta}} \tag{19}$$

where

$$\dot{\underline{p}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \\ \dot{\delta} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}, G(\underline{p}) = \begin{bmatrix} r\sin\phi_1 + \frac{yr}{2b+\delta}\cos\phi_1 & -\frac{yr}{2b+\delta}\cos\phi_2 & 0 & 0 \\ r\cos\phi_1 - \frac{(x+b)r}{2b+\delta}\cos\phi_1 & \frac{(x+b)r}{2b+\delta}\cos\phi_2 & 0 & 0 \\ -\frac{r}{2b+\delta}\cos\phi_1 & \frac{r}{2b+\delta}\cos\phi_2 & 0 & 0 \\ -r\sin\phi_1 & r\sin\phi_2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \dot{\underline{\eta}} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}$$

In the above equation $\dot{\underline{\eta}} \in \mathbb{R}^4$ is a vector of independent velocities. Note that in general $\dot{\underline{\eta}}$ can be a function that is smooth in $\underline{p}$, and linear in $\dot{\underline{p}}$, $\dot{\underline{\eta}}(\underline{p}, \dot{\underline{p}})$ [10]. The above equation indicates that velocities $\dot{\theta}_1, \dot{\theta}_2, \dot{\phi}_1, \dot{\phi}_2$ are sufficient to determine the instantaneous velocities of all generalized coordinates of the system. Also, note that the velocities calculated with Eq.(19) satisfy the nonholonomic constraints, since $G(\underline{p}) = [g_1(\underline{p}) \quad g_2(\underline{p}) \quad g_3(\underline{p}) \quad g_4(\underline{p})]$ is the right null space of the constraints. That is

$$A(\underline{p})g_j(\underline{p}) = 0$$

Equation(19) is referred as the *kinematic state-model* [13].

2.5 Dynamic Model

A dynamic model of the system is obtained by only considering the dynamics of the actuators. The dynamics due to the mass of the paper has been neglected since it is small. For the normal mode of operation we consider the kinematical model derived in section § 2.4. This system has a two-wheel driven, and two-wheel steered system. Each wheel can turn freely around its horizontal and vertical axis. The contact points between each of the wheels and the paper must satisfy pure rolling and non-slip conditions. The kinematical model of this system is described by Eq. (15), Eq. (16), Eq. (17) and Eq. (18). To get the dynamic model of the system we start by differentiating the kinematic equations, Eq. (15), Eq. (16), Eq. (17) and Eq. (18). For simplicity, we consider actuator dynamics of the following form

$$\ddot{\theta}_1 = u_1, \ddot{\theta}_2 = u_2, \dot{\phi}_1 = u_3, \dot{\phi}_2 = u_4.$$

This gives us the following dynamical system

$$\underline{\dot{x}} = \underline{f}_1(\underline{x}) + \underline{B}_1(\underline{x})\underline{u}_1, \tag{20}$$

where

$$\underline{x} = [x \quad \dot{x} \quad y \quad \dot{y} \quad \phi \quad \dot{\phi} \quad \delta \quad \dot{\delta} \quad \theta_1 \quad \dot{\theta}_1 \quad \theta_2 \quad \dot{\theta}_2 \quad \phi_1 \quad \phi_2]^T,$$

$$\underline{f}_1(\underline{x}) = \begin{bmatrix} f1_1 \\ f1_2 \\ f1_3 \\ f1_4 \\ f1_5 \\ f1_6 \\ 0 \\ f1_8 \\ 0 \\ f1_9 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \underline{B}_1(\underline{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b1_{21} & b1_{22} & b1_{23} & b1_{24} \\ 0 & 0 & 0 & 0 \\ b1_{41} & b1_{42} & b1_{43} & b1_{44} \\ 0 & 0 & 0 & 0 \\ b1_{61} & b1_{62} & b1_{63} & b1_{64} \\ 0 & 0 & 0 & 0 \\ b1_{81} & b1_{82} & b1_{83} & b1_{84} \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \underline{u}_1 = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}.$$

with
$f1_1 = \dot{x}$, $f1_2 = -\dot{\phi}\dot{y} + \frac{y\dot{\delta}\dot{\phi}}{2b+\delta}$, $f1_3 = \dot{y}$, $f1_4 = \dot{\phi}\dot{x} - \frac{(x+b)\dot{\delta}\dot{\phi}}{2b+\delta}$, $f1_5 = \dot{\phi}$, $f1_6 = -\frac{\dot{\delta}\dot{\phi}}{2b+\delta}$, $f1_7 = \dot{\delta}$, $f1_9 = \dot{\theta}_1$, $f1_{11} = \dot{\theta}_2$,$b1_{21} = \frac{yr}{2b+\delta}\cos\phi_1 + r\sin\phi_1$, $b1_{22} = -\frac{yr}{2b+\delta}\cos\phi_2$, $b1_{23} = r\cos\phi_1\dot{\theta}_1 - \frac{yr}{2b+\delta}\sin\phi_1\dot{\theta}_1$, $b1_{24} = +\frac{yr}{2b+\delta}\sin\phi_2\dot{\theta}_2$, $b1_{41} = r\cos\phi_1 - \frac{(x+b)r}{2b+\delta}\cos\phi_1$, $b1_{42} = \frac{(x+b)r}{2b+\delta}\cos\phi_2$, $b1_{43} = -r\sin\phi_1\dot{\theta}_1 + \frac{(x+b)r}{2b+\delta}\sin\phi_1\dot{\theta}_1$, $b1_{44} = -\frac{(x+b)r}{2b+\delta}\sin\phi_2\dot{\theta}_2$, $b1_{61} = -\frac{r}{2b+\delta}\cos\phi_1$, $b1_{62} = \frac{r}{2b+\delta}\cos\phi_2$, $b1_{63} = \frac{r}{2b+\delta}\sin\phi_1\dot{\theta}_1$, $b1_{64} = -\frac{r}{2b+\delta}\sin\phi_2\dot{\theta}_2$, $b1_{81} = -r\sin\phi_1$, $b1_{82} = r\sin\phi_2$, $b1_{83} = -r\cos\phi_1\dot{\theta}_1$, $b1_{84} = r\cos\phi_2\dot{\theta}$.

3 Control Modes

3.1 Normal Feedback Control

In this section we derive the control law for normal operation. During the derivation of the feedback dynamic system we will obtain conditions for mode changes.

In this mode we want to control the position of the paper and satisfy $\delta = 0$ at all time. This means that states (x, y, ϕ, δ) have to be controlled at all times, therefore the output vector function is

$$\underline{y} = h(\underline{x}) = \left[x\ y\ \phi\ \delta \right]^T . \tag{21}$$

The above system is a square Multi-Input Multi-Output(MIMO) system. It is called square because it has as many inputs as outputs. This MIMO system can be linearized by static state feedback. This is accomplished by differentiating the j_{th} output with respect to time, see [15] for details. After differentiating, the following equation is obtained

$$\ddot{\underline{y}} = \left[\ddot{x}\ \ddot{y}\ \ddot{\phi}\ \dot{\delta} \right]^T = \underline{C}_1(\underline{x}) + \underline{E}_1(\underline{x})\underline{u}_1, \tag{22}$$

where

$$\underline{C}_1(\underline{x}) = \begin{bmatrix} c1_1 \\ c1_2 \\ c1_3 \\ 0 \end{bmatrix}, \underline{E}_1(x) = \begin{bmatrix} e1_{11}\ e1_{12}\ e1_{13}\ e1_{14} \\ e1_{21}\ e1_{22}\ e1_{23}\ e1_{24} \\ e1_{31}\ e1_{32}\ e1_{33}\ e1_{34} \\ e1_{41}\ e1_{42}\ e1_{43}\ e1_{44} \end{bmatrix},$$

with
$c1_1 = -\dot{\phi}\dot{y} + \frac{y\dot{\delta}\dot{\phi}}{2b+\delta}$, $c1_2 = \dot{\phi}\dot{x} - \frac{(x+b)\dot{\delta}\dot{\phi}}{2b+\delta}$, $c1_3 = -\frac{\dot{\delta}\dot{\phi}}{2b+\delta}$, $e1_{11} = \frac{yr}{2b+\delta}\cos\phi_1 + r\sin\phi_1$, $e1_{12} = -\frac{yr}{2b+\delta}\cos\phi_2$, $e1_{13} = r\cos\phi_1\dot{\theta}_1 - \frac{yr}{2b+\delta}\sin\phi_1\dot{\theta}_1$, $e1_{14} = \frac{yr}{2b+\delta}\sin\phi_2\dot{\theta}_2$, $e1_{21} = r\cos\phi_1 - \frac{(x+b)r}{2b+\delta}\cos\phi_1$, $e1_{22} = \frac{(x+b)r}{2b+\delta}\cos\phi_2$, $e1_{23} = -r\sin\phi_1\dot{\theta}_1 + \frac{(x+b)r}{2b+\delta}\sin\phi_1\dot{\theta}_1$, $e1_{24} = -\frac{(x+b)r}{2b+\delta}\sin\phi_2\dot{\theta}_2$, $e1_{31} = -\frac{r}{2b+\delta}\cos\phi_1$, $e1_{32} = \frac{r}{2b+\delta}\cos\phi_2$, $e1_{33} = \frac{r}{2b+\delta}\sin\phi_1\dot{\theta}_1$, $e1_{34} = -\frac{r}{2b+\delta}\sin\phi_2\dot{\theta}_2$, $e1_{41} = -r\sin\phi_1$, $e1_{42} = r\sin\phi_2$, $e1_{43} = -r\cos\phi_1\dot{\theta}_1$, $e1_{44} = r\cos\phi_2\dot{\theta}_2$.
Then, choose the following state feedback law:

$$\underline{u}_1 = \underline{E}_1{}^{-1}(\underline{x})(\underline{v}_1 - \underline{C}_1(\underline{x})), \tag{23}$$

where

$$\underline{E}_1{}^{-1}(\underline{x}) = \begin{bmatrix} ie_{11}\ ie_{12}\ ie_{13}\ ie_{14} \\ ie_{21}\ ie_{22}\ ie_{23}\ ie_{24} \\ ie_{31}\ ie_{32}\ ie_{33}\ ie_{34} \\ ie_{41}\ ie_{42}\ ie_{43}\ ie_{44} \end{bmatrix}$$

with
$ie_{11} = \frac{\cos\phi_1\sin\phi_1}{r\cos\phi_1}$,$ie_{12} = \frac{1-\sin\phi_1^2}{r\cos\phi_1}$, $ie_{13} = \frac{-b-x-\sin\phi_1(-y\cos\phi_1-(b+x)\sin\phi_1)}{r\cos\phi_1}$, $ie_{21} = \frac{\cos\phi_2\sin\phi_2}{r\cos\phi_2}$, $ie_{22} = \frac{\cos\phi_2^2}{r\cos\phi_2}$, $ie_{23} = \frac{\cos\phi_2((b-x+\delta)\cos\phi_2+y\sin\phi_2)}{r\cos\phi_2}$, $ie_{24} = \frac{\cos\phi_2\sin\phi_2}{r\cos\phi_2}$, $ie_{31} = \frac{\cos\phi_1}{r\dot{\theta}_1}$, $ie_{32} = \frac{-sin\phi_1}{r\dot{\theta}_1}$, $ie_{33} = \frac{y\cos\phi_1+(b+x)\sin\phi_1}{r\dot{\theta}_1}$, $ie_{41} = \frac{y\cos\phi_2}{r\dot{\theta}_2}$, $ie_{42} = -\frac{sin\phi_2}{r\dot{\theta}_2}$, $ie_{43} = \frac{y\cos\phi_2-(b-x+\delta)\sin\phi_2}{r\dot{\theta}_2}$, $ie_{44} = \frac{\cos\phi_2}{r\dot{\theta}_2}$
This yields the linear closed loop system

$$\left[\ddot{x}\ \ddot{y}\ \ddot{\phi}\ \dot{\delta} \right]^T = \left[v_1\ v_2\ v_3\ v_4 \right]^T . \tag{24}$$

At this point we have four decoupled equations. This means that v_1, v_2, v_3 and v_4 only affect the outputs x,y,ϕ and δ respectably. Choose $v_1 = \ddot{x}_d + k_1\dot{\tilde{x}} + q_1\tilde{x}$,

$v_2 = \ddot{y}_d + k_2\dot{\tilde{y}} + q_2\tilde{y}$, $v_3 = \ddot{\phi}_d + k_3\dot{\tilde{\phi}} + q_3\tilde{\phi}$ and $v_4 = \ddot{\delta}_d + k_4\dot{\tilde{\delta}} + q_4\tilde{\delta}$ where $\tilde{x} = x_d - x$, $\tilde{y} = y_d - y$, $\tilde{\phi} = \phi_d - \phi$, $\tilde{\delta} = \delta_d - \delta$, and k_1, k_2, k_3, k_4, q_1, q_2, q_3, q_4 are positive constants. The choice of v_i with positive constants for k_i, q_i give exponentially decaying errors. The differential equation of these errors will be $\ddot{\tilde{x}}_i + k_1\dot{\tilde{x}} + q_1\tilde{x} = 0$, $\ddot{\tilde{y}}_i + k_2\dot{\tilde{x}} + q_2\tilde{x} = 0$, $\ddot{\tilde{\phi}}_i + k_3\dot{\tilde{\phi}} + q_3\tilde{\phi} = 0$ and $\ddot{\tilde{\delta}}_i + k_4\dot{\tilde{\delta}} + q_4\tilde{\delta} = 0$.

Note that this control law, $\underline{u}_1$, has terms with $\dot{\theta}_1$ and $\dot{\theta}_2$ in the denominator. This means that this matrix will be ill conditioned if either $\dot{\theta}_1$ or $\dot{\theta}_2$ is zero. This makes this control law unusable when $\dot{\theta}_1$ or $\dot{\theta}_2$ are close or equal to zero. Based on the above discussion we introduce the first discrete control mode q_1:

q_1: Trajectory tracking $\left[x_{ref}\ y_{ref}\ \phi_{ref}\ \delta_{ref}\right]^T$. This mode is conditioned on $\dot{\theta}_1 \geq \dot{\theta}_{min} \wedge \dot{\theta}_2 \geq \dot{\theta}_{min}$.

3.2 Feedback Control to Overcome Wheel Velocity Singularity

It was shown above that the normal feedback control fails when the velocity of either wheel is zero (angular velocity of the wheels in the direction parallel to the sheet). This comes from the fact the lateral error cannot be corrected unless the media is moving. For this case we derive another control law. We choose to track the longitudinal position and the change in nominal distance of point 1 and 2 along the sheet (x, δ). Also, during the tracking of x and δ we will maintain the orientation of the wheels fixed therefore $\dot{\phi}_1$ and $\dot{\phi}_2$ will be zero. For this mode the state space equation and the output function are

$$\dot{\underline{x}} = \underline{f}_2(\underline{x}) + \underline{B}_2(\underline{x})\underline{u}_2, \tag{25}$$

$$\underline{y} = h(\underline{x}) = \left[x\ \delta\right]^T \tag{26}$$

where

$$\underline{f}_2(\underline{x}) = \left[f2_1\ f2_2\ f2_3\ f2_4\ f2_5\ f2_6\ f2_7\ 0\ f2_9\ 0\ f2_{11}\ 0\ 0\ 0\right]^T,$$

$$\underline{B}_2 = \begin{bmatrix} 0\ b2_{12}\ 0\ b2_{14}\ 0\ b2_{16}\ 0\ b2_{18}\ 0\ 1\ 0\ 0\ 0\ 0 \\ 0\ b2_{22}\ 0\ b2_{44}\ 0\ b2_{26}\ 0\ b2_{28}\ 0\ 0\ 1\ 0\ 0\ 0 \end{bmatrix}^T, \underline{u}_2 = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

with

$f2_1 = \dot{x}$, $f2_2 = -\dot{\phi}\dot{y} + \frac{y\dot{\delta}\dot{\phi}}{2b+\delta}$, $f2_3 = \dot{y}$, $f2_4 = \dot{\phi}\dot{x} - \frac{(x+b)\dot{\delta}\dot{\phi}}{2b+\delta}$, $f2_5 = \dot{\phi}$, $f2_6 = -\frac{\dot{\delta}\dot{\phi}}{2b+\delta}$, $f2_7 = \dot{\delta}$, $f2_9 = \dot{\theta}_1$, $f2_{11} = \dot{\theta}_2$, $b2_{12} = \frac{yr}{2b+\delta}\cos\phi_1 + r\sin\phi_1$, $b2_{22} = -\frac{yr}{2b+\delta}\cos\phi_2$, $b2_{14} = r\cos\phi_1 - \frac{(x+b)r}{2b+\delta}\cos\phi_1$, $b2_{24} = \frac{(x+b)r}{2b+\delta}\cos\phi_2$, $b2_{16} = -\frac{r}{2b+\delta}\cos\phi_1$, $b2_{26} = \frac{r}{2b+\delta}\cos\phi_2$, $b2_{18} = -r\sin\phi_1$, $b2_{28} = r\sin\phi_2$.

Again differentiating the output until one of the inputs appears and cancelling the terms that have $\dot{\phi}_1$ and $\dot{\phi}_2$, we get the following output equation:

$$\underline{y} = \left[\ddot{x}\ \ddot{\delta}\right]^T = \underline{C}_2(\underline{x}) + \underline{E}_2(\underline{x})\underline{u}_2, \tag{27}$$

where

$$\underline{C}_2(\underline{x}) = \begin{bmatrix} -\dot{\phi}\dot{y} + \frac{y\dot{\delta}\dot{\phi}}{2b+\delta} \\ 0 \end{bmatrix}, \underline{E}_2(\underline{x}) = \begin{bmatrix} \frac{yr}{2b+\delta}\cos\phi_1 + r\sin\phi_1 & -\frac{yr}{2b+\delta}\cos\phi_2 \\ -r\sin\phi_1 & r\sin\phi_2 \end{bmatrix}.$$

The state feedback control law for this system is

$$\underline{u}_2 = \underline{E}_2{}^{-1}(\underline{x})(\underline{v}_2 - \underline{C}_2(\underline{x})), \tag{28}$$

where

$$\underline{E}_2{}^{-1}(\underline{x}) = \begin{bmatrix} \frac{(2b+\delta)\sin\phi_2}{d_2} & \frac{y\cos\phi_2}{d_2} \\ \frac{(2b+\delta)\sin\phi_1}{d_2} & \frac{y\cos\phi_1+(2b+\delta)\sin\phi_1}{d_2} \end{bmatrix}.$$

with

$$d_2 = r(-y\cos\phi_2\sin\phi_1 + (y\cos\phi_1 + (2b+\delta)\sin\phi_1)\sin\phi_2).$$

The above state feedback yields the following linear closed loop system

$$\ddot{\underline{y}} = \begin{bmatrix} \ddot{x} \ \ddot{\delta} \end{bmatrix}^T = \begin{bmatrix} v_1 \ v_2 \end{bmatrix}^T. \tag{29}$$

For this system we have two decoupled equations. This means that v_i only affects the output y_i, for $i = 1, 2$. Choosing $v_1 = \ddot{x}_d + k_1\dot{\tilde{x}} + q_1\tilde{x}$ and $v_2 = \ddot{\delta}_d + k_2\dot{\tilde{\delta}} + q_2\tilde{\delta}$ where $\tilde{x} = x_d - x$, $\tilde{\delta} = \delta_d - \delta$, k_1, k_2, q_1, and q_2 are positive constants, we get errors that are exponentially decaying. The differential equations of these errors will be $\ddot{\tilde{x}} + k_1\dot{\tilde{x}} + q_1\tilde{x} = 0$ and $\ddot{\tilde{\delta}} + k_2\dot{\tilde{\delta}} + q_2\tilde{\delta} = 0$. Note that the matrix in the control law (28) will be ill conditioned if $\phi_1 = \phi_2 = 0$. Therefore, this control law cannot be used under these conditions. This case occurs when the lateral errors have been corrected. Based on the above discussion the second discrete control mode is as follows:
q_2: Trajectory tracking $\begin{bmatrix} x_{ref} \ \delta_{ref} \end{bmatrix}^T$. This mode is condition on $(\dot{\theta}_1 \leq \dot{\theta}_{min} \vee \dot{\theta}_2 \leq \dot{\theta}_{min}) \wedge (\phi_1 \neq \phi_2 \neq 0)$.

3.3 Feedback Control with No Steering

The two wheels will have a zero steering angle when the lateral error has been corrected or at initial operation when the wheels are starting to move. This takes us to the third and fourth discrete control modes. In these modes of operation the wheels have zero steering angle. The control law used in these cases was developed by Kanayama [16]. Kanayama proposed a control law that makes use of reference velocities and current posture of the vehicle to control the vehicle position. Note that this problem is similar to the one of moving paper with fixed wheels. This control rule uses the linear and rotational reference velocities of the paper to correct for the position errors and follow a given trajectory. The kinematic model for this system is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos(\phi) & 0 \\ \sin(\phi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix}, \begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}. \tag{30}$$

The nonholonomic constraints of this system is

$$\dot{x}\sin(\phi) - \dot{y}cos(\phi) = 0. \tag{31}$$

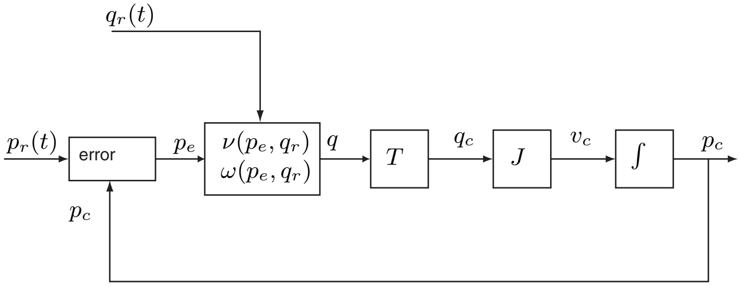

Fig. 4. Tracking Controller Block Diagram for Unsteerable Wheels

The error posture used by the controller is defined as

$$p_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\phi}_e \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} (p_r - p), \tag{32}$$

where p_r is the reference posture. The block diagram of this controller is shown in Figure 4. The law for the velocities is as follows:

$$q = \begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} \nu(p_e, q_r) \\ \omega(p_e, q_r) \end{bmatrix} = \begin{bmatrix} \nu_r \cos(\phi_e) + K_x x_e \\ \omega_r + \nu_r (K_y y_e + K_\phi \sin(\phi_e)) \end{bmatrix}, \tag{33}$$

where K_x, K_y and K_ϕ are positive constants, and ν_r and ω_r are the reference velocities. A proof of the stability of this control law can be found in [16]. The following Lyapounov function candidate (34) was used to show that the control law is asymptotically stable.

$$V = \frac{1}{2}(x_e^2 + y_e^2) + (1 - \cos(\phi_e))/K_y. \tag{34}$$

Based on the above discussion the third and fourth discrete control mode correspond to the case when the lateral errors have been corrected and when the wheels are starting to move from rest. They are respectively:

q_3: Trajectory tracking $\begin{bmatrix} x_{ref} \; y_{ref} \; \phi_{ref} \end{bmatrix}^T$. This mode is conditioned on $(\dot{\theta}_1 \leq \dot{\theta}_{min} \vee \dot{\theta}_2 \leq \dot{\theta}_{min}) \wedge (\phi_1 = \phi_2 = 0)$.

q_4: Trajectory tracking $\begin{bmatrix} x_{ref} \; y_{ref} \; \phi_{ref} \end{bmatrix}^T$. This mode is conditioned on $(\phi_1 = \phi_2 = 0) \wedge y \leq y_{min}$.

4 Control Using Hybrid Automata

In this section we construct a switched mode feedback controller which provides global finite-time convergence of the continuous time states to the origin. The switched mode feedback controller is constructed as a hybrid system comprising

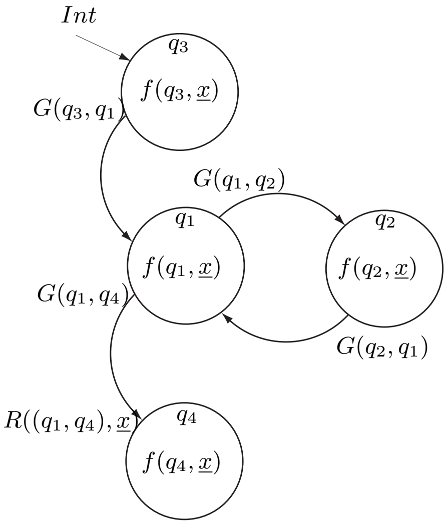

Fig. 5. Hybrid Automaton for a Steerable Nips System

of states at which the system will switch to during the tracking of the trajectories. A hybrid automaton of the steerable nips is shown in Figure 5. There are four discrete states in which the system can be at a given time. The state q_3 represents the initial state. In this state the velocity of the sheet is the only thing being tracked. The reason for tracking only the speed is that, at the start, the velocities of the wheels are zero. The full steerable nips system cannot be used since the control law requires that the speeds of the wheels are nonzero. Once both wheels have reached the minimum speed, $\dot{\theta}_{min}$, the normal control law, q_1, can be used. During the normal control mode the controller will track the references. Longitudinally, the paper will track a sinusoidal reference with a given magnitude. The lateral and angular references are zero. The reason to track a sinusoidal reference is that we want to oscillate between the final longitudinal position while the lateral and angular errors are being corrected. Tracking the sinusoidal reference means that at a given point the wheel velocities have to be zero. This is the point at which the second control mode, q_2, will be used. Finally, once the lateral errors have been corrected, the control mode will be switched from q_1 to q_4. At this point the lateral errors and angular errors have been corrected and the system needs to be driven to its final longitudinal positon $x = 0$. The hybrid automaton of this system is defined based on [17], that is,

- $Q = \{q_1, q_2, q_3, q_4\}$
- $\mathbb{X} : \underline{x} = \left[\, x \; \dot{x} \; y \; \dot{y} \; \phi \; \dot{\phi} \; \delta \; \dot{\delta} \; \theta_1 \; \dot{\theta}_1 \; \theta_2 \; \dot{\theta}_2 \; \phi_1 \; \phi_2 \,\right]$
- $Int = \{q\} \times \{\underline{x} \in \mathbb{R}^{14} : \dot{x} = \dot{y} = \dot{\phi} = \delta = \dot{\delta} = \theta_1 = \dot{\theta}_1 = \theta_2 = \dot{\theta}_2 = \phi_1 = \phi_2 = 0\}$
- $f(q_1, \underline{x}, \underline{u}_1) = f_1(\underline{x}) + \underline{B}_1 \underline{u}_1$
- $[\ddot{x} \quad \ddot{y} \quad \ddot{\phi} \quad \ddot{\delta}]^T = \underline{C}_1(\underline{x}) + \underline{E}_1(\underline{x})\underline{u}_1$
- $\underline{u}_1 = \underline{E}_1^{-1}(\underline{x})(\underline{v}_1 - \underline{C}_1(\underline{x}))$, $v_{11} = \ddot{x}_d + k_1\dot{\tilde{x}} + q_1\tilde{x}$, $v_{12} = \ddot{y}_d + k_2\dot{\tilde{y}} + q_2\tilde{y}$,

$v_{13} = \ddot{\phi}_d + k_3\dot{\tilde{\phi}} + q_3\tilde{\phi}$, $v_{14} = \ddot{\delta}_d + k_4\dot{\tilde{\delta}} + q_4\tilde{\delta}$, where $\tilde{x} = x_d - x$, $\tilde{y} = y_d - y$, $\tilde{\phi} = \phi_d - \phi$, $\tilde{\delta} = \delta_d - \delta$ and $k_1, k_2, k_3, k_4, q_1, q_2, q_3, q_4 \in \mathbb{R}_+$

- $f(q_2, \underline{x}, \underline{u}_2) = f_2(\underline{x}) + \underline{B}_2\underline{u}_2$
- $[\ddot{x} \quad \ddot{\delta}]^T = \underline{C}_2(\underline{x}) + \underline{E}_2(\underline{x})\underline{u}_2$
- $\underline{u}_2 = \underline{E}_2^{-1}(\underline{x})(\underline{v}_2 - \underline{C}_2(\underline{x}))$, $v_{21} = \ddot{x}_d + k_1\dot{\tilde{x}} + q_1\tilde{x}, v_{22} = \ddot{\delta}_d + k_4\dot{\tilde{\delta}} + q_4\tilde{\delta}$ where $\tilde{x} = x_d - x, \tilde{\delta} = \delta_d - \delta$ and $k_1, k_4, q_1, q_4 \in \mathbb{R}+$
- $f(q_3, \underline{x}, \underline{u}_3) \quad such \quad that$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos(\phi) & 0 \\ \sin(\phi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix}, \begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}, \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$p_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\phi}_e \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{ref} - x \\ y_{ref} - y \\ \phi_{ref} - \phi \end{bmatrix}$$

$$q = \begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} \nu(p_e, q_r) \\ \omega(p_e, q_r) \end{bmatrix} = \begin{bmatrix} \nu_r \cos(\phi_e) + K_x x_e \\ \omega_r + \nu_r(K_y y_e + K_\phi \sin(\phi_e)) \end{bmatrix}$$

with $x_{ref}(t) = At$, $A \in \mathbb{R}+$, $\nu_{ref} = \nu_{min}$, $\omega_{ref} = 0$.

- $f(q_4, \underline{x}, \underline{u}_4) = f(q_3, \underline{x}, \underline{u}_3)$ with $x_{ref} = 0$, $\nu_{ref} = 0$, $\omega_{ref} = 0$.
- $D(q_1) = \{\underline{x} \in \mathbb{X} : \dot{\theta}_1 \geq \dot{\theta}_{min} \wedge \dot{\theta}_2 \geq \dot{\theta}_{min}\}, D(q_2) = \{\underline{x} \in \mathbb{X} : \dot{\theta}_1 \leq \dot{\theta}_{min} \vee \dot{\theta}_2 \leq \dot{\theta}_{min} \wedge (\phi_1 \neq \phi_2 \neq 0)\}, D(q_3) = \{\underline{x} \in \mathbb{X} : \dot{\theta}_1 \leq \dot{\theta}_{min} \vee \dot{\theta}_2 \leq \dot{\theta}_{min} \wedge (\phi_1 = \phi_2 = 0)\}, D(q_4) = \{(\phi_1 = \phi_2 = 0) \wedge y \leq y_{min}\}$
- $E = \{(q_1, q_2), (q_2, q_1), (q_2, q_4), (q_3, q_2)\}$
- $G(q_1, q_2) = \{\underline{x} \in \mathbb{X} : \dot{\theta}_1 \leq \dot{\theta}_{min} \vee \dot{\theta}_2 \leq \dot{\theta}_{min} \wedge (\phi_1 \neq \phi_2 \neq 0)\}, G(q_2, q_1) = \{\underline{x} \in \mathbb{X} : \dot{\theta}_1 \geq \dot{\theta}_{min} \wedge \dot{\theta}_2 \geq \dot{\theta}_{min}\}, G(q_3, q_1) = \{\underline{x} \in \mathbb{X} : \dot{\theta}_1 \geq \dot{\theta}_{min} \wedge \dot{\theta}_2 \geq \dot{\theta}_{min}\}, G(q_1, q_4) = \{y \leq y_{min}\}$
- $R((q_1, q_2), \underline{x}) = R((q_2, q_1), \underline{x}) = R((q_3, q_1), \underline{x}) = \underline{x}$,
  $R((q_1, q_4), \underline{x}) = (x, \dot{x}, y, \dot{y}, \phi, \dot{\phi}, \delta, \dot{\delta}, \theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, 0, 0)$

5 Simulation Results

The model was simulated for the following initial conditions. $x_0 = -10mm, y_0 = 10mm, \phi = 4^o$. Simulation results are shown in Figs. 6 - 9. Figure 6 shows the trajectory of the paper as it goes from its initial position at $(x, y, \phi, \delta) = (-10mm, 10mm, 4^o, 0)$ to its final position at (x,y,ϕ,δ)=(0,0,0,0). Figure 7 shows the position of the paper and control modes used during the simulation. As shown the initial control mode, q_3 is used for less that 0.001 second. The reason is that the normal feedback control can be used only when the velocity of the wheels are non zero. This happens almost instantaneously. Also note that the normal feedback control q_1 is used most of the time during the correction of lateral

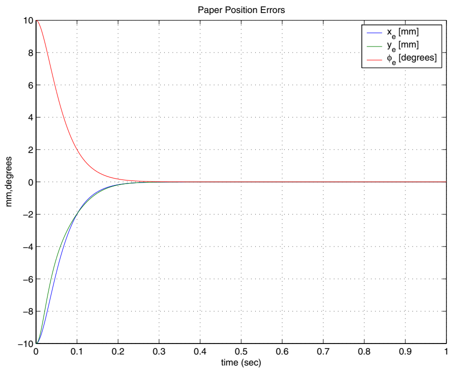

Fig. 6. Paper Position

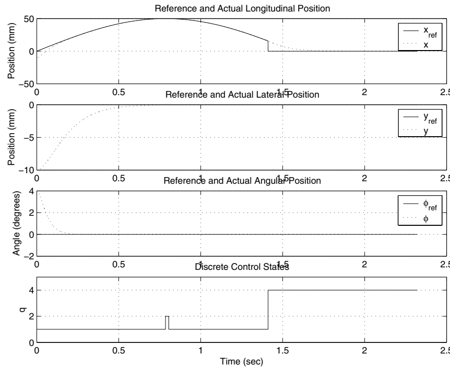

Fig. 7. Paper Position vs. Time

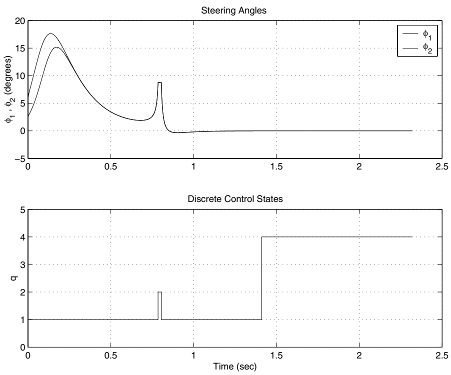

Fig. 8. Wheels Steering Angle vs. Time

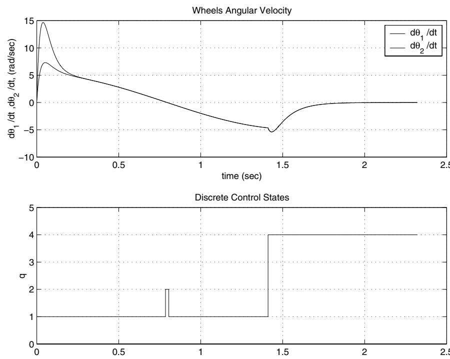

Fig. 9. Angular velocities of the Wheels vs. Time

displacement. The steering angles of both wheels are shown in Figure 8 and the velocity of both wheels are shown in Figure 9. The initial steering angles of the wheels are zero. They are steered immediately once the normal control mode is in use, this action will correct the lateral errors. The steering angle will become zero once the lateral errors have been corrected, and at this point the control mode switches from q_1 to q_4.

6 Conclusion

In this paper we have successfully used hybrid automata to fully control the position of a sheet. The use of hybrid automata was necessary to overcome singularities of the normal control action law. This singularity was due to the fact that the control law matrix has elements with angular velocities in the denominator. For this matrix not to be ill conditioned the angular velocities of the wheels cannot be zero. During the actuation of the paper the velocity of the wheels are zero. This happens at the beginning since the sheet starts at rest. Also, the longitudinal reference is sinusoidal with zero mean; therefore the velocity will be zero at one point during the actuation. Results obtained in this paper have shown that by using hybrid automata it is possible to drive the paper from rest to any other position also at rest. This can be accomplished by satisfying the nonholonomic constraints at all time.

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