2001 IEEE Intelligent Transportation Systems Conference Proceedings - Oakland (CA), USA - August 25-29, 2001

A Dynamic Tire/Road Friction Model for 3D Vehicle Control and Simulation

Xavier Claeys, Jingang Yi, Luis Alvarez, Roberto Horowitz, Carlos Canudas de Wit

Abstract— **A tire/road friction model based on the LuGre dry friction model and on tire dynamics is presented. The dynamics of the longitudinal and lateral forces, and the self-aligning torque are described by a set of first order differential equations. This model is suitable for 3D vehicle traction/braking simulation and control. A comparison of the forces and torque produced by this dynamic model with the well known "magic formula" is presented.**

Keywords— **Tire/road friction, LuGre model, vehicle control and simulation.**

I. Introduction

One of the main arguments for deploying intelligent transportation systems (ITS) is that the capacity of highways will be increased while maintaining or increasing actual levels of safety. A very important issue, which requires careful analysis, is the influence of tire/road friction on individual vehicle safety and therefore on the overall highway safety. This friction is a key factor in determining vehicles' braking capabilities. The tire/road interaction depends on many factors at the vehicle and roadside levels: vehicle conditions, type and quality of each tire, condition of pavement, weather, etc. If the goal of ITS is to maintain high levels of safety, it is desirable to make vehicles that are able to adjust their behavior to accommodate for changes in the road or vehicle conditions. For example, in the case of wet pavement, inter-vehicle spacing should be increased, as the tire/road friction is expected to decrease in this situation. An accurate and identifiable model of tire/road friction is therefore of prime importance for vehicle control and simulation.

Research in the area of tire/road friction models is significant. [2] and [6] proposed two pseudo-static frictions models that are widely used by researchers in the field. These models are parametric and are based on empirical curve fitting for the friction forces and torque. The parameters are calibrated through data collected in suitable experiments. Similar parametric approaches are presented in [5] and [10]. In particular, the model proposed in [2], commonly called the "magic formula", has been used in industry as a bench mark for tire/road friction models. However, these models have disadvantages. They are over-parameterized, difficult to calibrate and are not suitable for direct identification. In addition, both models ignore the physics and dynamics of the friction process. Their parameters lack any physical interpretation, making it difficult to integrate model dependencies on specific factors such as humidity of the road or tire tread temperature. Also, these models need to be re-calibrated for different road and vehicle conditions. There are some identifiable approximations to the models, proposed recently in [11] and [1]. The parameters in these models can be estimated in real-time. The model in [1] has an interesting estimation property, in that it can guarantee the underestimation of the tire/road coefficient of friction, which helps to maintain vehicle safety.

In an effort to overcome the limitations of the pseudo-static friction models, [8] and [4] recently proposed dynamic friction models. It was claimed that these models capture most of the important phenomena that occur during the friction process. These models are suitable for identification and friction compensation in mechanical systems. [9] extended the use of the dynamic friction models to tire/road interactions. The authors of [9] introduced a LuGre first-order dynamic friction model. In [7] this model was parameterized to allow direct identification of road conditions. Adaptive emergency braking control and comparison of this model with the "magic formula" are discussed in [13]. All the dynamic models that have been applied to tire/road friction are constrained to consider only longitudinal motion, i.e., they disregard the lateral forces.

This paper extends the work in [13] by proposing a three dimensional LuGre-type dynamic friction model. This kind of model is required for new developments in vehicle control design, for example, advanced braking control or observer-based road mon-

Research supported by California PATH grant MOU 373 and a research gift from Renault Inc.

Xavier Claeys and Canudas Canudas de Wit are with the Laboratoire d'Automatique de Grenoble, ENSIEG-INPG, France. Xavier Claeys is currently a visiting scholar at the University of California at Berkeley. E-mail: Xavier.Claeys@inpg.fr.

Jingang Yi and Roberto Horowitz are with the Department of Mechanical Engineering, University of California at Berkeley, CA, USA.

Luis Alvarez is with the Instituto de Ingeniería, Universidad Nacional Autónoma de México, México.

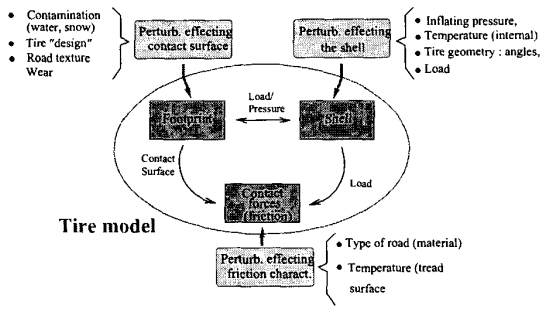

Fig. 1. Schematic representation summarizing the different problems in tire modeling.

itoring.

To analyze the tire/road interface, a description that separates the different elements affecting the dynamic behavior is proposed. Fig. 1 shows the three main factors: friction between tire and road, deformation of the shell, and contact patch area. This paper focuses only on tire/road friction. Although friction properties are only one part of the tire/road interaction, a thorough understanding is still necessary. For this reason a three dimensional model of tire/road contact forces and momentum is proposed. This model is compared numerically with the well-known "magic formula" and shows a very good fitting. This model is a dynamic description of the contact properties; and therefore it should be able to describe richer behavior than the pseudo-static models.

The paper is arranged as follows: in section II we define the coordinate systems and longitudinal and lateral slips which are used for modeling. A distributed friction model is presented in section III based on the LuGre model and the physical dynamics of the tire. Stationary properties of such a model are also presented. A lumped LuGre model is discussed in section IV. This lumped friction model is equivalent to the distributed model with certain boundary conditions but is easier to use for estimation and control purposes. Parameter calibration and a comparison with the "magic formula" are included in section V. Concluding remarks and future work are discussed in section VI.

II. Slip definitions

In this section we recall the definitions of slip that are commonly used in tire/road friction modeling. We use $V = [V_x, V_y]$ to denote the translational longitudinal velocity at the center of the contact patch O (see Fig. 2) in the wheel plane XOY. The angular velocity of the wheel is denoted by w, V_c is the equivalent wheel translational velocity at the point O and $V_c = \rho w$ where ρ is the free radius of the tire. When w is positive (the vehicle is moving forward), the wheel equivalent longitudinal velocity V_c is positive in the coordinate system R_0, where R_0 is defined as a moving frame with velocity V_x with the origin O in the center contact patch.

We define the slip velocity of the point O as $V_s = [V_{sx}, V_{sy}]$ in the wheel plane XOY. The slip angle is denoted by α. The slip ratios used to parameterize the friction model are defined as S_s and S_α, for longitudinal and lateral directions, respectively. Two conventions will be used to separate the braking and traction cases (see Fig. 2), since as usual, the pseudo-static braking curves are developed under constant velocity whereas pseudo-static traction curves are given for constant angular velocity. These conventions also prevent the slip from becoming undefined when either the wheel speed or the longitudinal speed reach zero.

- In the braking case, longitudinal slip S_s and lateral slip S_α are given by

$$\begin{aligned} S_s &= \frac{V_x - V_c}{V_x} = \frac{V_{sx}}{V_x} \\ S_\alpha &= \frac{V_{sy}}{V_x} = |\tan\alpha| \end{aligned}$$

  In braking $V_x - V_c > 0$, $V_x \neq 0$, then $1 \geq S_s > 0$.

- In the traction case, longitudinal slip S_s and lateral slip S_α are given by

$$\begin{aligned} S_s &= \frac{V_c - V_x}{V_c} = \frac{V_{sx}}{V_c} \\ S_\alpha &= \frac{V_{sy}}{V_c} = (1 - S_s)|\tan\alpha| \end{aligned}$$

  In traction, if $V_x - V_c < 0$, $w \neq 0$, then $1 \geq S_s > 0$.

Other conventions, like those used for the "magic formula" (see [3]), can easily be employed and do not change the final results. In this particular case, a specific definition needs to be considered when the longitudinal speed V_x or V_c tend to zero, in order to prevent a singularity in the definition of the slip.

III. Distributed dynamic tire/road friction model

Several dynamic tire/road friction models have already been studied which are restricted to only longitudinal motion [9], [4]. In this section we propose to extend the models to consider both longitudinal and lateral motions, and investigate the resulting forces and torque at the center of the contact patch.

A. Two dimensional model

Let $\delta\xi$ represent a small slice of the deformed belt crossing the contact patch at position ξ in coordi-

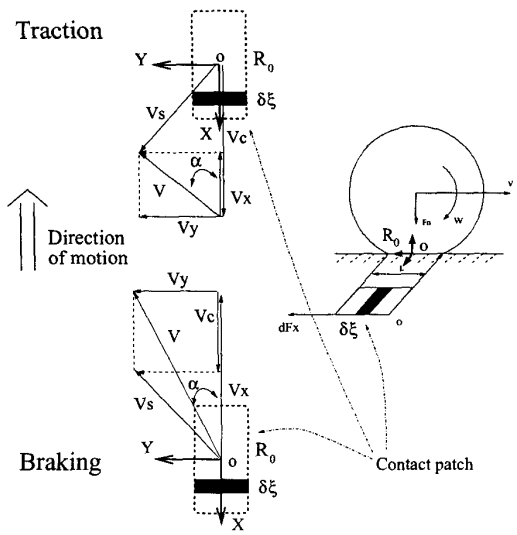

Fig. 2. Convention for the slip definition for both braking and traction cases

nate R_0 frame (see Fig. 2). The slice $\delta\xi$ is moving at the speed $V_{\delta\xi} = [V_x, V_y + \xi\dot{\varphi}]$ with $\dot{\varphi}$ the yaw speed of the rim [12]. We can model the dry friction present in each slice using the LuGre dynamic friction model. The contact between the two surfaces can be represented by microscopic bristle deflections with the coordinates $\delta z(\xi,t) = [\delta z_x(\xi,t), \delta z_y(\xi,t)]$, and the relative velocity of each slice at ξ with respect to O is given as $V_r(\xi,t) = [V_{rx}(\xi,t), V_{ry}(\xi,t)] = [-V_{sx}(t), -V_{sy}(t) - \xi\dot{\varphi}]$ (the direction of the total force is opposite to the slip vector). However, in this paper we treat the tire belt as a rigid body. The model might be extended in order to include dynamic properties of the rubber belt. In this case, the longitudinal velocity of each slice $V_{\delta\xi}$ of the contact patch would have a more complex expression that might include camber angle dependencies or other factors. These notions have already been explained in the literature [12], [10].

For the rigid tire belt model, the extended two dimensional distributed tire/road friction model is given by

$$\delta\dot{z}_x(\xi,t) = V_{rx} - \frac{\sigma_{0_x}}{g_x(V_{rx})}\delta z_x(\xi,t)|V_{rx}| \tag{1a}$$

$$\delta\dot{z}_y(\xi,t) = V_{ry} - \frac{\sigma_{0_y}}{g_y(V_{ry})}\delta z_y(\xi,t)|V_{ry}| \tag{1b}$$

and the friction forces

$$\delta F_x = \{\sigma_{0_x}\delta z_x(\xi,t) + \sigma_{1_x}\delta\dot{z}_x(\xi,t) + \sigma_{2_x}V_{rx}\}\,\delta F_n \tag{2a}$$

$$\delta F_y = \{\sigma_{0_y}\delta z_y(\xi,t) + \sigma_{1_y}\delta\dot{z}_y(\xi,t) + \sigma_{2_y}V_{ry}\}\,\delta F_n \tag{2b}$$

where σ_{j_i}, $i = x,y$; $j = 1,2,3$, are the dynamic coefficients of the LuGre friction model for lateral and longitudinal directions, known as the normalized rubber stiffness (σ_{0_i}), the normalized rubber damping (σ_{1_i}), and the normalized viscous relative damping (σ_{2_i}). The normal load δF_n is considered uniformly distributed over the patch along ξ direction, thus $\delta F_n = F_n/L$, and

$$g_x(V_{rx}) = \mu_{c_x} + (\mu_{s_x} - \mu_{c_x})e^{-|\frac{V_{rx}}{v_{s_x}}|^{1/2}} \tag{3a}$$

$$g_y(V_{ry}) = \mu_{c_y} + (\mu_{s_y} - \mu_{c_y})e^{-|\frac{V_{ry}}{v_{s_y}}|^{1/2}} \tag{3b}$$

are two functions that characterize the steady state properties of the friction, where μ_{c_i}, μ_{s_i}, v_{s_i} are, respectively, the Coulomb friction coefficient, the normalized static friction and the Stribeck relative velocity. The two LuGre models for longitudinal and lateral motions use different parameters since the friction properties of the contact tire/road are different in longitudinal and lateral directions. The fact that the tire has non-isotropic properties is well known in this area and has already been introduced in most of the current models [3], [5].

The system given in Eqs. (1) and (2) is both time and space dependent, and as a consequence the derivative of $\delta z_i(\xi,t)$, $i = x,y$, for longitudinal and lateral directions is a full derivative given by

$$\delta\dot{z}_i(\xi,t) = \frac{\partial\delta z_i(\xi,t)}{\partial\xi}\frac{\partial\xi}{\partial t} + \frac{\partial\delta z_i(\xi,t)}{\partial t} \tag{4}$$

The system composed of (1), (2) and (4) is difficult to solve analytically. However, the stationary case (i.e. pseudo-static case, V_x and w are constant) can be studied and compared with the available stationary tire models in this research area. This will be discussed in section III-C.

B. Self-aligning torque

The self-aligning torque is an important part of the tire model because the reaction force applied to the vehicle (steering wheel feedback force) is strongly dependent upon it. The self-aligning torque consists of two important elements, the yaw motion of the tire that creates a friction torque $\delta M_{z_1}(\xi,t)$, and the moment of the friction forces about the center of wheel frame $\delta M_{z_2}(\xi,t)$. Both of these effects generate the torque at the center of patch, known as the self-aligning torque. In the current paper we neglect the yaw motion of the tire $\delta M_{z_1}(\xi,t)$, due to the fact that the yaw motion effect is small [12]. Denoting φ as the yaw angle, this torque could easily be described by using a third dynamic friction model similar to the LuGre model given in section III (Eqs. (5) and (6)). Given a yaw bristle deformation,

$$\delta\dot{z}_z(\xi,t) = \dot{\varphi} - \frac{\sigma_{0_z}}{g_z(\varphi)}\delta z_z(\xi,t)|\dot{\varphi}| \tag{5}$$

the friction torque could be expressed by the following equation

$$\delta M_{z_1} = \{\sigma_{0_z}\delta z_z(\xi,t) + \sigma_{1_z}\delta\dot{z}_z(\xi,t) + \sigma_{2_z}\dot{\varphi}\}\,\delta F_n \tag{6}$$

The tire/road forces and torque are always calculated at the center of the patch O in Fig. 2. Consequently, a self-aligning torque is produced resulting from the non-symmetry of the contact patch deformation $\delta z(\xi,t)$, or forces $\delta F_y(\xi,t)$, over the contact patch length L. The equivalent forces and torque produced by a slice $\delta\xi$ at position ξ with respect to the center of the patch O is given by $[\delta F_x, \delta F_y, \delta M_z = \xi\delta F_y]$ and the total tire/road interaction in the patch center is now expressed by two forces F_x, F_y and the self-aligning torque M_z:

$$F_x = \int_{-\frac{L}{2}}^{\frac{L}{2}} \delta F_x(\xi,t)d\xi \tag{7a}$$

$$F_y = \int_{-\frac{L}{2}}^{\frac{L}{2}} \delta F_y(\xi,t)d\xi \tag{7b}$$

$$\begin{aligned} M_z &= \int_{-\frac{L}{2}}^{\frac{L}{2}} [\ \underbrace{\delta M_{z_1}(\xi,t)}_{\text{neglected}} + \delta M_{z_2}(\xi,t)\ ]d\xi \\ &= \int_{-\frac{L}{2}}^{\frac{L}{2}} \xi\delta F_y(\xi,t)d\xi \end{aligned} \tag{7c}$$

Remark 1 Eventually, we could also look for the non-symmetric force distribution along the width of the patch and add other components to the model. In particular, the self-aligning torque will then depend upon the longitudinal force $\delta F_x(\xi,t)$ as confirmed by the experiments. ∘

C. Stationary properties

The stationary characteristics of the tire are widely applied in this research area. To produce these characteristics, a complex experimental setup is usually utilized. These conditions are hard to obtain on a real vehicle, since the required maneuvers would be very severe for the passengers. Each point on the stationary curve is given for a constant slip and a constant wheel velocity or wheel angular velocity, therefore V_c and V remain constant and a slip angle α is obtained. The yaw motion of the rim is also not considered, i.e. $\dot{\varphi} = 0$. Therefore, during the stationary conditions, ξ and t are no longer independent because of constant velocity, thus we have $\delta z_i(\xi,t) = \delta z_i(t)$ if we desire a time varying solution, or $\delta z_i(\xi,t) = \delta z_i(\xi)$ if we require a spatial solution. We choose to calculate the spatial solution in the frame R_0 defined previously. Notice that, if $\dot{\xi} = V_c$ is constant during stationary conditions, we have

$$\frac{d}{dt}\delta z_i(\xi,t) = \frac{d\delta z_i(\xi)}{d\xi}\frac{d\xi}{dt} = \frac{d\delta z_i(\xi)}{d\xi}V_c$$

with spatial coordinates. As a consequence, the stationary bristle model with spatial coordinates becomes

$$\frac{d\delta z_i(\xi)}{d\xi}V_c = -V_{si} - \frac{\sigma_{0_i}}{g_i(V_{si})}\delta z_i(\xi)|V_{si}|$$

with $\delta z_i\left(-\frac{L}{2}\right) = 0$ as the boundary condition. The spatial solution is given by

$$\delta z_i(\xi) = \text{sign}(-V_{si})\frac{g_i(V_{si})}{\sigma_{0_i}}\left[1 - e^{-\frac{\sigma_{0_i}|V_{si}|/V_c}{g_i(V_{si})}(\xi+\frac{L}{2})}\right]$$

Integrating the forces and torque along the contact patch (using the formula (7)), we obtain three components for the stationary tire model F_x, F_y, M_z for the traction case:

$$\begin{aligned} \frac{F_x}{F_n} &= -\gamma_x(V_{sx})\text{sign}(V_{sx})g_x(V_{sx})\left[1 + \frac{g_x(V_{sx})}{L\sigma_0 S_s}\right. \\ &\quad \left.(e^{-\frac{\sigma_{x_0}LS_s}{g_x(V_{sx})}} - 1)\right] - (\sigma_{x_1} + \sigma_{x_2})V_{sx} \end{aligned} \tag{8a}$$

$$\begin{aligned} \frac{F_y}{F_n} &= -\gamma_y(V_{sy})\text{sign}(V_{sy})g_y(V_{sy})\left[1 + \frac{g_y(V_{sy})}{L\sigma_{0_y}S_\alpha}\right. \\ &\quad \left.(e^{-\frac{\sigma_{0_y}LS_\alpha}{g_y(V_{sy})}} - 1)\right] - (\sigma_{1_y} + \sigma_{2_y})V_{sy} \end{aligned} \tag{8b}$$

$$\begin{aligned} \frac{M_z}{F_n} &= -\frac{\gamma_y(V_{sy})\text{sign}(V_{sy})g_y^2(V_{sy})}{2\sigma_{0_y}^2 S_\alpha}\left[(e^{-\frac{\sigma_{0_y}LS_\alpha}{g_y(V_{sy})}} - 1) + \right. \\ &\quad \left.\left(\frac{2g_y(V_{sy})}{L\sigma_{0_y}S_\alpha}\right)\left(e^{-\frac{\sigma_{0_y}LS_\alpha}{g_y(V_{sy})}} - 1\right)\right] \end{aligned} \tag{8c}$$

The function γ_i defined as $\gamma_i(V_{s_i}) = 1 - \frac{\sigma_{1_i}|V_{s_i}|}{g_i(V_{s_i})}$, for $i = x, y$. Calibration of parameters and comparison with the "magic formula" are presented later in section V. For the braking case, we can find similar formulae for F_x, F_y, and M_z.

IV. Lumped dynamic tire/road friction model

Distributed models are difficult to use for estimation and control purposes. Thus, we will now develop a simplified lumped parametric representation. An approach has been given in [9] for deriving a lumped model, assuming null boundary conditions for the internal state are not possible as the deflection is not symmetrical with respect to the center of the patch O. This is an essential property to guarantee the existence of a self-aligning torque. In this paper, we

obtained the lumped model by defining lumped variables $\bar{z}_i$ as follows,

$$\bar{z}_i(t) = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} \delta z_i(\xi,t)d\xi, \qquad i = x, y$$

where L is defined as an "elementary surface length", which could be a *tread block element* or the *full contact patch length* between the tire and the road. Neglecting the yaw motion of the rim, i.e. $\dot{\varphi} = 0$, the distributed friction model becomes

$$\dot{\bar{z}}_i + \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\partial \xi}{\partial t}\frac{\partial \delta z_i(\xi,t)}{\partial \xi}d\xi = -V_{si} - \frac{\sigma_{0_i}}{g_i(V_{si})}\bar{z}_i|V_{si}| \quad (9)$$

where $\frac{\partial \xi}{\partial t} = V_c$ is assumed uniformly along the patch and $i = x, y$. Assuming $\delta z\left(-\frac{L}{2}, t\right) = 0$, the system turns into two first order differential equations which are similar to [13] ($\delta z(\frac{L}{2}, t) \neq 0$ because the solution is not symmetric). Finally the lumped model can be defined by

- **Internal states**

$$\delta \dot{z}_i\left(\frac{L}{2}, t\right) = -V_{si} - \frac{\sigma_{0_i}}{g_i(V_{si})}\delta z_i\left(\frac{L}{2}, t\right)|V_{si}|$$

$$\dot{\bar{z}}_i + \rho w \delta z\left(\frac{L}{2}, t\right) = -V_{si} - \frac{\sigma_{0_i}}{g_i(V_{si})}\bar{z}_i|V_{si}|$$

with $\bar{z}_i(0) = 0$ and $\delta z\left(\frac{L}{2}, 0\right) = 0$ because the patch is at free when $t = 0$.

- **Lumped forces**

$$F_i = \{\sigma_{i_0}\bar{z}_i + \sigma_{i_1}\dot{\bar{z}}_i + \sigma_{i_2}V_{ri}\}F_n$$

Using the new lumped internal states $\bar{z}_i$ and $\delta z_i\left(\frac{L}{2}, t\right)$, we can obtain

- **Lumped self-aligning torque**

$$M_z = \int_{-\frac{L}{2}}^{\frac{L}{2}} \xi \delta F_y(\xi,t)d\xi$$

Noticing that

$$\begin{aligned}\frac{\partial \delta F_y}{\partial t} &= \frac{F_n}{L}\Big\{\sigma_{y_0}\gamma_y(V_{sy})\frac{\partial \delta z_y(\xi,t)}{\partial t} \\ &\quad + \sigma_{y_0}\dot{\gamma}_y(V_{sy})\delta z_y(\xi,t) + (\sigma_{y_1} + \sigma_{y_2})\dot{V}_{ry}\Big\} \\ &= \frac{F_n}{L}\Big\{\sigma_{y_0}\gamma_y(V_{sy})\Big(V_{sy} \\ &\quad - \frac{\sigma_{0_y}|V_{sy}|}{g_y(V_{sy})}\delta z_y(\xi,t)V_c\frac{\partial \delta z_y(\xi,t)}{\partial \xi}\Big) \\ &\quad + \sigma_{y_0}\dot{\gamma}_y(V_{sy})\delta z_y(\xi,t) + (\sigma_{y_1} + \sigma_{y_2})\dot{V}_{ry}\Big\}\end{aligned}$$

we have

$$\begin{aligned}\dot{M}_z &= \int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\dot{\xi}\delta F_y(\xi,t) + \xi\frac{\partial \delta F_y(\xi,t)}{\partial t}\right]d\xi \\ &= V_cF_y + \left(\frac{\dot{\gamma}_y(V_{sy})}{\gamma_y(V_{sy})} - \frac{\sigma_{0_y}|V_{sy}|}{g_y(V_{sy})}\right)M_z \\ &\quad + \sigma_{y_0}\gamma_y(V_{sy})V_c(\bar{z}_y - \frac{\delta z_y(\frac{L}{2},t)}{2})F_n\end{aligned}$$

Remark 2 The dynamic tire/road friction model has several interesting properties:

- The model is an average over the patch length and depends only upon time.
- The model is not limited to stationary curves, we can describe the system when V_c and V are not constant.
- The bristle dynamics are relatively fast with respect to vehicle dynamics when slip velocity V_s is large. However, for small values of the slip velocities V_{sx} and V_{sy}, the friction dynamics become slower and should be considered in vehicle control and simulations. ∘

V. Parameter calibrations and numerical simulations

We need to calibrate the dynamic friction model parameters before the model can be used for friction estimation and vehicle control. Moreover, we should validate the model with experimental data. In this paper, we use the typical tire model 165-65R14 as an example. The "magic formula" has been calibrated for dry surface conditions for tires given by Michelin Inc. under a pure braking/corning maneuver. Fig. 3 shows the test results from the calibrated "magic formula".

We calibrated the parameters of our model using the nonlinear fitting methods in MATLAB. The model parameters are given in Table I and the force and torque curves in Fig. 3. The result shows a good fit between the model and the empirical approach. Moreover, we found that including the lateral force and the self-aligning torque facilitated the determination of some parameters, such as those in the y direction.

VI. Conclusion

In this paper we extended and derived a three dimensional dynamic tire/road friction model, based on previous work which had only considered longitudinal motions. Both distributed and lumped friction models were discussed. The lumped model can be used to identify the tire/road conditions and can be applied to vehicle control. A numerical example was presented to calibrate the model parameters and

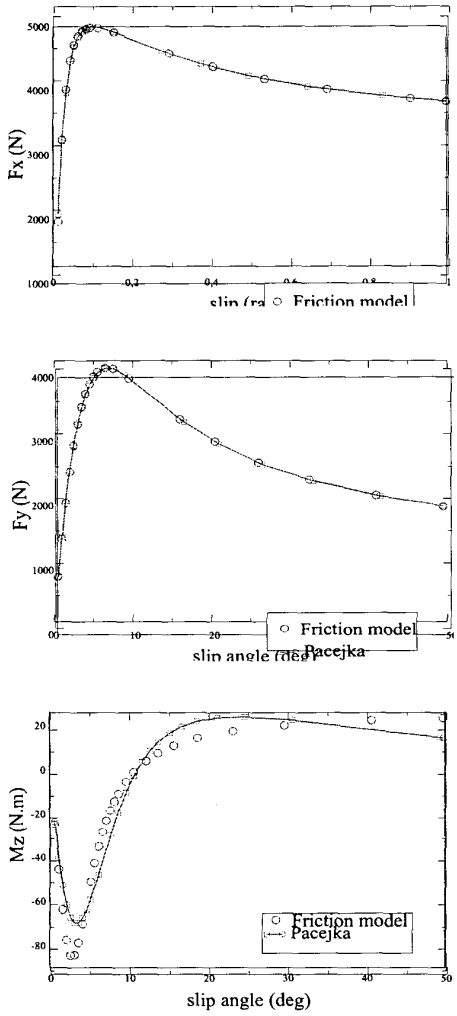

Fig. 3. Comparison of the stationary tire/road friction model and the "magic formula" (constant velocity during braking $v = 15m/s$).

TABLE I

FRICTION PARAMETERS FOR F_x AND F_y

Coefficients	$i = x$	$i = y$
σ_{0_i}	267	122
σ_{1_i}	1.33	0.327
σ_{2_i}	0.0001	0
μ_{c_i}	0.57	0.2675
μ_{s_i}	1.41	3.05
v_{s_i}	2.66	1.17

validate the model with respect to the widely used "magic formula". From the analysis and numerical results obtained, we found that the proposed friction model can capture the tire/road friction characteristics and can easily be used for friction estimation and control purposes. Integrating yaw motion and the coupling dynamics between longitudinal and lateral friction into the dynamic friction model is a topic of current research.

REFERENCES

[1] Luis Alvarez, Jingang Yi, and Roberto Horowitz. Adaptive Emergency Brake Control in AHS with Underestimation of Friction Coefficient. In *Proceedings of American Control Conference*, pages 574–579, Chicago, IL, 2000.

[2] E. Bakker, L. Nyborg, and H.B. Pacejka. Tyre Modelling for Use in Vehicle Dynamic Studies. Society of Automotive Engineers Paper # 870421, 1987.

[3] Egbert Bakker, Lars Nyborg, and Hans B. Pacejka. Tyre Modeling for Use in Vehicle Dynamics Studies. *SAE Transactions*, (870421):190–204, 1987.

[4] P.A. Bliman, T. Bonald, and M. Sorine. Hysteresis Operators and Tyre Friction Models: Application to Vehicle Dynamic Simulation. In *Proceedings of ICIAM'95*, Hamburg, Germany, 1995.

[5] M. Burckhardt. *ABS und ASR, Sicherheitsrelevantes, Radschlupf-Regel System. Lecture Scriptum*, University of Braunschweig, Germany, 1987.

[6] M. Burckhardt. *Fahrwerktechnik: Radschlupfregelsysteme.* Vogel-Verlag, Germany, 1993.

[7] Carlos Canudas de Wit and Roberto Horowitz. Observers for Tire/Road Contact Friction using only wheel angular velocity information. In *Proceedings of* 38th *IEEE Conference of Decision and Control*, Phoenix, AZ, 1999.

[8] Carlos Canudas de Wit, H. Olsson, K.J. Åström, and P. Lischinsky. A New Model for Control of Systems with Friction. *IEEE Transactions on Automatic Control*, 40(3):419–425, 1995.

[9] Carlos Canudas de Wit and Panagiotis Tsiotras. Dynamic Tire Friction Models for Vehicle Traction Control. In *Proceedings of* 38th *IEEE Conference of Decision and Control*, Phoenix, AZ, 1999.

[10] Gwanghun Gim and P.E. Nikravesh. An Analytical Model of Pneumatic Tyres for Vehicle Dynamic Simulations. Part I: Pure Slips. *International Journal of Vehicle Design*, 11(6):589–618, 1990.

[11] U. Kiencke. Realtime Estimation of Adhesion Characteristic Between Tyres and Road. In *Proceedings of the IFAC World Congress*, volume 1, 1993.

[12] M. Sorine and J. Szymanski. A new dynamic multi d.o.f. tire model. Preprint, 2000.

[13] J. Yi, L. Alvarez, R. Horowitz, and C. Canudas de Wit. Adaptive Emergency Braking Control in Automated Highway System Using Dynamic Tire/Road Friction Model. In *Proceedings of* 39th *IEEE Conference of Decision and Control*, Sydney, Australia, 2000.