Predictive Control Approach to Autonomous Vehicle Steering

Tamás Keviczky, Paolo Falcone, Francesco Borrelli, Jahan Asgari, Davor Hrovat

Abstract—A model predictive control (MPC) approach to active steering is presented for autonomous vehicle systems. The controller is designed to stabilize a vehicle along a desired path while rejecting wind gusts and fulfilling its physical constraints. Simulation results of a side wind rejection scenario and a double lane change maneuver on slippery surfaces show the benefits of the systematic control methodology used. A trade-off between the vehicle speed and the required preview on the desired path for vehicle stabilization is highlighted. The paper concludes with future research directions and the necessary steps for experimental validation of the approach.

I. INTRODUCTION

Recent trends in automotive industry point in the direction of increased content of electronics, computers, and controls with emphasis on the improved functionality and overall system robustness. While this affects all of the vehicle areas, there is a particular interest in active safety, which very nicely and effectively complements the passive safety counterpart. Passive safety is primarily focused on structural integrity of the vehicle. Active safety on the other hand is primarily used to avoid accidents and at the same time facilitate better vehicle controllability and stability especially in emergency situations, which may occur when suddenly encountering strong wind gusts and slippery parts of the road [1].

The progress of active safety related functions started by primarily focusing on longitudinal dynamics of motion, in particular, on more effective braking (ABS) and traction control. This was followed by work on different vehicle stability control systems [2] (which are also known under different acronyms such as Electronic Stability Program, ESP, Vehicle Stability Control, VSC, Interactive Vehicle Dynamics, IVD, and Dynamic Stability Control, DSC). Essentially, these systems use brakes on one side to stabilize the vehicle in extreme limit handling situations through controlling the yaw motion. The early efforts which found limited production applications were primarily focused on improved handling without explicit emphasis on active safety by itself. Similar can be said about the early introduction of active suspensions, which primarily focused on improved ride and handling. Recent systems include additional intervention with brakes but as a function of excessive vehicle roll, so that in this case the brakes are used to prevent vehicle roll over [3].

It is anticipated that future systems will be able to increase the effectiveness of active safety interventions beyond what is currently available. This will be facilitated not only by additional actuator types such as active steering, active suspensions, or active differentials, but also by additional sensor information, such as the increased inclusion of onboard cameras, as well as infrared and other sensor alternatives. All these will be further complemented by GPS information including pre-stored mapping. In this context, it is possible to imagine that future vehicles would be able to identify obstacles on the road such as an animal, a rock, or fallen tree/branch, and assist the driver by following the best possible path, in terms of avoiding the obstacle and at the same time keeping the vehicle on the road at a safe distance from incoming traffic.

At this stage, we assume this “ultimate” obstacle avoidance system will be possible sometime in the future and we propose a double lane change scenario on a slippery road, with a vehicle equipped with a fully autonomous steering system. We also study how the same system can be used to safely perform side wind rejection when the road surface conditions change abruptly. The control input is the front steering angle and the goal is to follow the desired trajectory or target as close as possible while fulfilling various constraints reflecting physical vehicle limits.

In these scenarios, a robot driver [2] can be used to perform active steering. The robot driver can also learn the desired path by first going very slowly through the double lane change maneuver. Subsequently, the robot driver could attempt to negotiate the same trajectory with increased entry speed, testing the vehicle’s overall stability and behavior on a slippery road surface. In addition, these examples can also serve as a precursor to fully autonomous vehicles. This approach can be used in military applications to transfer the load without any associated personnel casualties. A contemporary example of this is the “DARPA Grand Challenge” race, which has been undertaken through autonomous driving [4]. The main facilitators for this system are GPS, infrared sensors, cameras and path planning and following control routines.

Additional source of information can also come from surrounding vehicles and environments which may convey the information from the vehicle ahead about road condition, which can give a significant amount of preview to the controller. This is particularly useful if one travels on snow or ice covered surfaces. In this case, it is very easy to reach the limit of vehicle handling capabilities. We reemphasize that in the present paper we assume a given desired trajectory and we will design a controller that can best follow the trajectory.
on slippery road at the highest possible entry speed or largest side wind velocity.

Anticipating sensor and infrastructure trends toward increased integration of information and control actuation agents, it is then appropriate to ask what is the best and optimum way of controlling the vehicle maneuver for given obstacle avoidance situation or wind condition. This will be done in the spirit of Model Predictive Control, MPC [5], [6] along the lines of our ongoing internal research efforts dating from early 2000 [7]. We use a nonlinear model of the plant to predict the future evolution of the system [6]. Based on this prediction, at each time step t a performance index is optimized under operating constraints with respect to a sequence of future steering moves in order to best follow or maintain the given trajectory on a slippery road. The first of such optimal moves is the control action applied to the plant at time t. At time t + 1, a new optimization is solved over a shifted prediction horizon.

In this paper we use nonlinear MPC (NLMPC). This allows us to: (i) increase the stability boundary of the controlled system compared to linear controllers, (ii) test its computational complexity, (iii) create a benchmark controller against which other sub-optimal controllers can be compared. The simulation results presented in this work show the benefits of the systematic control methodology used. In particular, we show how complex steering maneuvers are relatively easily obtained as a result of the MPC feedback policy.

The paper is structured as follows. Section II describes the vehicle dynamical model used with a brief discussion of tire models. Section III formulates the control problem. Section IV briefly discussed the nonlinear optimization tools used. The side wind rejection scenario and simulation results are presented in Section V, while the double lane change problem is addressed in Section VI. Computational tools and new methodologies which may lead to real-time implementation of the proposed approach are discussed in Section VII. Concluding remarks in Section VIII highlight future research directions.

II. MODELING

This section describes the vehicle and tire model used for simulations and control design.

A. Vehicle model

We use a “bicycle model” to describe the dynamics of the car and assume constant normal tire load, i.e., $F_{yf}, F_{yr}, F_{zf}, F_{zr}$ constant. Such model captures the most relevant nonlinearities associated to lateral stabilization of the vehicle. Figure 1 depicts a diagram of the vehicle model, which has the following longitudinal, lateral and turning or yaw degrees of freedom (DOF)

\[
\begin{align*}
mx &= m\dot{x} = \dot{m}\psi + 2F_{xj} + 2F_{xr}, \\
m\dot{y} &= -m\dot{x}\psi + 2F_{yj} + 2F_{yr} + F_w, \\
I\ddot{\psi} &= 2aF_{yf} - 2bF_{yr} + M_w, \\
\end{align*}
\]  

where $F_w$ and $M_w$ represent the force and moment exerted by a side wind impacting the car. These quantities are assumed to be modelled according to

\[
\begin{align*}
F_w &= \frac{2.5\pi}{2}v_{w}^2, \\
M_w &= \left(\frac{2.5\pi}{2} - 3.3\left(\frac{\pi}{2}\right)^3\right)v_{w}^2 + \frac{a - b}{2}F_w,
\end{align*}
\]

where $v_w$ is wind velocity.

The vehicle’s equations of motion in an absolute inertial frame are

\[
\begin{align*}
\dot{Y} &= \dot{x}\sin\psi + \dot{y}\cos\psi, \\
\dot{X} &= \dot{x}\cos\psi - \dot{y}\sin\psi.
\end{align*}
\]

Longitudinal and lateral tire forces lead to the following forces acting on the center of gravity:

\[
\begin{align*}
F_y &= F_1\sin\delta + F_c\cos\delta, \\
F_x &= F_1\cos\delta - F_c\sin\delta.
\end{align*}
\]

Tire forces for each tire are given by

\[
\begin{align*}
F_1 &= f_1(\alpha, s, \mu, F_z), \\
F_c &= f_c(\alpha, s, \mu, F_z),
\end{align*}
\]

where $\alpha$ is the slip angle of the tire and $s$ is the slip ratio defined as

\[
s = \begin{cases} 
\frac{\dot{r} - \omega}{\dot{r} + \omega} - 1 & \text{if } v > r\omega, v \neq 0 \text{ for braking} \\
1 - \frac{\dot{r} - \omega}{\dot{r} + \omega} & \text{if } v < r\omega, \omega \neq 0 \text{ for driving}
\end{cases}
\]

The slip angle represents the angle between the wheel velocity and the direction of the wheel itself:

\[
\alpha = \tan^{-1} \frac{v_y}{v_x}.
\]

In equation (7), $v_x$ and $v_y$ are the lateral (or cornering) and longitudinal wheel velocities, respectively, which are expressed as

\[
\begin{align*}
v_l &= v_y\sin\delta + v_x\cos\delta, \\
v_c &= v_y\cos\delta - v_x\sin\delta,
\end{align*}
\]

and

\[
\begin{align*}
v_y &= \dot{y} + a\dot{\psi} \\
v_y &= \dot{y} - b\dot{\psi}, \\
v_x &= \dot{x} \\
v_x &= \dot{x}.
\end{align*}
\]
The parameter $\mu$ represents the road friction coefficient and $F_z$ is the total vertical load of the vehicle. This is distributed between the front and rear wheels based on the geometry of the car model, described by the parameters $a$ and $b$:

$$F_{zf} = \frac{bmg}{2(a + b)},$$

$$F_{zr} = \frac{amg}{2(a + b)}.$$  \hspace{1cm} (10a)

$$F_{zr} = \frac{amg}{2(a + b)}.$$  \hspace{1cm} (10b)

The terminology and nomenclature used for describing the tire orientation and forces is illustrated in Figure 2.

Using the equations (1)-(10), the nonlinear vehicle dynamics can be described by the following compact differential equation assuming a certain slip ratio $s$ and friction coefficient value $\mu$:

$$\dot{\xi} = f_{s, \mu}(\xi, u),$$

$$\eta = h(\xi),$$  \hspace{1cm} (11a)

where the state and input vectors are $\xi = [y, \dot{y}, \dot{x}, \psi, \dot{\psi}, Y, X]$ and $u = \delta_f$ respectively, and the outputs are $\eta = [\dot{\psi}, Y]$.

B. Tire model

Except for aerodynamic forces and gravity, all the forces that affect vehicle handling are produced by the tires. Tire forces provide the primary external influence and, because of their highly nonlinear behavior, cause the largest variation in vehicle handling properties throughout the longitudinal and lateral maneuvering range. As a result, it is important to use a realistic nonlinear tire model, especially when investigating large control inputs that result in responses near the limits of the maneuvering capability of the vehicle. In such situations, the lateral and longitudinal motions of the vehicle are strongly coupled through the tire forces, and large values of longitudinal slip and slip angle can occur simultaneously.

The many existing tire models are predominantly “semi-empirical” in nature, where model structure is determined through analytical considerations, but key parameters still depend on tire data measurements. These models range from extremely simple (where lateral forces are computed as a function of slip angle, given only a measured slope at $\alpha = 0$ and a measured value of the maximum lateral force) to relatively complex algorithms, which use tire data measured at many different loads and slip angles.

The model for tire tractive and cornering forces (5) used in this paper is described by a Pacejka model [9]. This is a complex, semi-empirical model that takes into consideration the interaction between the tractive force and the cornering force in combined braking and steering. The longitudinal and cornering forces are assumed to depend on the normal force, slip angle, surface friction, and longitudinal slip. Sample plots of predicted longitudinal and lateral force versus longitudinal slip and slip angle are shown in Figures 3-4. These plots are shown for the front tire of the “bicycle” model, which represents the two front tires of the actual car.

The dynamic effects of tires while negotiating sudden changes of road/drive condition [10] have been ignored in this first level of investigation. The modeling of tire dynamics may be important for the development of future high perfor-
mance ABS, traction control, and IVD systems. In addition, the use of dynamic models yields the advantage of avoiding the static tire model numerical difficulties at low vehicle speeds [10]. Our future work will include consideration of the above tire dynamic effects, as needed. However, for the present study, which serves to establish the main trends and approaches, the above static tire model is appropriate.

III. PROBLEM FORMULATION

Given (i) the tire model described in Section II-B, (ii) the nonlinear dynamics of the model equations (11), and (iii) a desired path, we seek to minimize deviations from their references of the heading angle $\psi$ and of the lateral distance $Y$. We optimize over $\delta_f$ when $\delta_r = 0$ (assuming Active/Augmented Front Steer, AFS or 2WS) while external signal evolutions of tire slip and road friction $(s,\mu)$ act on the vehicle dynamics and side wind may impact the car. We regard the slip history as an external input to the model we are considering (we assume that there is an independent traction controller yielding the slip signal evolution). The Four-Wheel Steering (4WS) optimization over $\delta_f$ and $\delta_r$ is the topic of current ongoing research.

In the sequel we describe how a nonlinear Model Predictive Controller (MPC) can be designed for the posed problem. The main concept of MPC is to use a model of the plant to predict the future evolution of the system [6], [11]–[14]. Based on this prediction, at each time step $t$ a certain performance index is optimized under operating constraints with respect to a sequence of future input moves. The first of such optimal moves is the control action applied to the plant at time $t$. At time $t+1$, a new optimization is solved over a shifted prediction horizon.

In order to obtain a finite dimensional optimal control problem we discretize the system dynamics (11) with the Euler method, to obtain

\[ \xi(k+1) = f_s^d(\xi(k), \Delta u(k)), \]
\[ \eta(k+1) = h(\xi(k)), \]

where $\Delta u$ formulation is used, i.e., $u(k) = u(k-1) + \Delta u(k)$ and $u(k) = \delta_f(k)$, $\Delta u(k) = \delta_f(k)$.

We consider the following cost function:

\[ J(\xi(t), \Delta u_t) = \sum_{i=1}^{H_P} \left[ \eta_{t+i,t} - \eta_{ref,t+i,t} \right]^2 + \sum_{i=0}^{H_P-1} \left\| \Delta u_{t+i,t} \right\| R, \]

where, as in standard MPC notation [6], [12], $\Delta u_t = \Delta u_{t,\ldots,t+H_P}, \ldots, \Delta u_{t+H_p-1,t}$ is the optimization vector at time $t$ and $\eta_{t+i,t}$ denotes the output vector predicted at time $t+i$ obtained by starting from the state $\xi_t = \xi(t)$ and applying to system (12) the input sequence $\Delta u_{t,\ldots,t+H_P}, \ldots, \Delta u_{t+H_p-1,t}$.

At each time step $t$ the following finite horizon optimal control problem is solved:

\[
\begin{align}
\min_{\Delta u_t} & \quad J(\xi(t), \Delta u_t) \\
\text{sub. to} & \quad \xi_{k+1,t} = f_s^d(\xi_{k,t}, \Delta u_{k,t}), \\
& \quad \eta_{k,t} = h(\xi_{k,t}), \\
& \quad k = t, \ldots, t + H_p \\
& \quad \delta_{f,\min} \leq u_{k,t} \leq \delta_{f,\max} \\
& \quad \Delta \delta_{f,\min} \leq \Delta u_{k,t} \leq \Delta \delta_{f,\max} \\
& \quad k = t, \ldots, t + H_c - 1 \\
& \quad u_{k,t} = u_{k-1,t} + \Delta u_{k,t}.
\end{align}
\]

We denote by $\Delta u^*_t \triangleq [\Delta u^*_{t,t}, \ldots, \Delta u^*_{t+H_c-1,t}]'$ the sequence of optimal input increments computed at time $t$ by solving (14) for the current observed states $\xi_{t,t}$, assuming wind, slip and friction coefficient values constant and equal to the estimates/given values at time $t$ over the prediction horizon.

The resulting state feedback control law is

\[ \delta_f(t)(\xi(t)) = \delta_f(t-1) + \Delta u^*_{t,t}(\xi(t)). \]

It is well known that stability is not ensured by the MPC law (14)–(15). Usually the problem (14) is augmented with a terminal cost $l_N$ and a terminal constraint set $X_f$, chosen to ensure closed-loop stability. We assume that the reader is familiar with the basic concept of MPC and its main issues, a treatment of sufficient stability conditions goes beyond the scope of this work and can be found in the survey [6].

IV. NONLINEAR PROGRAMMING SOLVERS

Linear and nonlinear MPC has been a success in the process industry [15]. However, MPC implementation requires significant computing infrastructure which might not be available on processes with fast sampling time and limited computational resources. Recently, thanks to the combination of new research results and faster computing units, it has been possible to extend the implementation of MPC design to new areas such as aerospace and automotive (see the recent NLMPc experimental test carried out on an autonomous modified T-33 Aircraft [16], [17], and the experimental test of a hybrid MPC controller on a Ford vehicle [14]).

We used the commercial NPSOL software package [18] for solving the nonlinear programming problem (14). NPSOL is a set of Fortran subroutines for minimizing a smooth function subject to constraints, which may include simple bounds on the variables, linear constraints and smooth nonlinear constraints. The user provides subroutines to define the objective and constraint functions and (optionally) their first derivatives. NPSOL uses a sequential quadratic programming (SQP) algorithm, in which each search direction is the solution of a QP subproblem. Bounds, linear constraints and nonlinear constraints are treated separately. NPSOL requires relatively few evaluations of the problem functions. Hence it
is especially effective if the objective or constraint functions are expensive to evaluate.

V. SIDE WIND REJECTION USING ACTIVE STEERING

In the first simulation scenario considered in this paper, the NL MPC controller described in Section III was used to evaluate the effect of an external side wind on the vehicle. The study allowed to estimate the maximum wind speed that an MPC-based active steering system is able to contain so that the vehicle remains stable. Section V-A describes details of the simulation scenario, while Section V-B presents our most important results and findings.

A. Scenario Description

In this scenario, the vehicle is assumed to be going straight on a flat surface at 15 m/s, with zero slip ratio (i.e., neither accelerating, nor braking). At time $t = 0.5$ sec, a side wind gust impacts the vehicle with velocity $v_w$, generating the force and moment given by the equations in (2). The wind gust is assumed to be persistent throughout the simulation. In the meantime, at time $t = 2$ sec, the road friction coefficient $\mu$ changes from 0.9 to 0.1 (from cement to ice) and an exogenous traction control system is assumed to produce the slip history shown in Figure 5.

![Slip history of the front wheel](image)

Fig. 5. Slip history of the front wheel.

The objective is to keep the vehicle along its straight path, hence the lateral position reference value $\eta_{ref}$ is zero. The controllers minimize the $\psi$ and $Y$ deviations from zero by optimizing for the front steering angle $\delta_f$.

B. Simulation Results

The NMPC active steering controller performed well in this lateral stabilization problem and could reject wind gusts up to 10.1 m/s. This limit was imposed by the saturation of lateral forces on the rear wheels.

Figure 6 shows the results obtained in the 2WS setup with a wind speed of 10 m/s. In this scenario, steady state errors of $-0.5$ degrees on $\psi$ and $0.01$ m on $Y$ were achieved. The average NPSOL computational time was $0.13$ sec, with prediction and control horizons equal to 10 and 4 steps, respectively. The worst-case NPSOL computational time was $0.31$ sec. Figure 6 shows that in steady state the front steering angle solution oscillates with a magnitude of approximately 0.2 degrees.

These oscillations were reduced by constraining the steering angle rate of change to $\pm 25$ deg/s. In addition to this, the steering angle was constrained artificially to $\pm 2$ deg in order to illustrate constraint enforcement. The effect is shown by the dashed signals in Figure 6. Due to the presence of active constraints (steering angle saturation), the NPSOL average and worst-case computational times increased to $0.17$ sec and $0.38$ sec, respectively.

VI. DOUBLE LANE CHANGE ON SNOW USING ACTIVE STEERING

The nonlinear MPC steering controller described in Section III was also implemented to perform a sequence of double lane change maneuvers at increasing entry speed. The desired path negotiates the vehicle in between and around cones as shown in the lower right subplot of subsequent figures. Section VI-A describes details of the simulation scenario, and Section VI-B presents our most important results and findings. It should be pointed out that the same controller is being used here to control the vehicle during different maneuvers, as in the side wind gust rejection example of the previous section.

A. Scenario description

This test represents an obstacle avoidance emergency maneuver in which the vehicle is entering a double lane change maneuver on snow with a given initial forward speed. The control inputs in a 2WS scenario are the front steering angle and the goal is to follow the desired path as close as possible by minimizing the vehicle deviation from the target trajectory. The use of front and rear steering angles in a 4WS scenario is the topic of current ongoing research. The experiment is repeated with increasing entry speeds until the vehicle loses control. It should be noticed that the vehicle
is coasting during the maneuver, i.e., no braking or traction torque has been applied to the wheels.

Unless specified otherwise, the following parameters have been used in the design of the 2WS NLMPC controller described in Section III:

- sample time: $T = 0.05$ sec;
- constraints on maximum and minimum steering angles $-30$ deg $\leq \delta_f \leq 30$ deg
- constraints on maximum and minimum changes in steering angles $-20$ deg/s $\leq \Delta\delta_f \leq 20$ deg/s

The controller tuning parameters at a given longitudinal vehicle speed are the prediction horizon $H_p$, control horizon $H_u$ and the weighting matrices $Q$ and $R$.

### B. Simulation results

The NLMPC (14)-(15) has been tested for different vehicle entry speed ranging from 5 m/s to 17 m/s. The minimum length of prediction and control horizons required for vehicle stabilization and acceptable performance has been estimated and reported on the diagonal elements of Table I. For the controllers marked as “Unstable”, a stabilizing tuning (in term of matrices $Q$ and $R$) working at the specified vehicle speed has not been found. For instance, with a vehicle speed of 10 m/s, the minimum required prediction and control horizons for stabilizing the vehicle are seven and two steps, respectively. The empty cells in the table denote controllers that provide better performance than the controllers on the diagonal with additional computational load.

### Table I

**Summary of results using different controllers at different vehicle speeds.**

<table>
<thead>
<tr>
<th>Vehicle speed [m/s]</th>
<th>Prediction horizon ($H_p$)</th>
<th>2</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$H_u = 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Unstable $\forall H_u \leq 2$</td>
<td>$H_u = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Unstable $\forall H_u \leq 2$</td>
<td>Unstable $\forall H_u \leq 7$</td>
<td>$H_u = 4$</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Unstable $\forall H_u \leq 2$</td>
<td>Unstable $\forall H_u \leq 7$</td>
<td>$H_u = 7$</td>
<td></td>
</tr>
</tbody>
</table>

The vehicle started to lose control at the beginning of the second lane change where the maneuver exhibited some elements similar to counter-steering. It should be pointed out that the NLMPC tuning at 15 m/s requires less effort than the 17 m/s case.

The effects of active constraints on the computational time and on maneuver policy have also been analyzed. To this aim, the double lane change maneuver has been performed at 12 m/s, with $H_p = 10$, $H_u = 4$ and by tightening the limits on the steering angle to $\pm 2$ degrees.

Figures 11-12 show the unconstrained and constrained case, respectively. The constrained case uses a different maneuver with an unavoidable higher error on the lateral deviation $Y$ and on the yaw angle $\psi$. We could observe that the computational time required by the nonlinear program solver increases when constraints on the steering angle become active.

As an independent test of above results, it should be pointed out that they are comparable with the corresponding results obtained through the optimization done at Ford Research Laboratories using SOCS [19] and iSIGHT [20] software.

### VII. Real-time implementation

In this section we discuss possible approaches which can provide suboptimal and real-time implementable MPC schemes. This is the topic of current ongoing research directed towards the experimental validation of the results presented in this paper. Also, increasing power of computational infrastructures and improvements of optimization tools will be relevant for the future development of this work and are not considered here.

The critical point in the demonstrated approach is the complexity associated to the solution of a constrained nonlinear optimization problem. Linear and piecewise-linear vehicle models might lead to suboptimal control problems with faster solution time and acceptable performance.

### A. Piecewise-linear models

Following a similar approach to the works of [14], [21], [22] in the automotive field, the model equations (11) can be piecewise-linearized. This would enable the use of Mixed Logical Dynamical (MLD) models [23] to describe the vehicle dynamics. The optimization problem becomes a mixed-integer quadratic programming (MIQP) or a mixed-integer linear programming (MILP) problem depending on the norm used in the cost function. The MPC controller requires the MILP or MIQP to be solved on-line at each sampling time. In a second phase, if the solution of a MILP or MIQP is prohibitive, the explicit piecewise affine form of the MPC law can be computed off-line by using multiparametric mixed integer programming (mp-MILP or mp-MIQP) solvers [24]. The resulting control law has the piecewise affine form and although the resulting piecewise affine control action is 

**identical** to the MPC designed, the on-line complexity is reduced to the simple evaluation of a piecewise affine function.
Fig. 7. Double lane change maneuver at 10 m/s with $H_p = 7$ and $H_u = 2$.

Fig. 9. Double lane change maneuver at 15 m/s with $H_p = 10$ and $H_u = 4$.

Fig. 11. Simulation results with inactive constraints at 12 m/s.

Fig. 8. Double lane change maneuver at 10 m/s with $H_p = 7$ and $H_u = 2$.
NPSOL computation time, yaw rate, tire forces and slip angles.

Fig. 10. Double lane change maneuver at 15 m/s with $H_p = 10$ and $H_u = 4$.

Fig. 12. Simulation results with active constraints at 12 m/s.
The use of piecewise-linear models allows nonlinear predictions in the MPC control design. In particular, for long prediction horizons, it might be critical to predict that the lateral tire forces do not increase linearly with the tire slip angle but they start decreasing beyond a certain peak. On the other hand, this approach requires the generation of piecewise linear approximations to the tire model described in Section II-B and shown in Figure 3-4. Such approximation might require a high number of affine terms in the model, which could lead to an explosion of the mp-MIQP or mp-MILP solution complexity. Nevertheless, it has been possible to obtain piecewise affine explicit solutions in the field of direct torque engines for highly nonlinear multidimensional systems [25].

B. Linear models

A further reduction can be obtained by linearizing the vehicle dynamics around a certain control input and state trajectory. This would lead to a linear time-varying (LTV) prediction model, where the linear state matrices change at every future time step. The problem complexity is reduced to a quadratic program, which requires less computational resources and could be implemented in real time. However, efficient linearization of the nonlinear vehicle dynamics around the chosen trajectory presents an additional challenge. Analytic expressions could be obtained for the Jacobian matrices using the equations of motion. Another approach is to calculate the Jacobian numerically for all future time steps online, which leads to a more significant computational burden.

As an alternative approach, one can locally linearize the optimization problem (14), and compute local explicit solution to multiparametric quadratic programs, as proposed in [26]. In particular, the method uses quadratic approximations to the nonlinear cost and linear approximations to the constraints equations. This allows to approximate the NLP (14) solution with a set of quadratic programs (QP), for which explicit equivalent solution functions can be calculated by means of multiparametric quadratic programs (mp-QP). The advantage is that these approximations can be computed off-line as an explicit, piecewise linear function of the state. The online computational burden is reduced to evaluation of a simple piecewise linear look-up table. Possible issues may arise from the interplay between the desired approximation accuracy and the solution complexity.

Finally, two major drawbacks deriving from the use of local linear models should be pointed out. First, predictions will be based on linear models, i.e., over a given prediction horizon the MPC controller will not be able to predict a change of slope in the cornering tire curve. Second, the state constraints, if present, might not be fulfilled for the initial nonlinear model, unless robust MPC formulations are used [27]–[29].

VIII. CONCLUDING REMARKS AND FUTURE WORK

We have presented a novel MPC-based approach for active steering control design. The approach was used to develop, in a systematic way, a feedback controller for a side wind rejection and double lane change maneuver on slippery surfaces such as snow covered roads. Simulation results showed that complex steering maneuvers are relatively easily obtained as a result of the MPC feedback policy, leading to the capability of stabilizing the vehicle up to wind speeds of 10.1 m/s in the wind rejection scenario and vehicle speed of 17 m/s in double lane change maneuvers, respectively. The results correlate well with those obtained through Ford internal research activities.

In addition, the trade-off between vehicle speed and prediction horizon $H_p$ has been highlighted. The minimum prediction horizon which provides acceptable performance and vehicle stability has been quantified at several vehicle speeds. The role of constraints has been discussed as well. We showed how the introduction of steering constraints can be handled in a systematic way by the MPC design. A loss of performance, without loss of vehicle stability, has been observed in the case of vehicle entry speed of 12 m/s when reducing the constraints on the steering angle from ±30 deg to ±2 deg.

The computational complexity of the feedback control policy has been highlighted and simulation results have shown the possibility of experimentally validating the methodology at low vehicle speed, typically below 10 m/s. For high vehicle speeds we are currently investigating new methodologies and computational tools for approximating the nonlinear MPC with a real-time implementable control scheme. Such sub-optimal MPC approaches have been proposed in Section VII-B.

Additional comments on future research issues are summarized below:

- The experiment setups rely on the assumption that the steering angles are the only control inputs. A vehicle dynamic control algorithm could be based on other control inputs as well, such as brakes, transmission, longitudinal vehicle speed, electrically controlled differentials and others.
- Actual implementation of the analyzed control schemes will have to incorporate state estimation from available measurements. This is a significant area of research on its own. In this paper, we assumed that the states associated to the prediction model can be measured on the actual testbed.
- As indicated in Section I, the present work assumes autonomous driving conditions that can be achieved via a steering robot driver. In the future, one could extend the proposed approach to the case where Active Front and Rear Wheel Steer are primarily used to assist a driver, especially in critical, safety-oriented maneuvers. In this context, the driver and vehicle form a combined plant to be controlled, where typical driver models (or in a more advanced stage, an on-line adaptive/learning driver) could be used.
- An additional degree of flexibility can be provided in the double lane change scenario by optimizing for the reference trajectory as well, instead of following a pre-
specified path. For example, this type of optimization would incorporate the knowledge about potential obstacles (facilitated by cameras, GPS and vehicle to vehicle communications) and the constrained dynamics of the vehicle. This would result in a reference trajectory that would avoid obstacles and optimize drivability within the given road surface conditions and vehicle speed.

REFERENCES