A Multi-rate Nonlinear State Estimator for Hard Disk Drives

Keitaro OHNO* and Roberto HOROWITZ**

* Autonomous System Laboratory, Fujitsu Laboratories Limited, Atsugi, Japan, k.ohno@jp.fujitsu.com
** Department of Mechanical engineering, University of California at Berkeley, USA, horowitz@me.berkeley.edu

Abstract

In this paper, we propose a new multi-rate state estimator for seeking control of hard disk drives, which has a proportional, integral, and discontinuous error feedback structure to improve robustness. Conventional state estimators do not have enough accuracy under the existence of external disturbances and model uncertainty. With the proposed estimator, the seeking control system of hard disk drives can have better robustness. Our main focus is on the discontinuous action, which alleviates the estimation error caused by, for example, actuator gain variations. Integral action is secondarily introduced not only to cancel out a quasi-constant disturbance and uncertainty, but also to reduce the chattering motion caused by the discontinuous feedback. Simulations and experiments have been carried out to validate our proposed multi-rate state estimator. Finally, an experimental example of seeking control for a single stage actuated hard disk drive is demonstrated.

1. Introduction

Since the utilization of multi-rate feedback control for hard disk drives (HDDs) was proposed [1], many authors have discussed this issue and it is widely used in order to achieve smoother control input and higher control bandwidth under lower sampling frequency. Among them, some have concentrated on developing a multi-rate state estimator [2] that yields smoother inter-sample estimation, while the others have proposed a multi-rate H_∞ controller [3][4] on the ground that this control design method can handle the robustness against parameter uncertainty. However, these considerations basically lie in passive robustness, and, for practical seeking control, active robustness against parameter uncertainty is required concerning future trends in HDD.

One possible alternative to achieve such active robustness is to introduce an adaptive control scheme as the one that [5] developed for a settling controller, however, these schemes usually take a certain time for adaptation and it is not always effective for short distance seeking. Another possibility is to introduce a variable structure control scheme [6][7], and many efforts have been concentrated on the application to the HDDs. However, few efforts have been concentrated on utilizing variable structure control in a multi-rate framework, and this control scheme is not easily applicable to HDDs.

In this paper, we combine the basic idea of sliding mode observer and integral observer, each of which are respectively introduced by [6] and [8], and put them into a multi-rate state estimator framework, as originally proposed by [2], in order to reduce the problem of chattering motion. Moreover, we consider the following system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w_c(x,u,t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^m$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and we assume that the pair (A,B) is controllable, the pair (A,C) is observable, B and C are both full rank. Time delay is ignored for simplicity. The equivalent input multi-rate system can be described by:

$$\begin{aligned} x(k,i+1) &= \Phi_m x(k,i) + \Gamma_m u(k,i) + w(x,u,k) \\ y(k,i) &= Cx(k,i) \end{aligned} \tag{2}$$

of which $\Phi_m \in \mathbb{R}^{n\times n}$ and $\Gamma_m \in \mathbb{R}^{n\times m}$ can be obtained as follows:

$$\Phi_m = e^{AT_m}, \Gamma_m = \int_0^{T_m} e^{At}dtB \tag{3}$$

where T_m is a control input updating time that can be described by $T_m = T_s / r$, of which T_s is a measurement sampling time, and r is a multi-rate ratio. (k,i) denotes the time $t = kT_s + iT_m$ for $i = 0,1,...,r-1$. $w \in \mathbb{R}^n$ represents any uncertainty and nonlinearity belonging to the range space of some matrix, $\Theta \in \mathbb{R}^{n\times q}$, with restriction, $q \le p$, and it can be thought of as comprising the average and fluctuating behaviors. Thus w can be described as follows:

$$w(x,u,k) = \Theta\{h + \delta h(x,u,k)\} \tag{4}$$

where $h \in \mathbb{R}^q$ is a quasi-constant vector that represents the average uncertainty behavior, and $\delta h \in \mathbb{R}^q$ represents the fluctuating behavior. In addition, we assume

$$\|\delta h(x,u,k)\| < h^+(x,u,k), \tag{5}$$

where $h^+ \in \mathbb{R}^q$ is a known scalar function. We also assume that δh is slowly varying. Practically, by slowly varying, we mean its frequency is less than, at least, a quarter of possible update frequency of variable feedback structure.

Throughout this paper, $\rho(\bullet)$ denotes the spectral radius, $\|\bullet\|$ denotes the Euclidian norm for vectors and the spectral norm for matrices, and $\lambda_{\min}(\bullet)$ denotes the smallest eigenvalue.

2. Estimator with integral feedback

The conventional multi-rate state estimator [2] can be described as follows:

This work was conducted at the Computer Mechanics Labora- tory of the University of California at Berkeley.

$$\bar{x}(k,i) = \begin{cases} \Phi_m \hat{x}(k-1,r-1) + \Gamma_m u(k-1,r-1) \cdots (i=0) \\ \Phi_m \hat{x}(k,i-1) + \Gamma_m u(k,i-1) \cdots (i=1,...,r-1) \end{cases} \quad (6)$$

$$\hat{x}(k,i) = \bar{x}(k,i) + L_i(y(k,0) - C\bar{x}(k,0)) \quad (7)$$

$$\hat{e}(k,i) = x(k,i) - \hat{x}(k,i), \ \bar{e}(k,i) = x(k,i) - \bar{x}(k,i) \quad (8)$$

where $\bar{x}$ is a prediction estimate of x, $\hat{x}$ is a current estimate of x, $\hat{e},\bar{e}$ are estimation error for $\hat{x},\bar{x}$, respectively, and $L_i \in \mathbb{R}^{n\times p}$ is a i-th state estimator gain. Then, from the above equations, we can derive the prediction estimation-error equation as follows, from which we can easily understand how w causes an estimation error:

$$\bar{e}(k,i+1) = \Phi_m \bar{e}(k,i) - \Phi_m L_i C\bar{e}(k,0) + w(x,u,k) \quad (9)$$

Now we add an integrator to the estimation error feedback structure, in order to alleviate the detrimental effect of h in Eq. (4), as follows:

$$\begin{cases} \bar{x}(k,i+1) = \Phi_m \hat{x}(k,i) + \Gamma_m u(k,i) \\ \bar{z}(k+1,0) = \hat{z}(k,0) \end{cases}$$
$$\begin{cases} \hat{x}(k,i) = \bar{x}(k,i) + L_i\{y(k,0) - C\bar{x}(k,0)\} + \Theta_o \hat{z}(k,0) \\ \hat{z}(k,0) = \bar{z}(k,0) + K\{y(k,0) - C\bar{x}(k,0)\} \end{cases} \quad (10)$$

where $z \in \mathbb{R}^j$ denotes an integral state, $K \in \mathbb{R}^{j\times p}$ denotes a corresponding gain matrix, and $\Theta_o \in \mathbb{R}^{n\times j}$ denotes a weighting matrix that allows us to arbitrarily choose the desired feedback channel. It should be pointed out that this structure has flexibility with respect to the size of integral state, as we have used the notation $j: 1 \le j \le p$ for the definition of the size of each associated matrix and vector.

Then from Eq. (2) and Eq. (10), the prediction estimation error dynamics can be described as follows:

$$\begin{bmatrix} \bar{e}(k+1,0) \\ \bar{z}(k+1,0) \end{bmatrix} = \begin{bmatrix} M & N \\ KC & I \end{bmatrix} \begin{bmatrix} \bar{e}(k,0) \\ \bar{z}(k,0) \end{bmatrix} + \sum_{j=0}^{r-1} \Phi_m^j \begin{bmatrix} w \\ 0 \end{bmatrix}, \quad (11)$$

where

$$M(i) \triangleq \Phi_m^i - \sum_{j=1}^{i} \Phi_m^j (L_{i-j} + \Theta_o K)C, \ M \triangleq M(r)$$
$$N(i) \triangleq -\sum_{j=1}^{i} \Phi_m^j \Theta_o, \ N \triangleq N(r) \quad , \quad (12)$$

and its characteristics equation is as follows:

$$\det \begin{bmatrix} zI - M & -N \\ -KC & zI - I \end{bmatrix} = 0 \quad (13)$$

It is of interest to see from Eq. (11) that, by letting $\Theta_o = \Phi_m^{-1}\Theta$, the uncertainty term w can be canceled out by the integral state z through N. In this case, $z \in \mathbb{R}^q$, $K \in \mathbb{R}^{q\times p}$, and $\Theta_o \in \mathbb{R}^{n\times q}$.

On the other hand, regarding the current estimation, the following relation is firstly considered using Eq. (10):

$$\bar{e} = \hat{e} - \bar{x} + x\hat{}$$
$$\therefore \bar{e}(k,0) = X^{-1}\hat{e}(k,0) + X^{-1}\Theta_o \hat{z}(k-1,0) \quad (14)$$
$$X \triangleq (I - L_0 C - \Theta_o KC)$$

We can also have the following relation with respect to $k+1$:

$$\bar{e}(k+1,0) = X^{-1}\hat{e}(k+1,0) + X^{-1}\Theta_o KCX^{-1}e\hat{(}k,0)$$
$$+ (X^{-1}\Theta_o + X^{-1}\Theta_o KCX^{-1}\Theta_o) z(k-1,0) \quad (15)$$

By substituting Eq. (14) and Eq. (15) for Eq. (11), we have

$$\begin{bmatrix} \hat{e}(k+1,0) \\ \hat{z}(k,0) \end{bmatrix} = \begin{bmatrix} XMX^{-1} - \Theta_o KCX^{-1} & R \\ KCX^{-1} & I + KCX^{-1}\Theta_o \end{bmatrix} \begin{bmatrix} e\hat{(}k,0) \\ z\hat{(}k-1,0) \end{bmatrix}$$
$$= \begin{bmatrix} X & -\Theta_o \\ 0 & I \end{bmatrix} \begin{bmatrix} M & N \\ KC & I \end{bmatrix} \begin{bmatrix} X & -\Theta_o \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \hat{e}(k,0) \\ \hat{z}(k-1,0) \end{bmatrix} \quad .(16)$$
$$R \triangleq \left(XN + XMX^{-1}\Theta_o - \Theta_o KCX^{-1}\Theta_o - \Theta_o \right)$$

Note that we have ignored the uncertainty term here because our aim is to calculate the closed-loop characteristic equation,

$$\det \begin{bmatrix} zI - XMX^{-1} + \Theta_o KCX^{-1} & -R \\ -KCX^{-1} & zI - I - KCX^{-1}\Theta_o \end{bmatrix}$$
$$= \det\left(\begin{bmatrix} X & -\Theta_o \\ 0 & I \end{bmatrix} \begin{bmatrix} zI - M & -N \\ -KC & zI - I \end{bmatrix} \begin{bmatrix} X & -\Theta_o \\ 0 & I \end{bmatrix}^{-1} \right). \quad (17)$$
$$= \det \begin{bmatrix} zI - M & -N \\ -KC & zI - I \end{bmatrix}$$

Thus, the characteristic equation for the current estimation error is equivalent to that for the prediction estimation error.

To design the estimator gain, Eq. (11) can be rewritten as follows:

$$\begin{bmatrix} \bar{e}(k+1,0) \\ z(k+1,0) \end{bmatrix} = \begin{bmatrix} \Phi_m^r & N \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{e}(k,0) \\ z(k,0) \end{bmatrix}$$
$$- \begin{bmatrix} \sum_{j=1}^{r} \Phi_m^j (L_{i-j} + \Theta_o K) \\ -K \end{bmatrix} \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \bar{e}(k,0) \\ z(k,0) \end{bmatrix} \quad (18)$$

Therefore, we can use conventional pole placement method. In addition, we may use the same proportional estimator gain for all L_i s, which can be calculated by using the design result of single-rate estimator as follows:

$$L_{mr} (= L_0 = L_1 = \cdots = L_{r-1}) + \Theta_o K$$
$$= \left(\sum_{j=1}^{r} \Phi_m^j \right)^{-1} \Phi_m^r \cdot L_{sr} \quad (19)$$

where L_{mr} is a single-rate estimator gain. In this case the integral gain, K, of both multi-rate and single-rate are

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same. Note that it does not necessarily imply it is the best way of selection of multi-rate estimation gain.

3. Discontinuous feedback

For notational convenience, let us now define the matrix $\Phi_{obs}(i)$ as follows:

$$\begin{aligned}\Phi_{obs}(i) &\triangleq \begin{bmatrix}\Phi_{obs11}(i) & \Phi_{obs12}(i)\\ \Phi_{obs21}(i) & \Phi_{obs22}(i)\end{bmatrix}\\ &\triangleq \begin{bmatrix}X & -\Theta_o\\ 0 & I\end{bmatrix}\begin{bmatrix}M(i) & N(i)\\ KC & I\end{bmatrix}\begin{bmatrix}X & -\Theta_o\\ 0 & I\end{bmatrix}^{-1}\\ e_z^T &\triangleq \begin{bmatrix}e & z\end{bmatrix}\end{aligned} \tag{20}$$

where $\Phi_{obs11}\in\mathbb{R}^{n\times n},\Phi_{obs21}\in\mathbb{R}^{q\times n},\Phi_{obs12}\in\mathbb{R}^{n\times q},\Phi_{obs22}\in\mathbb{R}^{q\times q}$.

Instead of using the current estimation update of Eq. (10), here we propose a multi-rate estimator with a discontinuous structure, which can be described as follows:

$$\begin{cases}\hat{x}(k,i) = \overline{x}(k,i)\\ \quad + L_i\left\{y(k,0) - C\overline{x}(k,0)\right\} + \Theta_o\hat{z}(k,0) + \Theta_o\nu(k,0)\\ \hat{z}(k,0) = \overline{z}(k,0) + K\left\{y(k,0) - C\overline{x}(k,0)\right\}\end{cases} \tag{21}$$

$$\nu(k,0) = \begin{cases}-\kappa(k,x,u)\dfrac{DC_z\overline{e}_z(k,0)}{\left\|DC_z\overline{e}_z(k,0)\right\|}, if \;\; DC_z\overline{e}_z \neq 0\\ 0 \qquad\qquad\qquad\qquad\quad , otherwise\end{cases} \tag{22}$$

where $\kappa\in\mathbb{R}$ is a design positive scalar function that satisfies $\kappa \geq h^+$, $C_z = \begin{bmatrix}C & 0\end{bmatrix}$, and $D\in\mathbb{R}^{q\times p}$ is some matrix that satisfies the following structural constraint:

$$\begin{aligned}&C_z^T D^T = \Phi_{obs}^T P_z \Theta_z,\\ &\Theta_z^T \triangleq [Y\Theta_o \quad 0],\; Y \triangleq \Omega^{-1}\sum_{j=0}^{r-1}\Phi_m^j\end{aligned} \tag{23}$$

where P_z is a positive definite matrix which is the unique solution to the following Lyapunov function,

$$\Phi_{obs}P_z\Phi_{obs}^T - P_z = -Q_z, \tag{24}$$

where $Q_z\in\mathbb{R}^{(n+q)\times(n+q)}$ is a symmetric positive definite design matrix.

Regarding this structural constraint, it should be pointed out that, for a single output HDD, D in Eq. (23) can be set to one. It is because $p=1$ implies $q=1$ from the dimensional restriction $q\leq p$, and $D\in\mathbb{R}$. Thus, it is obvious from Eq. (22) that the matrix D can be set to one. Further discussion related to the selection of D in Eq. (23) for MIMO system can be found in [6].

Theorem 1: Suppose that L and K are designed such that $\rho\left(\Phi_{obs}\right) < 1$. Then there always exist an ultimately bounded sliding mode observer.

Proof : By taking a coordinate transformation, $\hat{e}_z \rightarrow \hat{e}_{zw}^T \triangleq \left[e^{\wedge} \;\; \hat{z} - X\Theta h\right]$, and also adding the discontinuous and uncertainty terms, Eq. (16) can be rewritten as follows:

$$e_{zw}(k+1) = \Phi_{obs}e_{zw}(k) + \Theta_z\delta h(k) + \Theta_z\nu(k) \tag{25}$$

Now define the Lyapunov candidate function,

$$V_{obs}(k) = e_{zw}^T P_z e_{zw}, \tag{26}$$

and evaluate along the trajectories of Eq. (25), we have

$$\begin{aligned}&V_{obs}(k+1) - V_{obs}(k)\\ &< -\lambda_{\min}(Q_z)\left\|e_{zw}\right\|^2 + \left\|\Theta_z^T P_z\Theta_z\right\|\left(h^+ + \kappa\right)^2.\end{aligned} \tag{27}$$

Since Q_z is positive definite, it is upper convex with respect to e_{zw} and there always exists ultimately bounded sliding mode. The size of boundary can be obtained by solving above equation with respect to $\left\|e_{zw}\right\|$ as follows:

$$\left\|e_{zw}\right\| < \sqrt{\frac{\lambda_{\max}\left(\Theta_z^T P_z \Theta_z\right)}{\lambda_{\min}(Q_z)}}\left(h^+ + \kappa\right) \tag{28}$$

Physical meaning of above boundary is that, if the signum of switching function is different from that of the uncertainty, the discontinuous term is fedback so that states go to the same direction as the uncertainty. This can be easily understood by thinking of a fast varying uncertainty whose frequency is almost the same as Nyquist frequency. ■

In practical design, for this reason, we may apply a band-limit filter to κ of Eq. (22). In addition, we may introduce the saturation function or other kind of function so as to alleviate the chattering motion.

4. Equivalent linear state feedback

Although we may invoke a conventional design method for designing a state feedback gain as a regulator, here we derive a sliding mode based linear multi-rate state feedback gain.

First, the discrete-time switching function is defined with a reference input $x_r\in\mathbb{R}^n$ as follows:

$$S(k,i) = G\{x(k,i) + x_r(k,i)\} \tag{29}$$

where $G\in\mathbb{R}^{m\times n}$ is a sliding hyperplane matrix. Usually G is designed to be a static matrix, which means the control bandwidth is not satisfactorily taken into consideration and it may lead to a so-called spillover problem. For this reason, several methods of designing the dynamic hyperplane so that it has some dynamics in itself have been exploited [10] in an attempt to reduce the spillover problem. This concept works well. In this paper, an alternate simple dynamic structure is derived without augmenting the state space. To do so, the following condition is considered:

$$\begin{aligned}&S(k+1) = \alpha S(k)\\ &\alpha = diag(\alpha_1,\alpha_2,\cdots,\alpha_m), 0\leq \alpha_1,\alpha_2,\cdots,\alpha_m < 1\end{aligned} \tag{30}$$

where S was defined in Eq. (29). The introduction of the parameter α is to reduce the spillover problem as discussed above. In this case, sliding hyperplane itself does not have dynamics, but the switching function can be thought of as having a first order dynamics without the need to introduce an additional state. Therefore, α can be designed as follows:

$$\alpha_j = \exp(-2\pi f_j T_m) \tag{31}$$

where f_j denotes the cut-off frequency of j-th element of switching function (or control input).

The equivalent control input that holds the states onto the settling sliding surface can be obtained by letting

$$\begin{aligned} &S(k,i+1) = \alpha S(k,i) \\ &Gx(k,i+1) + Gx_r(k,i+1) = \alpha Gx(k,i) + \alpha Gx_r(k,i) \\ &G\{\Phi_m x(k,i) + \Gamma_m u(k,i)\} + Gx_r(k,i+1) \\ &\qquad = \alpha Gx(k,i) + \alpha Gx_r(k,i) \end{aligned} \tag{32}$$

From Eq. (32), we obtain the following equivalent control input:

$$\begin{aligned} u_{eq}(k,i) &= -(G\Gamma_m)^{-1}(G\Phi_m - \alpha G)\hat{x}(k,i) \\ &\quad - (G\Gamma_m)^{-1}\{Gx_r(k,i+1) - \alpha Gx_r(k,i)\} \end{aligned} \tag{33}$$

Note that the current state estimator is used in the above equation.

Regarding the equivalent dynamics of multi-rate system, by substituting u of Eq. (2) for Eq. (33) as well as letting w of Eq. (2) and x_r of Eq. (33) be 0, we have the following equation:

$$\begin{aligned} &x(k,i+1) \\ &= \Phi x(k,i) - \Gamma_m (G\Gamma_m)^{-1}(G\Phi_m - \alpha G)\hat{x}(k,i) \end{aligned} \tag{34}$$

For notational simplicity, let us define:

$$K_{eq} \triangleq -(G\Gamma_m)^{-1}(G\Phi_m - \alpha G),\ K_{eq} \in \mathbb{R}^{m\times n} \tag{35}$$

Then, from Eq. (34) and Eq. (35), we have:

$$x(k+1,0) = \Phi_{eq}^r x(k,0) - \sum_{j=0}^{r-1} \Phi_{eq}^{r-1-j}\Gamma_m K_{eq}\hat{e}(k,j) \tag{36}$$

In the above equation, the following matrix was defined:

$$\Phi_{eq} \triangleq \left(\Phi_m + \Gamma_m K_{eq}\right),\ \Phi_{eq} \in \mathbb{R}^{n\times n} \tag{37}$$

By substituting the estimation error in Eq. (36) for Eq. (16), we have:

$$\begin{aligned} x(k+1,0) &= \Phi_{eq}^r x(k,0) \\ &\quad - \sum_{j=0}^{r-1} \Phi_{eq}^{r-1-j}\Gamma_m K_{eq}\Phi_{obs11}(j)\hat{e}(k,0) \\ &\quad - \sum_{j=0}^{r-1} \Phi_{eq}^{r-1-j}\Gamma_m K_{eq}\Phi_{obs12}(j)z(k-1,0) \end{aligned} \tag{38}$$

Therefore, the considered dynamics of Eq. (16) and Eq. (38) can be written as follows:

$$\begin{bmatrix} \hat{e}(k+1,0) \\ \hat{z}(k,0) \\ \hline x(k+1,0) \end{bmatrix} = \begin{bmatrix} \Phi_{obs} & 0 \\ E & \Phi_{eq}^r \end{bmatrix} \begin{bmatrix} \hat{e}(k,0) \\ \hat{z}(k-1,0) \\ \hline x(k,0) \end{bmatrix}$$

$$E \triangleq -\sum_{j=0}^{r-1} \Phi_{eq}^{r-1-j}\Gamma_m K_{eq}\left[\Phi_{obs11}(j) \quad \Phi_{obs12}(j)\right] \tag{39}$$

$$E \in \mathbb{R}^{n\times n+p}$$

This eigenstructure shows that G and L can be designed separately. Note that G can be designed such that $G\Gamma$ is nonsingular without any serious difficulty since B is full rank by assumption. Also it should be pointed out that G and α can be designed separately.

Theorem 2: Suppose that G is designed such that $\rho(\Phi_m - \Gamma_m(G\Gamma_m)^{-1})G\Phi_m) < 1$. Then there always exist an asymptotically stable system for parameter $0 \le \alpha_j < 1$ such that $\rho(\Phi_{eq}) < 1$. In this case, G and α can be designed separately.

Proof: The system can be transformed into a canonical form using some coordinate transformation matrix defined as follows:

$$T = \begin{bmatrix} I_{n-m\times n-m} & O_{n-m\times m} \\ G_1 & G_2 \end{bmatrix} \tag{40}$$

where I denotes the identity matrix, and $G = [G_1\, G_2]$. Note that the system is pre-transformed into the controllable canonical form before applying T such that the system and control distribution matrix have the form

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix},\ \Gamma = \begin{bmatrix} O \\ \Gamma_2 \end{bmatrix}. \tag{41}$$

Then, Eq. (37) can be transformed to

$$T\Phi_{eq}T^{-1} = \begin{bmatrix} \Phi_{11} - \Phi_{12}G_2^{-1}G_1 & \Phi_{12}G_2^{-1} \\ O & \alpha \end{bmatrix}. \tag{42}$$

The eigenvalues of equivalent system matrix consist of that of $\Phi_{11} - \Phi_{12}G_2^{-1}G_1$ which is designed to be stable by the hyperplane matrix G, and α of which each element is less than one. Therefore the equivalent system is stable for $0 \le \alpha_j < 1$, and G and α can be designed separately.

■

The hyperplane matrix G can be though of as a state feedback gain, and it can be determined in the H_2 sense in order to minimize the error from the ideal sliding hyperplane by solving the following discrete algebraic Riccati equation [11]:

$$\begin{aligned} &\Phi_\varepsilon^T P\Phi_\varepsilon - P - \Phi_\varepsilon^T P\Gamma_m(\Gamma_m^T P\Gamma_m)^{-1}\Gamma_m^T P\Phi_\varepsilon + W = 0 \\ &G = \Gamma_m^T P \end{aligned} \tag{43}$$

where P is the unique solution of Eq. (43), W is a weighting, and Φ_ε is a modified system matrix [12] with stability margin ε defined as follows:

$$\Phi_\varepsilon \triangleq \Phi_m + \varepsilon I,\ 0 \le \varepsilon < 1 \tag{44}$$

With this modified system matrix associated with ε, we can design G such that the spectral radius of the closed-loop system other than α be less than $1-\varepsilon$. This implies that the cutoff frequency of associated modes can be described with ε as follows:

$$f_{co} \ge -\frac{\log_e(1-\varepsilon)}{2\pi T_m} \tag{45}$$

5. Design Example and Experiments

First, we consider a double integrator model for a VCM in a single stage actuated HDD. Table 1 shows the design parameters.

Table 1: Design parameters

Sampling Freq	$1/T_s$	15kHz
Control updating Frequency	$1/T_m$	45kHz
Multi-rate ratio	r	3
VCM gain	K_f	1e4

Multi-rate equivalent model can be described as follows:

$$\Phi = \begin{bmatrix} 1 & T_m \\ 0 & 1 \end{bmatrix}, \Gamma = K_f \begin{bmatrix} T_m^2/2 \\ T_m \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (46)$$

Here we invoke Ackerman's formula [13] to place the poles at:

$$p=\begin{bmatrix} 0.2593+0.3192i & 0.2593-0.3192i & 0.9590 \end{bmatrix} \quad (47)$$

Then, from Eq. (18), we have:

$$L_{mr} = \begin{bmatrix} 3.5287e-1 & 3.3119e3 \end{bmatrix}, K = 6.0045e2 \quad (48)$$

Regarding the plant uncertainty, we consider here a VCM gain perturbation of 30%. Thus, the discontinuous term in Eq. (22) can be described as follows:

$$\nu(k,0) = -0.3|u| \frac{C\bar{x}(k,0) - y(k,0)}{|C\bar{x}(k,0) - y(k,0)| + \delta} \quad (49)$$

where δ is a design parameter for alleviating the undesired chattering motion.

For the regulator, we let f in (31), f_o in Eq. (45), and W in Eq. (43) be 1kHz, 1kHz, and diag(1,1), respectively, and obtain the following hyperplane matrix:

$$G = \begin{bmatrix} 2.7920e4 & 3.4652 \end{bmatrix} \quad (50)$$

Moreover, we applied a series of conventional second order notch filters to compensate for VCM resonant modes at around 6kHz and 8kHz.

Using these design parameters, we conducted an experiment on a 3.5 inch 7200rpm low profile HDD driven by a floating point DSP (TMS320C6711) equipped with 14bit ADC, 12bit DAC.

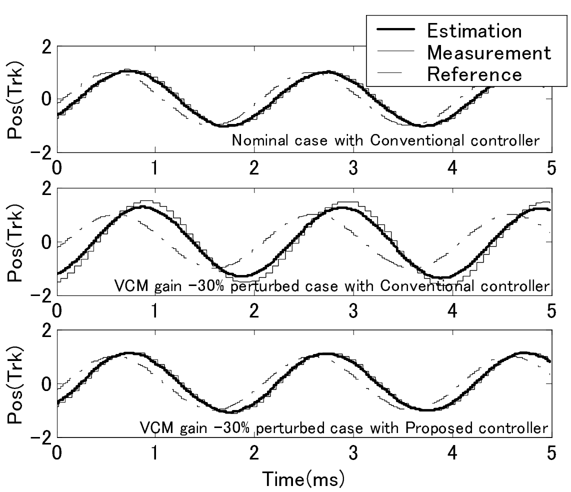

Figure 1: Sinusoidal Response at 500Hz

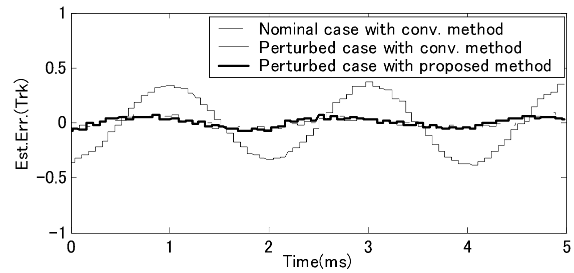

Figure 2: Comparison of Estimation Error

In figure 1, we compared the typical sinusoidal response to a 500Hz reference signal for conventional controller against that for our proposed controller. As it can be seen from this figure, with the conventional controller, the estimation error cannot be ignored when the VCM gain is perturbed. Our proposed controller achieved almost perfect tracking compared with the conventional one, even under such conditions, and estimation error is suppressed to the same level as the nominal case as shown in figure 2.

We then conducted eight tracks seeking experiments on the same HDD mentioned above, and compared the result for conventional controller against the controller proposed here.

Figure 3 and 4 show the forward and backward seeking responses (five times each) for the conventional and proposed controllers with -30% perturbation of VCM actuator gain. Our proposed method achieves a 1ms seek time, while conventional controller takes almost 2.5ms because of the presence of residual vibrations, which is caused by larger estimation error. Figure 5 compares the estimation error of each of Figure 3 and Figure 4, where the results of both the backward and forward seeking operation are depicted on the same figure. As can be seen from this figure, the estimation error is much improved. In this experiment, we only used the SMART [14] trajectory as a reference input and no other method was applied. It should be noted that the average behavior h of the uncertainty w considered in the above experiments is much smaller than the fluctuating behavior δh. Therefore, the discontinuous action is the main contributing factor to the observed improvement.

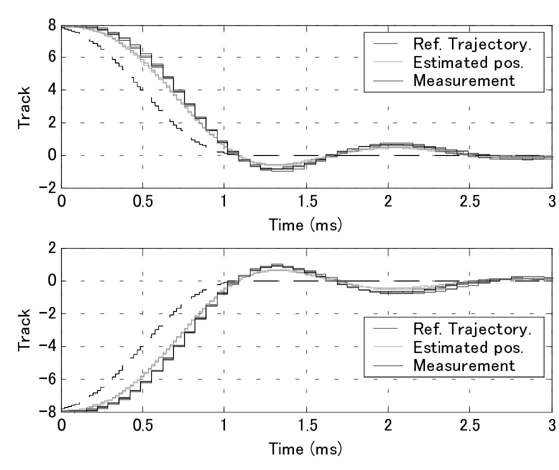

Figure 3: 8-Pos Seek with conventional controller

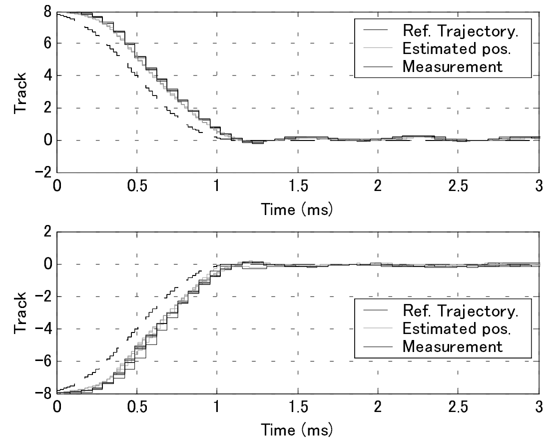

Figure 4: 8-Pos Seek with proposed controller

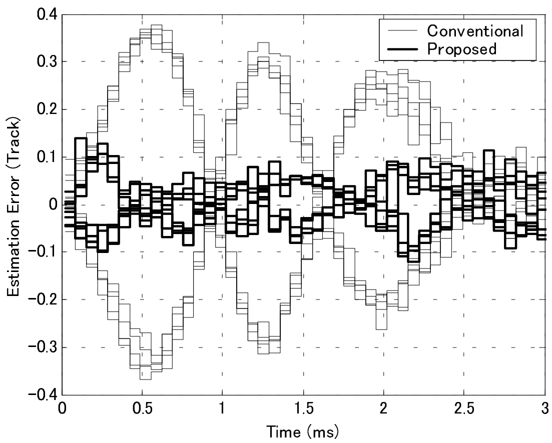

Figure 5: Comparison of Estimation Error During Track Seeking Operation

We also carried out a seeking control experiment for a dual stage actuated HDD, which is the same HDD mentioned above but has a PZT actuator on tip of the arm, and have confirmed the similar results. The best achievable seeking time for the SMART trajectory driven one track seeking was 0.2ms.

6. Conclusion

A new multi-rate state estimator for seeking control of hard disk drives, which has a proportional, integral, and discontinuous estimation error feedback structure, was proposed to improve controller robustness against disturbance inputs and model parametric variation. The proposed controller was validated by carrying out some experiments. Simulation and experimental results confirmed that the robustness of the feedback system against actuator gain perturbation was improved as compared to conventional servoing method.

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