Abstract: This paper presents an active safety system for avoiding obstacles and preventing road departures by means of assistance in steering and braking. A Nonlinear Model Predictive Controller (NMPC) is designed with the goal of using the minimum control intervention to keep the driver safe. The NMPC uses a nonlinear vehicle and driver steering model to predict a collision or a roadway departure. The first part of the paper presents the models and the control design approach. In the second part, simulations demonstrate the effectiveness of the proposed approach.

1. INTRODUCTION

Modern passenger vehicles are equipped with computational, sensing and actuating capabilities which enable the design of systems that can autonomously control the vehicle motion in order to assist the driver. In this paper we develop an integrated safety system which utilizes a predictive controller, in combination with a driver steering model, in order to assist the driver in avoiding obstacles and remaining in the lane. This is a combined lane keeping, stability control and collision avoidance problem. Distner et al. (2009); Mellinghoff et al. (2009); Cerone et al. (2009); Van Zanten (2000); Gao et al. (2010).

Threat-assessment is a challenging aspect in the design of such a system. “Threat-assessment” usually refers to the real-time evaluation of a function which quantifies if and when an accident is unavoidable. In lane keeping systems, steering interventions occur when the time to line crossing passes a certain threshold, Mammar et al. (2006). Equivalently, the time to collision has been used to activate braking interventions in collision avoidance systems, Jansson (2005), and in stability control systems interventions are activated once the vehicle deviates from a nominal behavior, Van Zanten (2000). In Falcone et al. (2011) vehicle and driver models are used to compute a set of safe states, from which the vehicle can safely evolve, and assisting interventions are activated once the vehicle state is outside this set. By including a model of the driver behavior, interventions can be activated based on limitations of the estimated behavior of the driver rather than just the limitations set by the vehicle dynamics. All these are examples of threat-assessment design.

In this paper, because of its capability to systematically handle system nonlinearities and constraints, Model Predictive Control (MPC) is the preferred choice for control design. MPC in vehicle applications has mainly been used for generation and tracking of feasible trajectories, as in Gray et al. (2012) and Gao et al. (2010), rather than for determining whether a driver is in need of assistance.

In the approach presented in Anderson et al. (2010) the solution to the MPC problem is used to guide the vehicle through a safe corridor which is constructed based on the road boundaries and obstacles. The MPC problem is formulated as a quadratic program by restricting the intervention to steering only, linearizing the vehicle dynamics around a constant vehicle speed, and assuming linear tire characteristics. The output of the optimization problem is used to form a simple threat metric that is used to shift control from the driver and to the controller. Nonlinear MPC has been used for combined steering and braking in Falcone et al. (2008). However, incorporating obstacles as constraints while allowing the MPC controller to control the vehicle speed requires the online solution of a complex mixed-integer program. We have proposed a method to simplify these constraints in Gao et al. (2012) and allow for a real-time implementation.

This paper focuses both on improving the performance of safety systems for avoiding roadway departures and increasing the scope of such technology to avoid or mitigate collisions with obstacles. We formulate a single combined optimization problem that handles both problems. In particular, a predictive optimal control problem is formulated which simultaneously uses predicted driver behavior and determines the least intrusive intervention that will keep the vehicle in a region of the state space where the driver is deemed safe. We also present a transformation which transforms the time-dependent vehicle dynamics into spatial-dependent dynamics. This transformation leverages the incorporation of the obstacle avoidance constraints while maintaining the MPC controller’s authority to influence both steering and braking. The resulting controller is always active and only applies the correcting control action that is necessary to avoid violation of the safety constraints.

The rest of the paper is organized as follows: in Section 2 we introduce the vehicle dynamics and driver models. In Section 3 we introduce the safety constraints as requirements that the vehicle stays in a collision free path in
the lane while operating in a region of the state space where the driver is deemed capable of maneuvering the vehicle. In Section 4, the proposed predictive controller is presented and in Section 5 we present validation results of the proposed method. Finally, in Section 6 we provide some concluding remarks and outline future work.

2. MODELING

In this section, we present the mathematical models used for the combined threat assessment and control design.

2.1 Vehicle model

Consider the vehicle sketch in Figure 1. We use the following set of differential equations to describe the vehicle motion within the lane:

\[ m\ddot{v}_x = mv_y \dot{\psi} + \sum_{i=1}^{4} F_{xi}, \quad (1a) \]

\[ m\ddot{v}_y = -mv_x \dot{\psi} + \sum_{i=1}^{4} F_{yi}, \quad (1b) \]

\[ J_\psi \ddot{\psi} = l_f (F_{y1} + F_{y2}) - l_r (F_{y3} + F_{y4}) + \frac{w_t}{2} (-F_{x2} + F_{x3} + F_{x4}), \quad (1c) \]

\[ \dot{\psi} = \ddot{\psi} - \ddot{\psi}_{x}, \quad (1d) \]

\[ \dot{v}_x = v_y \cos(\psi) + v_x \sin(\psi), \quad (1e) \]

\[ \dot{v}_y = v_y \cos(\psi) + v_x \sin(\psi), \quad (1f) \]

where \( m \) and \( J_\psi \) denote the vehicle mass and yaw inertia, respectively, \( l_f \) and \( l_r \) denote the distances from the vehicle center of gravity to the front and rear axles, respectively, and \( w_t \) denotes the track width. \( \dot{v}_x \) and \( \dot{v}_y \) denote the vehicle longitudinal and lateral accelerations, respectively, and \( \dot{\psi} \) is the turning rate around a vertical axis at the vehicle’s center of gravity. \( e_\psi \) and \( e_y \) in Figure 1 denote the vehicle orientation and lateral position, respectively, in a road aligned coordinate frame and \( \psi_s \) is the angle of the tangent to the road centerline in a fix coordinate frame. \( F_{yi} \) and \( F_{xi} \) are tire forces acting along the vehicle lateral and longitudinal axis, respectively, and \( f_{yi} \), \( f_{xi} \) are forces acting along the tire lateral and longitudinal axis, respectively.

The longitudinal and lateral tire force components in the vehicle body frame are modeled as,

\[ F_{xi} = f_{xi} \cos(\delta_i) - f_{yi} \sin(\delta_i), \quad (2a) \]

\[ F_{yi} = f_{xi} \sin(\delta_i) + f_{yi} \cos(\delta_i), \quad i \in \{1,2,3,4\}, \quad (2b) \]

where \( \delta_i \) is the steering angle at wheel \( i \). We introduce the following assumption on the steering angles,

*Assumption 1.* Only the steering angles at the front wheels can be controlled and the steering angles at the right and left wheels of each axle are assumed to be the same, i.e., \( \delta_1 = \delta_2 = \delta \) and \( \delta_3 = \delta_4 = 0 \). In addition, an actuator which corrects the driver commanded steering angle, such that \( \delta = \delta_d + \delta_e \), is available, where \( \delta_d \) is the driver commanded steering angle and \( \delta_e \) is the correcting steering angle component. This can be realized by means of a planetary gear and electric motor.

We introduce the following assumption on the braking forces:

*Assumption 2.* Pedal braking, distributes braking forces according to the following relation,

\[ f_{x1} = f_{x2} = \sigma \frac{F_y}{2}, \quad f_{x3} = f_{x4} = (1 - \sigma) \frac{F_y}{2}, \quad (3) \]

where \( \sigma \) is a constant (vehicle dependant) distribution parameter and \( F_y \) is the total braking force. An actuator capable of augmenting the braking of the driver is assumed available.

The force component \( f_{yi} \) is computed using a simplified version of the Pacejka magic tire formula Pacejka (2005). We let \( \alpha_i \) denote the tire slip angle, \( \mu_i \) denote the friction coefficient, \( F_{zi} \) denote the vertical load at each wheel and write the tire formula as

\[ f_{yi} = \sqrt{(\mu_i F_{zi})^2 - f_{x_i}^2} \sin(C_i \arctan(B_i \alpha_i)), \quad (4) \]

where \( C_i, B_i \) are tire parameters calibrated using experimental data.

The tire slip angles \( \alpha_i \) in (4) are approximated as,

\[ \alpha_1 = \frac{v_y + l_f \dot{\psi}}{v_x + \frac{l_f}{2} \dot{\psi}} - \delta, \quad \alpha_2 = \frac{v_y + l_f \dot{\psi}}{v_x + \frac{l_f}{2} \dot{\psi}} - \theta, \quad (5a) \]

\[ \alpha_3 = \frac{v_y - l_f \dot{\psi}}{v_x - \frac{l_f}{2} \dot{\psi}}, \quad \alpha_4 = \frac{v_y - l_f \dot{\psi}}{v_x - \frac{l_f}{2} \dot{\psi}}. \quad (5b) \]

We make use of the following assumptions:

*Assumption 3.* In equation (4) the vertical forces \( F_{zi} \) are assumed constant and determined by the vehicle’s steady state weight distribution when no lateral or longitudinal accelerations act at the vehicle center of gravity.

*Assumption 4.* The signal \( \dot{\psi}_s \) is treated as an exogenous disturbance signal. Every time instant, an estimate of \( \dot{\psi}_s \) is available over a finite time horizon. See, e.g., Jansson (2005); Bertozzi et al. (2000); Eidehall et al. (2007) for sensing technologies that can be used to obtain this signal.
2.2 Driver Model

In this problem formulation the Nonlinear MPC controller utilizes a driver steering model. Generally, an accurate description of the driver’s behavior requires complex models accounting for a large amount of exogenous signals, Cacciaue (2007), however, we are interested in very simple model structures, enabling the design of low complexity model-based threat assessment and control. The driver steering model we utilize is an explicit equation for steering angle as a function of road geometry and vehicle state. The parameters are estimated online based on observed driver behavior.

Define the orientation error \( e^p_\psi \) w.r.t. a look-ahead point as in Figure 1,

\[
e^p_\psi = \psi - \hat{\psi}^p = e_\psi + \Delta \psi_s,
\]

where \( \hat{\psi}^p \) is the heading of the lane centerline at time \( t + t^p \), with \( t \) the current time, \( \Delta \psi_s = \psi_s - \hat{\psi}^p \) and \( t^p \) the preview time that can be mapped into the current preview distance \( s_{tp} \).

Denote by \( w_{or}, w_{ol} \), the width of an obstacle located at the right and left lane borders, respectively, which are zero if no obstacle is present. We denote the position error

\[
e^p_y = e_y = \frac{1}{2}w_{or} + \frac{1}{2}w_{ol},
\]

as the distance of the vehicle’s center of gravity from the center of the free portion of the lane.

We compute an estimate of the driver commanded steering angle \( \delta_d \) as,

\[
\delta_d = K_y e^p_\psi + K_\psi e^p_\psi
\]

with \( K_y \) and \( K_\psi \) as gains that are, in general, time varying and are updated online. Clearly, \( \Delta \psi_s \) in (6) and \( w_{or}, w_{ol} \) in (7) depend on the preview time \( t^p \), that, in our modeling framework, is considered as a parameter of the driver model. Note that the steering model (8) is velocity dependant since \( \Delta \psi_s \) also depends on the vehicle speed \( v_y \).

Remark 1. The obstacle widths \( w_{or}, w_{ol} \) are included in (7) to model the driver’s attempt to avoid road side obstacles, which is a realistic assumption when the driver is attentive. Nevertheless, if it can be established that the driver is distracted, as in Vasudevan et al. (2012), this can be accounted for by setting the widths \( w_{or}, w_{ol} \) to zero.

Estimation results of driver model parameters, \( K_y \) and \( K_\psi \), in the case of no obstacles, are presented in Falcone et al. (2011) for both normal and aggressive driving styles.

We write the model (1)-(8) in the following compact form,

\[
\xi(t) = f(\xi(t), u(t), w(t)),
\]

where \( \xi = [e_x, e_y, \hat{\psi}, e_\psi, e_y]^T \), \( u = [\delta_d, F_b]^T \) and \( w = [\mu, \dot{\psi}_d, \Delta \psi_d, w_{ol}, w_{or}]^T \) are the state, input and disturbance vectors, respectively.

2.3 Spatial Model

The semi-autonomous controller derived in Section 4 utilizes both braking and steering to keep the vehicle on a collision-free path. In order to formulate the constraints for obstacles and road bounds while maintaining the ability to control both steering and braking, we introduce a spatial-vehicle model where the independent variable, with respect to which the system states are differentiated, is the traveled distance along the lane centerline \( s \). We have experimentally validated the proposed transformation in a real-time implementation in Gao et al. (2012).

The following kinematic equations can be derived from Figure 1,

\[
v_s = (R_s - e_y) \cdot \dot{\psi} = v_x \cdot \cos(e_\psi) - v_y \cdot \sin(e_\psi).
\]

where \( v_s \) is the projected vehicle speed along direction of the lane centerline and \( R_s \) is the radius of curvature. The vehicle’s velocity along the path \( \dot{s} = \frac{dv_y}{ds} \) is then given by

\[
\dot{s} = R_s \cdot \dot{\psi} = \frac{R_s}{R_s - e_y} \cdot (v_x \cdot \cos(e_\psi) - v_y \cdot \sin(e_\psi)).
\]

Using the fact that \( \frac{dv_x}{ds} = \frac{dv_x}{dt} \cdot \frac{dt}{ds} \) we write the spatial model in the following compact form,

\[
\xi' = f(\xi, u, w) \cdot \frac{1}{s} = f_s(\xi, u, w),
\]

where \((\cdot)'\) denotes a variable’s derivative with respect to \( s \).

Remark 2. The time as a function of \( s \), \( t(s) \), can be retrieved by integrating \( t' \), \( t(s) = \int_{s_0}^{s} \frac{1}{s} \, ds \).

3. SAFETY CONSTRAINTS

We recall that the overall aim of the safety system proposed in this paper is to keep the vehicle on a collision free path within the lane while maintaining a stable vehicle motion. In this section we express the requirements that the vehicle stays in the lane and avoids obstacles, while operating in a stable operating region, as constraints on the vehicle state, input and disturbance variables.

Let \( e_i, i \in \{1,2,3,4\} \) be the distances of the four vehicle corners from the lane centerline. The requirement that the vehicle stays in the free portion of the lane is then expressed,

\[
-e_{i \text{max}} + w_{or} \leq e_i \leq e_{i \text{max}} - w_{ol}, i \in \{1,2,3,4\}.
\]

In addition to staying in the lane, we require that the vehicle operates in a region of the state space where the vehicle is easily maneuverable by a normally skilled driver.

This requirement can be ensured by limiting the tire slip angles \( \alpha_i \) as,

\[
\alpha_{i \text{min}} \leq \alpha_i \leq \alpha_{i \text{max}}, i \in \{1,2,3,4\}.
\]

For limited slip angles the vehicle behavior is predictable by most drivers and Electronic Stability Control (ESC) systems are inactive Gillespie (1992); Tseng et al. (1999).

The constraints (13)-(14) can be compactly written as,

\[
h(\xi, u, w) \leq 0,
\]

where \( \mathbf{0} \) is a vector of zeros with appropriate dimension.

4. PREDICTIVE CONTROL PROBLEM

In this section we formulate the threat assessment and control problems as a single Model Predictive Control Problem (MPC). At each sampling time instant an optimal
input sequence is calculated by solving a constrained finite-distance optimal control problem for the transformed system (12). The computed optimal control input sequence is only applied to the plant during the following sampling interval. At the next time step the optimal control problem is solved again, using new measurements.

We discretize the system (12) with a fixed sampling distance $ds$ to obtain,

$$
\xi_{s+k} = f^{ds}(\xi_s, u_s, w_s),
$$

and formulate the optimization problem, to be solved at each step, as

$$
\min_{u_{s+k}} \sum_{k=0}^{H_p-1} ||u_{s+k}||^2_R + \rho c
$$

subject to

$$
\begin{align}
\xi_{s+k+1} &= f^{ds}(\xi_{s+k}, u_{s+k}, w_{s+k}), & k = 0, \ldots, H_p - 1 \\
\epsilon_k(\xi_{s+k}, u_{s+k}, w_{s+k}) &\leq 1, & k = 0, \ldots, H_p \\
u_{s+k} &\leq \epsilon_{s+k} + \nu_{s+k-1}, & (17c) \\
u_{\min} &\leq u_{s+k} \leq u_{\max}, & (17d) \\
\Delta u_{\min} &\leq \Delta u_{s+k} \leq \Delta u_{\max}, & k = 0, \ldots, H_p - 1 \\
\Delta u_{s+k+1} &= 0, & (17e) \\
\xi_{s,s} &= \xi(s), & (17f)
\end{align}
$$

where $s$ denotes the current position along the curve and $\xi_{s+k}$ denotes the predicted state at step $s+k$ obtained by applying the control sequence $u_s = [u_{s}, \ldots, u_{s+k}]$ to the system (16) with $\xi_s = \xi(s)$. $H_p$ denotes the prediction horizon and $H_c$ denotes the control horizon. The safety constraints (15) have been imposed as soft constraints, by introducing the slack variable $\epsilon$ in (17a) and (17d). $R$ and $\rho$ are weights of appropriate dimension penalizing control action and violation of the soft constraints.

We note that no penalty on deviation from a tracking reference is imposed in the cost function (17a). The objective here is to ensure that the safety constraints (15) are not violated, while utilizing minimal control action. If the driver steering model (8) is alone capable of steering the vehicle without violating the safety constraints (15), no control action will be applied and the optimal cost will thus be zero.

In addition to the soft constraints we have imposed hard constraints. (17e)-(17g) reflect limitations set by the actuators. The constraint (17h) enables $H_p$ to be chosen larger than $H_c$ and the control kept constant during the prediction time beyond $H_c$. This constraint is useful for real-time execution when computational resources are limited.

5. RESULTS

In this section we validate the behavior of the proposed active safety system. We first show a situation where the driver is attentive and capable of avoiding an obstacle. In this situation, the safety system correctly detects that the driver is attentive and capable of performing the driving task and does not intervene. Next, we demonstrate the ability of the adopted approach to detect critical situations and adequately assist the driver in avoiding accidents. We also consider a scenario with multiple obstacles and demonstrate the ability of the controller to avoid collisions in such challenging situations.

For the results presented next, the estimation algorithm used in Falcone et al. (2011) is implemented to estimate parameters of the driver model (8) and the vehicle and design parameters in Tables 1 and 2 are used to implement the predictive controller (17).

<table>
<thead>
<tr>
<th>$m$ (kg)</th>
<th>$\sigma$</th>
<th>$u_t$ (m/s)</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$C_1$</th>
<th>$C_2$</th>
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<td>2050</td>
<td>0.7</td>
<td>1.63</td>
<td>10.5</td>
<td>0.5</td>
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<table>
<thead>
<tr>
<th>$J_F$ (kgm$^2$)</th>
<th>$l_f$</th>
<th>$l_r$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$C_3$</th>
<th>$C_4$</th>
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<tbody>
<tr>
<td>3344</td>
<td>1.43</td>
<td>1.47</td>
<td>-12.7</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The driver estimation algorithm adapts and updates the parameters of the driver model as new data becomes available. Since the estimation is conducted in nominal driving conditions, the resulting driver model is expected to be representative of the nominal behavior of the driver. The implications of this are discussed next, as the behavior of the suggested predictive controller is analyzed for the considered scenarios.

We first show a situation where the driver is attentive. Consider Figure 2 which shows a situation where an attentive driver is negotiating a curve and encounters an obstacle in the path. The dashed line shows the path traversed by the driver. The driver has no problems avoiding the obstacle. The shaded vehicles illustrate the trajectory that is predicted by the MPC controller when the vehicle is in the position shown with a darker color. In this situation the driver behavior, modeled by (8), is capable of avoiding the obstacle without assistance from the MPC controller.

![Fig. 2. A situation where an attentive driver encounters an obstacle in the path.](image-url)
that the driver can maintain a safe trajectory and the decision to not intervene in this situation is correct.

Next we consider a scenario where the driver is distracted. Consider Figure 7 where the vehicle is approaching an obstacle. In this situation the driver does not change his behavior to account for the obstacle. The driver model in the MPC controller has been modified to account for this, as suggested in Remark 1. The inset in Figure 3 shows a snapshot of the moment the second obstacle was encountered and intervention was required. In this case a driver monitoring system has detected that the driver is distracted. This paper presents the integrated design of an active safety system to successfully navigate around multiple obstacles while minimizing the interference to the driver.

Next we consider the scenario in Figures 5-6, where a distracted driver is approaching multiple obstacles. The inset in Figure 5 shows a snapshot of the moment the second obstacle was encountered and intervention was required. A driver monitoring system has detected that the driver is distracted. Consider the scenario shown in Figure 7 which shows a scenario where information about driver’s distraction is not utilized. Figure 8 shows the control signals in the same scenario. The inset in Figure 7 shows a comparison between predicted trajectories as in Figure 3. We note that, at this position, even though the driver is assumed attentive and will try to avoid the obstacle, the vehicle is already in a state where the driver would have to deviate from the nominal behavior, described by the model (8), in order to avoid the obstacle. The MPC controller, therefore, assists the driver with as much control action as necessary to avoid the obstacle while minimizing the cost function (17a). By comparing Figures 3-4 and 7-8 we note that, in the case where no driver monitoring system is utilized, the intervention comes later and more control action is required however the safety system manages to avoid the obstacles in both cases.

6. CONCLUSIONS

This paper presents the integrated design of an active safety system for prevention of collisions with roadside obstacles and roadway departures. The resulting predictive controller is constrained to stay within the safe region.


