Proceedings of
2001 ASME International Mechanical Engineering Congress and Exposition
November 11-16, 2001, New York, NY

IMECE2001/DSC-24528

# A New 3D Dynamic Tire/Road Friction Model for Vehicle Control and Simulation*

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## Abstract

In this paper a 3D dynamic tire/road friction model is presented. This model is based on the LuGre dry friction model and tire dynamics. The longitudinal and lateral forces, and tire self-aligning torque are determined through several first-order dynamic systems. The main advantage of this model is that it can be easily used and identified for vehicle control and simulation. Furthermore, calibration of model parameters and comparison with "magic formula", which is widely used in area, are presented to validate the model.

**Keywords:** Tire/road friction, LuGre model, vehicle control and simulation.

## 1 Introduction

In recent years, important research has been undertaken both in manual and automated vehicles to investigate the safety of manual traffic and Automated Highway Systems (AHS) when highway densities are significantly increased. One important issue that requires more careful analysis is the influence of tire/road interaction on vehicles' braking capabilities and, therefore, on the overall highway safety. The interaction of the vehicle with the road depends on both road and vehicle conditions. The road/vehicle interaction is also dependent on the type and quality of each individual tire. To maintain the high level of safety, it is necessary that vehicles are able to adjust their braking behavior according to the current road and vehicle conditions. Consequently, an accurate and identifiable tire/road friction model is important for vehicle controls and simulations.

There is a significant amount of research in tire/road friction model and estimation for individual vehicles. The two friction models that have received the most attention by researchers in the field are those proposed by [2] and [6]. Both models are using parametric formulas for friction forces and torque and empirical curve fitting are used to calibrate the parameters by experimental data. A similar parametric approach is discussed in [5] and [10]. The model given in [2], called "magic formula", fits well the experimental results and is used widely in automotive research and industries. However, these models are complicated and the parameters are difficult to calibrate and identify. They do not explain the friction effects between the tire and road interface due to the curve fitting. Moreover, some of these tire/road friction models are over-parameterized and do not include the physical or geometric parameters in the models. Therefore it is difficult to integrate the dependencies of external conditions such as humidity of the road. Thus, for different tire or road conditions, we need to recalibrate all model parameters. Recently, in [11] and [1] identifiable pseudo-static parametric friction models are presented. The parameters in these models can be identified through on-line adaptations though the parameters lack of physical interpretation. A very nice property of the model given in [1] is the under-estimation of the friction coeffi-

*Research supported by California PATH grant MOU 373 and a research gift from Renault Inc.

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cient in the normal driving conditions, which can guarantee the vehicle safety.

In recent years, some dynamic friction models, such as in [8] and [4], are proposed to capture the friction mechanism. These models are used successfully to identify and compensate the friction in mechanical systems. In [9] the first-order friction dynamic model, called *LuGre* model, was first introduced into tire/road interface. Such model was intensively investigated to estimate the tire/road friction coefficient under different road conditions [7], and adaptive braking control [13]. The calibration of model parameters and comparison with "magic formula" are also discussed in [13]. However, all those models are discussed under an assumption that the vehicle does not have lateral motion, i.e. only longitudinal motion.

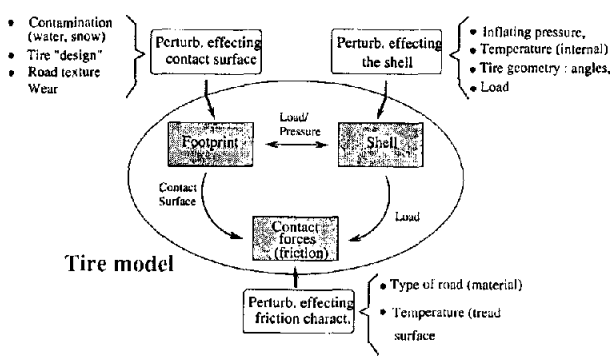


**Figure 1:** Schematic representation underlining the different problems in tire modeling.

In this paper we extend previous work in [13] and propose a new three dimensional tire/road friction model based on the LuGre dynamic friction model. These results pertain to applications where new automatic vehicle features are to be designed, for example, brake control or observer-based road monitoring systems. A new physical interpretation can be discussed based on the dynamic tire/road friction models (see [9]). A first step in the analysis is to design a description of the tire, which is capable of separating the different elements that affect the tire/road interaction dynamics (see Fig. 1). Three aspects of tire modeling are presented: the friction between tire and road, the deformation of the shell, and the contact patch area. The contact interface is only a consequence of the deformation of the shell, but these changes affect the effective dry friction surface and interaction with the friction properties. In this paper we will focus on the tire/road friction properties which are only part of the problem but have not been understood very well. A new three dimensional model of tire/road contact forces and momentum is derived. This new model is numerically compared with the well-known "magic formula" showing a very good fitting. Finally, as this new model is a dynamic description of the contact properties, it should be able to describe more than the commonly addressed stationary behavior of the tire, i.e. pseudo-static behavior.

The paper is arranged as following: in section 2 we define the coordinate systems and longitudinal and lateral slips which are used for modeling. A distributed friction model is presented in section 3 based on LuGre model and physical dynamics of tire. Stationary properties of such a model are also presented. A lumped LuGre model is discussed in section 4. This lumped friction model is equivalent to the distributed model but more easily used for estimation and control purposes. Parameter calibration and comparison with "magic formula" are in section 5. Concluding remarks and future work are presented in section 6.

## 2 Slip definitions

In this section we recall the definitions of slip that are commonly used in tire/road friction modeling. We use $V = [V_x, V_y]$ to denote the translational longitudinal velocity at the center of the contact patch $O$ (see Fig. 2) in the wheel plane $XOY$. The angular velocity of the wheel is denoted by $w$, $V_c$ is the equivalent wheel translational velocity at the point $O$ and $V_c = \rho w$ where $\rho$ is the free radius of the tire. When $w$ is positive (the vehicle is moving forward), the wheel equivalent longitudinal velocity $V_c$ is positive in the coordinate system $R_0$, where $R_0$ is defined as a moving frame with velocity $V_x$ with the origin $O$ in the center contact patch.

We define the slip velocity of the point $O$ as $V_s = [V_{sx}, V_{sy}]$ in the wheel plane $XOY$. The slip angle is denoted by $\alpha$. The slip ratios used to parameterize the friction model are defined as $S_s$ and $S_\alpha$, for longitudinal and lateral directions, respectively. Two conventions will be used to separate the braking and traction cases (see Fig. 2), since as usual, the pseudo-static braking curves are developed under constant velocity whereas pseudo-static traction curves are given for constant angular velocity. These conventions also prevent the slip from becoming undefined when either the wheel speed or the longitudinal speed reach zero.

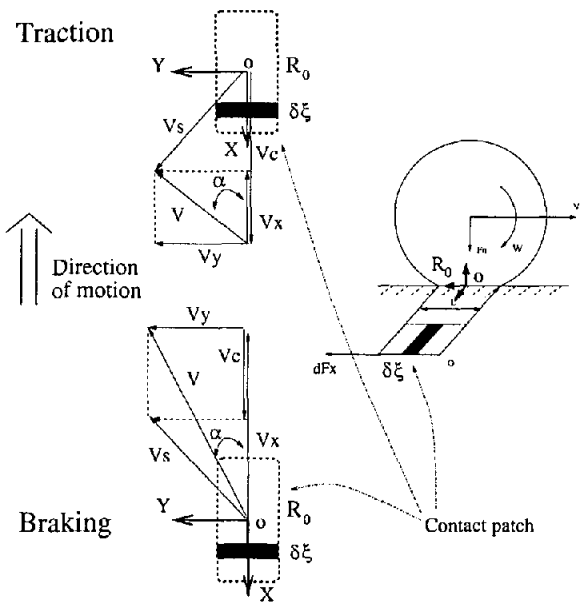


**Figure 2:** Convention for the slip definition for both braking and traction cases

- In the braking case, longitudinal slip $S_s$ and lateral slip $S_\alpha$ are given by

$$S_s = \frac{V_x - V_c}{V_x} = \frac{V_{sx}}{V_x}$$
$$S_\alpha = \frac{V_{sy}}{V_x} = |\tan\alpha|$$

In braking $V_x - V_c > 0$, $V_x \neq 0$, then $1 \geq S_s > 0$.

- In the traction case, longitudinal slip $S_s$ and lateral slip $S_\alpha$ are given by

$$S_s = \frac{V_c - V_x}{V_c} = \frac{V_{sx}}{V_c}$$
$$S_\alpha = \frac{V_{sy}}{V_c} = (1 - S_s)|\tan\alpha|$$

In traction, if $V_x - V_c < 0$, $w \neq 0$, then $1 \geq S_s > 0$.

Other conventions, like those used for the "magic formula" (see [3]), can easily be employed and do not change the final results. In this particular case, a specific definition needs to be considered when the longitudinal speed $V_x$ or $V_c$ tend to zero, in order to prevent a singularity in the definition of the slip.

## 3 Distributed dynamic tire/road friction model

Several dynamic tire/road friction models have already been studied which are restricted to only longitudinal motion [9, 4]. In this section we propose to extend the models to consider both longitudinal and lateral motions, and investigate the resulting forces and torque at the center of the contact patch.

### 3.1 Two dimensional model

Let $\delta\xi$ represent a small slice of the deformed belt crossing the contact patch at position $\xi$ in coordinate $R_0$ frame (see Fig. 2). The slice $\delta\xi$ is moving at the speed $V_{\delta\xi} = [V_x, V_y + \xi\dot{\varphi}]$ with $\dot{\varphi}$ the yaw speed of the rim [12]. We can model the dry friction present in each slice using the LuGre dynamic friction model. The contact between the two surfaces can be represented by microscopic bristle deflections with the coordinates $\delta z(\xi,t) = [\delta z_x(\xi,t), \delta z_y(\xi,t)]$, and the relative velocity of each slice at $\xi$ with respect to $O$ is given as $V_r(\xi,t) = [V_{rx}(\xi,t), V_{ry}(\xi,t)] = [-V_{sx}(t), -V_{sy}(t) - \xi\dot{\varphi}]$ (the direction of the total force is opposite to the slip vector). However, in this paper we treat the tire belt as a rigid body. The model might be extended in order to include dynamic properties of the rubber belt. In this case, the longitudinal velocity of each slice $V_{\delta\xi}$ of the contact patch would have a more complex expression that might include camber angle dependencies or other factors. These notions have already been explained in the literature [12, 10].

For the rigid tire belt model, the extended two dimensional distributed tire/road friction model is given by

$$\delta\dot{z}_x(\xi,t) = V_{rx} - \frac{\sigma_{0_x}}{g_x(V_{rx})}\delta z_x(\xi,t)|V_{rx}| \quad (1a)$$
$$\delta\dot{z}_y(\xi,t) = V_{ry} - \frac{\sigma_{0_y}}{g_y(V_{ry})}\delta z_y(\xi,t)|V_{ry}| \quad (1b)$$

and the friction forces

$$\delta F_x = \{\sigma_{0_x}\delta z_x(\xi,t) + \sigma_{1_x}\delta\dot{z}_x(\xi,t) + \sigma_{2_x}V_{rx}\}\,\delta F_n \quad (2a)$$
$$\delta F_y = \{\sigma_{0_y}\delta z_y(\xi,t) + \sigma_{1_y}\delta\dot{z}_y(\xi,t) + \sigma_{2_y}V_{ry}\}\,\delta F_n \quad (2b)$$

where $\sigma_{j_i}$, $i = x, y$; $j = 1, 2, 3$, are the dynamic coefficients of the LuGre friction model for lateral and longitudinal directions, known as the normalized rubber stiffness ($\sigma_{0_i}$), the normalized rubber damping ($\sigma_{1_i}$), and the normalized viscous relative damping ($\sigma_{2_i}$). The normal load $\delta F_n$ is considered uniformly distributed over the patch along $\xi$ direc-

tion, thus $\delta F_n = F_n/L$, and

$$g_x(V_{rx}) = \mu_{c_x} + (\mu_{s_x} - \mu_{c_x})e^{-|\frac{V_{rx}}{v_{s_x}}|^{1/2}} \quad (3a)$$

$$g_y(V_{ry}) = \mu_{c_y} + (\mu_{s_y} - \mu_{c_y})e^{-|\frac{V_{ry}}{v_{s_y}}|^{1/2}} \quad (3b)$$

are two functions that characterize the steady state properties of the friction, where $\mu_{c_i}$, $\mu_{s_i}$, $v_{s_i}$ are, respectively, the Coulomb friction coefficient, the normalized static friction and the Stribeck relative velocity. The two LuGre models for longitudinal and lateral motions use different parameters since the friction properties of the contact tire/road are different in longitudinal and lateral directions. The fact that the tire has non-isotropic properties is well known in this area and has already been introduced in most of the current models [3, 5].

The system given in Eqs. (1) and (2) is both time and space dependent, and as a consequence the derivative of $\delta z_i(\xi, t)$, $i = x, y$, for longitudinal and lateral directions is a full derivative given by

$$\delta \dot{z}_i(\xi, t) = \frac{\partial \delta z_i(\xi, t)}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial \delta z_i(\xi, t)}{\partial t} \quad (4)$$

The system composed of (1), (2) and (4) is difficult to solve analytically. However, the stationary case (i.e. pseudo-static case, $V_x$ and $w$ are constant) can be studied and compared with the available stationary tire models in this research area. This will be discussed in section 3.3.

### 3.2 Self-aligning torque

The self-aligning torque is an important part of the tire model because the reaction force applied to the vehicle (steering wheel feedback force) is strongly dependent upon it. The self-aligning torque consists of two important elements, the yaw motion of the tire that creates a friction torque $\delta M_{z_1}(\xi, t)$, and the moment of the friction forces about the center of wheel frame $\delta M_{z_2}(\xi, t)$. Both of these effects generate the torque at the center of patch, known as the self-aligning torque. In the current paper we neglect the yaw motion of the tire $\delta M_{z_1}(\xi, t)$, due to the fact that the yaw motion effect is small [12]. Denoting $\varphi$ as the yaw angle, this torque could easily be described by using a third dynamic friction model similar to the LuGre model given in section 3 (Eqs. (5) and (6)). Given a yaw bristle deformation,

$$\delta \dot{z}_z(\xi, t) = \dot{\varphi} - \frac{\sigma_{0_z}}{g_z(\varphi)}\delta z_z(\xi, t)|\dot{\varphi}| \quad (5)$$

the friction torque could be expressed by the following equation

$$\delta M_{z_1} = \{\sigma_{0_z}\delta z_z(\xi, t) + \sigma_{1_z}\delta \dot{z}_z(\xi, t) + \sigma_{2_z}\dot{\varphi}\}\,\delta F_n \quad (6)$$

The tire/road forces and torque are always calculated at the center of the patch $O$ in Fig. 2. Consequently, a self-aligning torque is produced resulting from the non-symmetry of the contact patch deformation $\delta z(\xi, t)$, or forces $\delta F_y(\xi, t)$, over the contact patch length $L$. The equivalent forces and torque produced by a slice $\delta\xi$ at position $\xi$ with respect to the center of the patch $O$ is given by $[\delta F_x, \delta F_y, \delta M_z = \xi \delta F_y]$ and the total tire/road interaction in the patch center is now expressed by two forces $F_x$, $F_y$ and the self-aligning torque $M_z$:

$$F_x = \int_{-\frac{L}{2}}^{\frac{L}{2}} \delta F_x(\xi, t)d\xi \quad (7a)$$

$$F_y = \int_{-\frac{L}{2}}^{\frac{L}{2}} \delta F_y(\xi, t)d\xi \quad (7b)$$

$$M_z = \int_{-\frac{L}{2}}^{\frac{L}{2}} [\ \underbrace{\delta M_{z_1}(\xi, t)}_{\text{neglected}} + \delta M_{z_2}(\xi, t)\ ]d\xi$$

$$= \int_{-\frac{L}{2}}^{\frac{L}{2}} \xi \delta F_y(\xi, t)d\xi \quad (7c)$$

**Remark 1** Eventually, we could also look for the non-symmetric force distribution along the width of the patch and add other components to the model. In particular, the self-aligning torque will then depend upon the longitudinal force $\delta F_x(\xi, t)$ as confirmed by the experiments. ◦

### 3.3 Stationary properties

The stationary characteristics of the tire are widely applied in this research area. To produce these characteristics, a complex experimental setup is usually utilized. These conditions are hard to obtain on a real vehicle, since the required maneuvers would be very severe for the passengers. Each point on the stationary curve is given for a constant slip and a constant wheel velocity or wheel angular velocity, therefore $V_c$ and $V$ remain constant and a slip angle $\alpha$ is obtained. The yaw motion of the rim is also not considered, i.e. $\dot{\varphi} = 0$. Therefore, during the stationary conditions, $\xi$ and $t$ are no longer independent because of constant velocity, thus we have $\delta z_i(\xi, t) = \delta z_i(t)$ if we desire a time varying solution, or $\delta z_i(\xi, t) = \delta z_i(\xi)$ if we require a spatial solution. We choose to calculate the spatial

solution in the frame $R_0$ defined previously. Notice that, if $\dot{\xi} = V_c$ is constant during stationary conditions, we have

$$\frac{d}{dt}\delta z_i(\xi,t) = \frac{d\delta z_i(\xi)}{d\xi}\frac{d\xi}{dt} = \frac{d\delta z_i(\xi)}{d\xi}V_c$$

with spatial coordinates. As a consequence, the stationary bristle model with spatial coordinates becomes

$$\frac{d\delta z_i(\xi)}{d\xi}V_c = -V_{si} - \frac{\sigma_{0_i}}{g_i(V_{si})}\delta z_i(\xi)|V_{si}|$$

with $\delta z_i\left(-\frac{L}{2}\right) = 0$ as the boundary condition. The spatial solution is given by

$$\delta z_i(\xi) = \text{sign}(-V_{si})\frac{g_i(V_{si})}{\sigma_{0_i}}\left[1 - e^{-\frac{\sigma_{0_i}|V_{si}|/V_c}{g_i(V_{si})}(\xi+\frac{L}{2})}\right]$$

Integrating the forces and torque along the contact patch (using the formula (7)), we obtain three components for the stationary tire model $F_x$, $F_y$, $M_z$ for the traction case:

$$\frac{F_x}{F_n} = -\gamma_x(V_{sx})\text{sign}(V_{sx})g_x(V_{sx})\left[1 + \frac{g_x(V_{sx})}{L\sigma_0 S_s}\left(e^{-\frac{\sigma_{x0}LS_s}{g_x(V_{sx})}} - 1\right)\right] - (\sigma_{x_1} + \sigma_{x_2})V_{sx} \quad (8a)$$

$$\frac{F_y}{F_n} = -\gamma_y(V_{sy})\text{sign}(V_{sy})g_y(V_{sy})\left[1 + \frac{g_y(V_{sy})}{L\sigma_{0_y}S_\alpha}\left(e^{-\frac{\sigma_{0_y}LS_\alpha}{g_y(V_{sy})}} - 1\right)\right] - (\sigma_{1_y} + \sigma_{2_y})V_{sy} \quad (8b)$$

$$\frac{M_z}{F_n} = -\frac{\gamma_y(V_{sy})\text{sign}(V_{sy})g_y^2(V_{sy})}{2\sigma_{0_y}^2 S_\alpha}\left[\left(e^{-\frac{\sigma_{0_y}LS_\alpha}{g_y(V_{sy})}} - 1\right) + \left(\frac{2g_y(V_{sy})}{L\sigma_{0_y}S_\alpha}\right)\left(e^{-\frac{\sigma_{0_y}LS_\alpha}{g_y(V_{sy})}} - 1\right)\right] \quad (8c)$$

The function $\gamma_i$ defined as $\gamma_i(V_{s_i}) = 1 - \frac{\sigma_{1_i}|V_{s_i}|}{g_i(V_{s_i})}$, for $i = x, y$. Calibration of parameters and comparison with the "magic formula" are presented later in section 5. For the braking case, we can find similar formulas for $F_x$, $F_y$, and $M_z$.

## 4 Lumped dynamic tire/road friction model

Distributed models are difficult to use for estimation and control purposes. Thus, we will now develop a simplified lumped parametric representation. An approach has been given in [9] for deriving a lumped model, assuming null boundary conditions for the internal state are not possible as the deflection is not symmetrical with respect to the center of the patch $O$. This is an essential property to guarantee the existence of a self-aligning torque. In this paper, we obtained the lumped model by defining lumped variables $\bar{z}_i$ as follows,

$$\bar{z}_i(t) = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} \delta z_i(\xi,t)d\xi, \qquad i = x, y$$

where $L$ is defined as an "elementary surface length", which could be a *tread block element* or the *full contact patch length* between the tire and the road. Neglecting the yaw motion of the rim, i.e. $\dot{\varphi} = 0$, the distributed friction model becomes

$$\dot{\bar{z}}_i + \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\partial\xi}{\partial t}\frac{\partial\delta z_i(\xi,t)}{\partial\xi}d\xi = -V_{si} - \frac{\sigma_{0_i}}{g_i(V_{si})}\bar{z}_i|V_{si}| \quad (9)$$

where $\frac{\partial\xi}{\partial t} = V_c$ is assumed uniformly along the patch and $i = x, y$. Assuming $\delta z\left(-\frac{L}{2}, t\right) = 0$, the system turns into two first order differential equations which are similar to [13] ($\delta z(\frac{L}{2}, t) \neq 0$ because the solution is not symmetric). Finally the lumped model can be defined by

- **Internal states**

$$\delta\dot{z}_i\left(\frac{L}{2}, t\right) = -V_{si} - \frac{\sigma_{0_i}}{g_i(V_{si})}\delta z_i\left(\frac{L}{2}, t\right)|V_{si}|$$

$$\dot{\bar{z}}_i + \rho w\delta z\left(\frac{L}{2}, t\right) = -V_{si} - \frac{\sigma_{0_i}}{g_i(V_{si})}\bar{z}_i|V_{si}|$$

with $\bar{z}_i(0) = 0$ and $\delta z\left(\frac{L}{2}, 0\right) = 0$ because the patch is at free when $t = 0$.

- **Lumped forces**

$$F_i = \{\sigma_{i_0}\bar{z}_i + \sigma_{i_1}\dot{\bar{z}}_i + \sigma_{i_2}V_{ri}\}F_n$$

Using the new lumped internal states $\bar{z}_i$ and $\delta z_i\left(\frac{L}{2}, t\right)$, we can obtain

- **Lumped self-aligning torque**

$$M_z = \int_{-\frac{L}{2}}^{\frac{L}{2}} \xi\delta F_y(\xi,t)d\xi$$

Noticing that

$$\begin{aligned}\frac{\partial \delta F_y}{\partial t} &= \frac{F_n}{L}\left\{\sigma_{y_0}\gamma_y(V_{sy})\frac{\partial \delta z_y(\xi,t)}{\partial t}\right. \\ &\quad \left. +\sigma_{y_0}\dot{\gamma}_y(V_{sy})\delta z_y(\xi,t)+(\sigma_{y_1}+\sigma_{y_2})\dot{V}_{ry}\right\} \\ &= \frac{F_n}{L}\left\{\sigma_{y_0}\gamma_y(V_{sy})\left(V_{sy}\right.\right. \\ &\quad \left. -\frac{\sigma_{0_y}|V_{sy}|}{g_y(V_{sy})}\delta z_y(\xi,t)V_c\frac{\partial \delta z_y(\xi,t)}{\partial \xi}\right) \\ &\quad \left. +\sigma_{y_0}\dot{\gamma}_y(V_{sy})\delta z_y(\xi,t)+(\sigma_{y_1}+\sigma_{y_2})\dot{V}_{ry}\right\}\end{aligned}$$

we have

$$\begin{aligned}\dot{M}_z &= \int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\dot{\xi}\delta F_y(\xi,t)+\xi\frac{\partial \delta F_y(\xi,t)}{\partial t}\right]d\xi \\ &= V_cF_y+\left(\frac{\dot{\gamma}_y(V_{sy})}{\gamma_y(V_{sy})}-\frac{\sigma_{0_y}|V_{sy}|}{g_y(V_{sy})}\right)M_z \\ &\quad +\sigma_{y_0}\gamma_y(V_{sy})V_c(\bar{z}_y-\frac{\delta z_y(\frac{L}{2},t)}{2})F_n\end{aligned}$$

**Remark 2** The dynamic tire/road friction model has several interesting properties:

- The model is an average over the patch length and depends only upon time.
- The model is not limited to stationary curves, we can describe the system when $V_c$ and $V$ are not constant.
- The bristle dynamics are relatively fast with respect to vehicle dynamics when slip velocity $V_s$ is large. However, for small values of the slip velocities $V_{sx}$ and $V_{sy}$, the friction dynamics become slower and should be considered in vehicle control and simulations. ○

## 5 Parameter calibrations and numerical simulations

We need to calibrate the dynamic friction model parameters before the model can be used for friction estimation and vehicle control. Moreover, we should validate the model with experimental data. In this paper, we use the typical tire model 165-65R14 as an example. The "magic formula" has been calibrated for dry surface conditions for tires given by Michelin Inc. under a pure braking/corning maneuver. Fig. 3 shows the test results from the calibrated "magic formula".

We calibrated the parameters of our model using the nonlinear fitting methods in MATLAB. The model parameters are given in Table 1 and the force and torque curves in Fig. 3. The result shows a good fit between the model and the empirical approach. Moreover, we found that including the lateral force and the self-aligning torque facilitated the determination of some parameters, such as those in the $y$ direction.

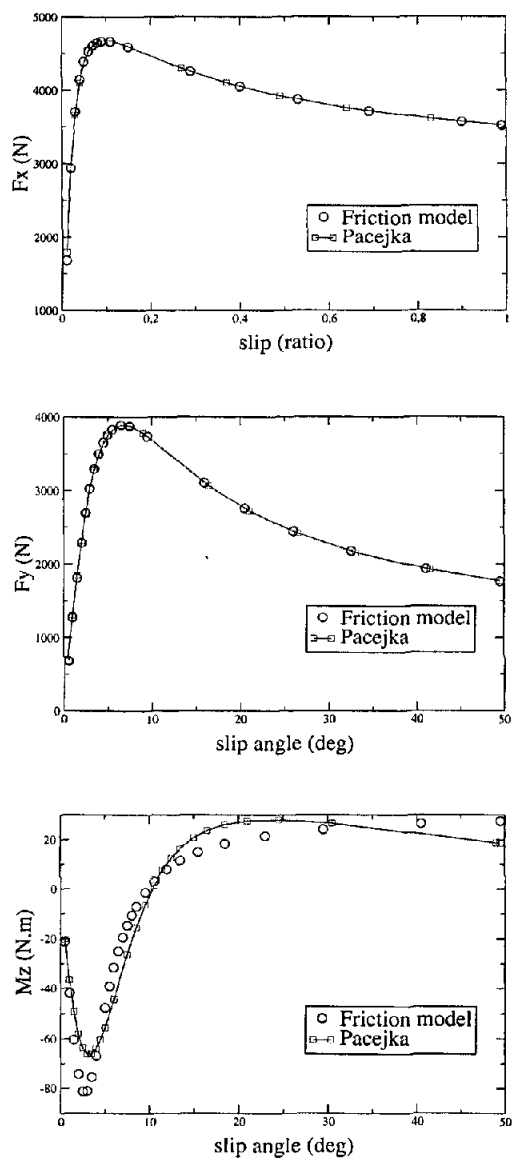


**Figure 3:** Comparison of the stationary tire/road friction model and the "magic formula" (constant velocity during braking $v = 15m/s$).

## 6 Conclusion

In this paper we extended and derived a three dimensional dynamic tire/road friction model, based

**Table 1:** Friction parameters for $F_x$ and $F_y$

| Coefficients | $i = x$ | $i = y$ |
|---|---|---|
| $\sigma_{0_i}$ | 267 | 122 |
| $\sigma_{1_i}$ | 1.33 | 0.327 |
| $\sigma_{2_i}$ | 0.0001 | 0 |
| $\mu_{c_i}$ | 0.57 | 0.2675 |
| $\mu_{s_i}$ | 1.41 | 3.05 |
| $v_{s_i}$ | 2.66 | 1.17 |

on previous work which had only considered longitudinal motions. Both distributed and lumped friction models were discussed. The lumped model can be used to identify the tire/road conditions and can be applied to vehicle control. A numerical example was presented to calibrate the model parameters and validate the model with respect to the widely used "magic formula". From the analysis and numerical results obtained, we found that the proposed friction model can capture the tire/road friction characteristics and can easily be used for friction estimation and control purposes. Integrating yaw motion and the coupling dynamics between longitudinal and lateral friction into the dynamic friction model is a topic of current research.

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