Scenario Model Predictive Control for Lane Change Assistance on Highways

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Abstract—This paper presents a new algorithm for detecting the safety of lane changes on highways and for computing safe lane change trajectories. This task is considered as a building block for driver assistance systems and autonomous cars. The presented algorithm is based on recent results in Scenario Model Predictive Control (SCMPC). It accounts for the uncertainty in the traffic environment via a small number of future scenarios, which can be generated by any model-based or data-based approach. The paper describes the SCMPC design as well as the integration with scenario-based traffic predictions. The design procedure is simple and can be generalized to other control situations. An extensive case study demonstrates the effectiveness of the proposed SCMPC algorithm and its performance in lane change situations.

I. INTRODUCTION

The past decade has witnessed a rising interest in Advanced Driver Assistance Systems (ADAS), backed by new sensor technology and the increasing capacity of computational hardware. ADAS can support drivers in many situations, e.g., collision avoidance, lane change assistance, or adaptive cruise control. Future developments will further increase the level of automation, eventually paving the way to fully autonomous cars.

This paper presents a new concept of scenario-based control design that can be integrated for ADAS and/or autonomous cars. The approach is applied to lane change assistance on highways.

A. Traffic Prediction Models

A key factor for further advances in ADAS is the estimation and prediction of traffic scenes. The estimation is based on measurements of the vehicle’s environment via laser scanners, radars, cameras, ultrasonic sensors, vehicle-to-vehicle or vehicle-to-infrastructure communication. Most ADAS also require predictions about the future development of the current traffic scene [1], [2]. These traffic predictions comprise the joint trajectory prediction for all relevant vehicles, called target vehicles (TVs). Difficulties arise from complex vehicle dynamics, intricate tire-road interactions, and uncertainty in a human’s driving actions.

The simplest approaches use physics-based models, which extrapolate the vehicle’s movement based on the fundamental laws of motion and friction limits [3]. These models do not account for the driver’s control actions. Hence they are accurate only up to a human’s reaction time, i.e., about 1 second. Longer prediction times require the use of maneuver-based models, which are based on motion primitives for the types of maneuvers the driver may perform [4]. The classification and identification of these motion primitives works well in cases where the level of interaction with the surrounding traffic is small. In other cases, interaction-aware models are needed for a higher level of realism. These models base their predictions on the modeling of a joint feedback mechanism of all vehicles in the traffic scene. However, they need to cope with a high dimensional space of possible scenarios.

B. Model Predictive Control

Model Predictive Control (MPC) is a powerful approach for optimal control of multi-variable systems with constraints on the inputs and states [5], [6]. MPC has been used successfully in a variety of industrial applications [7].

Recent studies have applied MPC also for vehicle control. One benefit of MPC is the easy integration of forward information and constraints, resulting from traffic predictions or road geometry. The most basic approach, Deterministic MPC (DMPC), is based on deterministic traffic predictions. DMPC has successfully been implemented for path following under constraints [8], in absence of other traffic. A more advanced approach, Stochastic MPC (SMPC), uses probabilistic traffic predictions. SMPC has been tested for highway driving in a simulation study [9], by considering chance constraints to keep the vehicle within safe regions.

This paper introduces a new Scenario MPC (SCMPC) approach [10], [11]. SCMPC is based on traffic predictions via scenarios, rather than a probabilistic model. The advantages are that scenario-based predictions are simple, intuitive, and amenable to numerical processing. Moreover, the scenarios can be sampled from any kind of model or data set.

Recent studies have revealed a direct connection between the number of scenarios and chance constraints on the system states [11], [12], using principles from scenario optimization [13], [14]. The number of scenarios required by SCMPC is low, typically less than a hundred for chance constraint probabilities of a few percent.

C. New SCMPC Algorithm

The SCMPC algorithm presented in this paper consists of two components: the generation of traffic scenarios (Section II) and the MPC formulation (Section III). It will be applied for detecting and performing safe lane change trajectories on highways. A safe trajectory is considered to be one that observes all actuator, comfort, and safety constraints. The algorithm requires that the lane change is initiated by a human driver, e.g., via the turn signal (using it for ADAS), or a higher-level path planner (using it for autonomous driving).
**Assumption 1 (Lane Change Initiation)** (a) The vehicle stays in its current lane until a lane change is initiated. (b) The initiation of a lane change is given as an external signal to the SCMPC controller.

**II. SCENARIO GENERATION**

Given the present traffic scene, SCMPC requires an uncertainty model via future traffic scenarios. Both the scenario generation and the controller assume discretized time steps \( t = 0, 1, 2, \ldots, \) using a common sample time \( t_s \).

**A. Background**

Different model types have been used for uncertainty description in traffic predictions [2]. Deterministic models simply provide a single prediction trajectory for all objects in the traffic scene [15]. As such, they do not reflect the significant uncertainty associated with the possibility for different actions taken by the human drivers. Stochastic models use standard distribution functions to model the driver’s behavior [1]. This approach, however, is restricted to a set of standard distributions, typically Gaussian mixtures [16]. Furthermore, it is difficult to adjust to a specific road geometry, the traffic rules, and the interaction with other vehicles.

Scenario-based models do not require an explicit probability distribution for all possibilities in a traffic scene [3], [17]. This may facilitate the uncertainty description significantly, as illustrated in Section II-B.

**Remark 2 (Scenario Generation Model)** The SCMPC algorithm (presented in Section III) can be based on any generic model-based or data-based method for scenario generation, if fulfills some minimal assumptions (as stated below).

Let \( N \) be the time horizon for the traffic predictions. Denote all uncertainty involved in the traffic predictions over the horizon by an abstract variable \( \delta_t \in \Delta_t \), where the support \( \Delta_t \) has an arbitrary nature and dimensions. The following technical assumption on the scenario generation is needed to derive the theoretical properties in Section IV.

**Assumption 3 (Traffic Scenarios)** (a) At every time step \( t \), an arbitrary number \( K \) of samples of the uncertainty \( \delta_t \in \Delta_t \) over the horizon \( N \) (‘scenarios’) can be drawn from the traffic prediction model. (b) These sampled scenarios \( \delta_t^{(1)}, \ldots, \delta_t^{(K)} \) are independent and identically distributed (i.i.d.) with the real uncertainty outcome \( \delta_t \).

Assumption 3 requires only that sufficient scenarios are available and that they are representative of the actual uncertainty in the traffic scene. Typically, the number of scenarios \( K \) turns out to be relatively small, as it will be seen later in Section IV. The availability of representative data is a minimal requirement for any uncertainty prediction.

Note that the scenario-based uncertainty model can be time-varying and hence be conditioned on the latest information available at each time step, unlike many other MPC approaches [18]. Neither the current joint probability distribution of \( \delta_t \) nor the support \( \Delta_t \) need to be known explicitly. These are further advantages over similar approaches using Stochastic MPC [9] or Robust MPC [19].

**B. Scenario Generation Model**

This paper employs an interaction-aware model for predicting the possible trajectories of each target vehicle (TV). The scenarios for individual TVs may be drawn directly from collected real-world driving data. Hence it does not require the fitting of an uncertainty model or any probabilistic assumptions. If a sampled trajectory causes a collision, it is rejected and replaced with another sample (rejection sampling) [3], [17]. Hence the resulting combined scenarios of all TVs in the traffic scene follows the implicit distribution of collision-free vehicle trajectories.

To obtain the speed profile of a TV \( j \), first a target speed \( v_{\text{ref},j} \) is sampled. This target speed equals to current speed of the TV with some high probability, but it may also differ with a non-zero probability. The acceleration or braking \( a_{j,t} \) at time \( t \) is then obtained by a proportional controller on the current velocity \( v_{j,t} \):

\[
a_{j,t} = K_{\text{lon},j}(v_{\text{ref},j} - v_{j,t}) ,
\]

which is saturated at the acceleration / braking limits (5). The gain of this controller, \( K_{\text{lon},j} \), is also sampled and capture the aggressiveness of the acceleration or braking.

The steering profile of a TV \( j \) is obtained in two steps. First, a random variable decides whether the TV intends to stay in its current lane or attempts a lane change to the left or the right. The centerline of the target lane is the reference for the lateral position, \( y_{\text{ref},j} \). Second, the reference is tracked by a steering angle \( \delta_{j,t} \) at time \( t \) computed via proportional control on the current lateral position \( y_{j,t} \):

\[
\delta_{j,t} = K_{\text{lat},j}(y_{\text{ref},j} - y_{j,t}) ,
\]

which is saturated according to steering constraints (3), (4) and lateral acceleration constraints (7). The gain \( K_{\text{lat},j} \) is again sampled to capture the aggressiveness of lane change maneuver.

**III. SCENARIO MODEL PREDICTIVE CONTROL**

**A. Vehicle Model**

The SCMPC uses a kinematic bicycle model [20, Sec. 2.2] for the controlled vehicle (CV). The model has been shown to yield a good trade-off between model complexity and prediction accuracy for controller design [8], [21].

The kinematic bicycle model is illustrated in Fig. 1. The center of gravity (CoG) follows a set of nonlinear ordinary differential equations:

\[
\dot{x} = v \cos(\psi + \beta) ,
\]

\[
\dot{y} = v \sin(\psi + \beta) ,
\]

\[
\dot{\psi} = \frac{v \cos(\beta)}{L_f + L_r} \tan(\gamma) ,
\]

\[
\dot{v} = a ,
\]
where the dot indicates a derivative with respect to time and
\[
\beta = \tan^{-1}\left(\frac{l_f}{l_f + l_r} \tan(\gamma)\right).
\]
Here \((\xi, \eta), \psi, \) and \(v\) represent the vehicle’s coordinates, orientation, and speed, respectively. The parameters \(l_f\) and \(l_r\) denote the lengths from the vehicle’s center of gravity (CoG) to the front and rear axes, respectively. The steering angle of the front wheel \(\gamma\) and the acceleration \(a\) are the control inputs to the system.

For the integration of the kinematic bicycle model into SC MPC, the differential equations in (1) are discretized using a forward Euler method. The state at time step \(t = 0, 1, 2, \ldots\) is denoted \(x_t := [\xi_t \ \eta_t \ \psi_t \ v_t]^T\) and the input \(u_t := [\gamma_t \ a_t]^T\).

The following vector-based notation is used for the resulting discrete-time nonlinear dynamics:
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x_{t+1} = f(x_t, u_t).
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In highway traffic, all drivers are required to maintain a safety distance to the vehicle driving in their front [22, pp.56 ff.] [23, §4(1)]. This safety distance must be large enough to allow drivers ample reaction time in case of braking maneuvers or unexpected lane changes.

**Assumption 4 (Safety Distances)** (a) If CV stays on its current lane, it is only responsible to maintain a safety distance \(d_{CV}\) to its front vehicle. (b) In an overtaking situation, the CV must respect also the safety distance of the target vehicle \(d_{TV}\) that drives behind it on the target lane. (c) These safety distances may occasionally be violated (e.g., if the TVs perform emergency braking maneuvers).

In a particular scenario \(\delta_t^{(k)}\), the safety distances translate directly into constraints on the longitudinal and lateral position of the CV. As illustrated in Fig. 2, the current lane (cl) and target lane (tl) are considered separately under each scenario. The safety distances on lane \(l \in \{cl, tl\}\) under scenario \(k \in \{1, \ldots, K\}\) are based on the trajectories of all TVs, as given by \(\delta_t^{(k)}\).

TVs in the back require the CV to maintain a safety distance \(d_{TV}\), depending on the corresponding TV’s speed. TV’s in the front require the CV to maintain a safety distance \(d_{CV}\) that varies with the CV’s speed. Hence the longitudinal position of the CV (\(\xi\) coordinate) is constrained by a safety interval \([\xi_{min,t}^{(k)}, \xi_{max,t}^{(k)}]\), as shown in Fig. 2. The lateral position of the CV (\(\eta\) coordinate) is constrained by the boundaries of the current lane (\(l = cl\)) and/or the target lane (\(l = tl\)), \([\eta_{min,t}^{(k)}, \eta_{max,t}^{(k)}]\).

**D. Lane Change Timing**

The SC MPC algorithm has a prediction horizon of \(N\) time steps, i.e., a total preview time of \(N \cdot t_s\). It consists of an outer loop and an inner loop. The outer loop iterates to find the earliest lane change time \(t_{bc}\) within the interval \([0, N \cdot t_s]\).

**Assumption 5 (Lane Change Constraints)** (a) When integrated into the current (or target) lane, CV must observe the...
The resulting longitudinal and lateral state constraints are

\[
x_{\min, t} \leq x_t \leq x_{\max, t},
\]

where \(x_{\min, t} := [\xi_{\min, t} \eta_{\min, t} - \infty 0]^T\) and \(x_{\max, t} := [\xi_{\max, t} \eta_{\max, t} + \infty v_{\max}]^T\), and \(v_{\max}\) denotes the speed limit.

Given the sampled values of all scenarios \(\delta_t^{(1)}, \ldots, \delta_t^{(K)}\), the safety intervals for each lane and hence the state constraints (13) can be readily computed for each time step \(t\).

\section*{F. Multi-Stage Scenario Program}

For the inner loop of SCMC, the lane change time \(t_{lc}\) is fixed. The control inputs \(u_t, \ldots, u_{t+N-1}\) over the prediction horizon are then computed by solving the following Multi-Stage Scenario Program (MSSP):

\[
\min_{u_T \in \mathcal{F}(x_t)} \sum_{\tau=1}^{T} (x_{\tau+1} - x_{ref, \tau+1})^T Q(x_{\tau+1} - x_{ref, \tau+1}) + u_T^T R u_T, \tag{14a}
\]

s.t. \(x_{\tau+1} = f(x_{\tau}, u_{\tau}) \quad \forall \tau \in T\), \(u_{\min}(x_{\tau}) \leq u_{\tau} \leq u_{\max}(x_{\tau}) \quad \forall \tau \in T\), \(u_{\min} \leq u_{\tau+1} - u_{\tau} \leq u_{\max} \quad \forall \tau \in T\), \(x_{\min, \tau+1} \leq x_{\tau+1} \leq x_{\max, \tau+1} \quad \forall \tau \in T\). \tag{14b}

Here \(T := \{t, \ldots, t+N-1\}\), and \(Q \in \mathbb{R}^{4x4}\), \(R \in \mathbb{R}^{2x2}\) are positive definite weighting matrices for tuning. The reference trajectory \(x_{ref, t+1}, \ldots, x_{ref, t+N}\) for the CV is constructed according to the reference speed and the lane change time \(t_{lc}\). Since the SCMC chooses the final trajectory in a safe and dynamically feasible manner, the exact shape of the reference is secondary. Here it is chosen as a simple step change in the lateral coordinate.

The MSSP is a nonlinear program, which can be solved by standard algorithms [24], [25]. The objective function (14a) is a convex quadratic function that penalizes the deviation of the state from the reference as well as the input usage. The dynamic constraints (14b) are nonlinear. The input constraints (14c,d) can be linearized by substituting the initial state \(x_{\tau} = x_t\). The state constraints (14e) are implemented as soft constraints, to ensure feasibility of the program.

\section*{IV. BOUND ON SAFETY DISTANCE VIOLATIONS}

So far, no indication has been made for the selection of the sample size \(K\). This section uses the framework of \textit{scenario-based optimization} [12]–[14] to establish a connection between \(K\) and the satisfaction of chance constraints, in the sense defined below [11].

\textbf{Definition 6 (Chance Constraints)} The safety distance of each CV and TV may be violated in no more than a fraction \(\varepsilon \in (0, 1)\) of all time steps \(t = 0, 1, \ldots\).

Here \(\varepsilon \in (0, 1)\) is a tuning parameter that represents the conservatism of the driving style. A small \(\varepsilon\) corresponds to a low-risk driving style, a large \(\varepsilon\) makes the controller perform more aggressive lane change maneuvers. To derive the theoretical results, the following technical assumption is required [11].
Assumption 7 (Feasibility) (a) If no lane change is performed, there always exists a feasible solution to the MSSP. (b) The outer loop iterations are replaced by a fixed lane change time $t_{lc}$; e.g., the highest value in $T_{lc}$.

Let the lane $l \in \{l_1, l_2\}$ and time step $t \in \{0, 1, \ldots\}$ be fixed and consider the front safety distance $\tau \in \{1, \ldots, N\}$ steps into the future (the back safety distance works analogously). The violation of the front safety distance can occur only if $\xi_{\text{max}, t+\tau}$ has been chosen higher than

$$\hat{\xi}_{\text{max}, t+\tau}(\delta_t),$$

which is the unknown true longitudinal position for the CV under the unknown true scenario $\delta_t$. In other words,

$$P[V] \leq P[\hat{\xi}_{\text{max}, t+\tau}(\delta_t) < \xi_{\text{max}, t+\tau}]$$

where $P[.]$ denotes the probability measure and $V$ is the event where the front safety distance is violated. Therefore, to upper bound $P[V]$ by $\varepsilon$, it suffices to upper bound the right-hand side of (16). This can be accomplished by a basic sampling lemma [26].

Theorem 8 (Violation Bound) Let Assumptions 3, 4, 5, 7 hold and (14b) be an exact model of the controlled vehicle. Then

$$P[\hat{\xi}_{\text{max}, t+\tau}(\delta_t) < \xi_{\text{max}, t+\tau}] \leq \frac{1}{K+1},$$

and hence the sample size $K \geq \frac{1}{\varepsilon} - 1$ ensures satisfaction of the chance constraints of Definition 6.

Proof: All scenarios $\delta_t^{(1)}, \ldots, \delta_t^{(K)}$ are independent and identically distributed with $\delta_t$ (Assumption 3), so each one is equally likely to generate the tightest bound for the safety distance [26], and hence (17). State constraint satisfaction follows by Assumption 7 along the lines of [11].

The performance of the SCMP can be enhanced by extensions to the presented basic SCMP approach [11].

Remark 9 (Extensions) (a) The SCMP can include a control parameterization by any nonlinear disturbance feedback for performance improvements [11, Rem. 7]. (b) The SCMP can be extended by an algorithm for a-posteriori sample removal, to reduce the effect of sample outliers [11, Sec. 3.3].

V. SIMULATION RESULTS

This paper considers a highway case study as a proof of concept for the SCMP algorithm. In the considered traffic scene, the CV is driving on the right of two lanes, with two TVs (green, violet) in the front and two TVs (gray, blue) in the back. The exact positions and speeds of the TVs vary between different configurations. The speed limit on the highway is 22.4 meters per second ($\frac{v_0}{s}$). The TVs actually keep their lane and retain constant speed, yet this is not known to the controller, which must consider different scenarios for the TVs.

The simulations start at time $t = 0$, when the CV receives a lane change command. The prediction horizon is $N = 50$ steps for a sampling time of $t_s = 0.1$ s, giving the SCMP a total preview time of $N \cdot t_s = 5$ s. Eight candidate lane change times $T_{lc} = \{0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5\}$ seconds are considered. The earliest lane change time is preferred.

The reference speed $v_{\text{ref}}$ equals $22.4 \frac{m}{s}$. The lane change is commanded via a reference step in the lateral position $y_{\text{ref}}$. Small weights on the control inputs $a$ and $\delta$ are used to make the control actions smooth and energy efficient. The number of scenarios has been selected as $K = 19$, corresponding to a chance constraint level of $\varepsilon = 5\%$.

The safe trajectories for three different configurations of the traffic scene are shown in Fig. 3. A sample scenario for the TVs is also depicted and serves as a reference. The depicted scenario assumes constant speed and no lane changes for each of the TVs. Fig. 4 shows the corresponding speed profile of the safe trajectory in each of the three cases, as well as each of the TVs speed.

In case 1, the CV attempts to overtake the slower TV 1 (green) in its front. In order not to violate the front safety distance of TV 4 (blue), the CV needs to accelerate before merging into the faster lane. The SCMP finds that the earliest lane change time $t_{lc} = 1$ s, up to which the CV needs to accelerate on its own lane; see Fig. 3(a).

In case 2, the CV intends to merge into a slower lane with TV 2 (violet) limiting its speed. In order not to violate its own front safety distance, the CV needs to slow down before merging into the target lane. The SCMP finds that a lane change is possible immediately, $t_{lc} = 0$ s, and it adjusts to the speed of TV 2 thereafter; see Fig. 3(b).

In case 3, the CV attempts to merge into a faster lane, blocked by TV 2 (violet) and TV 4 (blue) going at higher speeds. The CV would need to accelerate before merging into the target lane. However, the SCMP concludes that this is not possible within the preview time of $5$ s and therefore remains in the original lane; see Fig. 3(c).

REFERENCES


(a) Case 1: Merge into a faster lane.

(b) Case 2: Merge into a slower lane.

(c) Case 3: Lane change is rejected.

Fig. 3: Computed safe trajectories by SCMPC for three configurations of the considered traffic scene. The controlled vehicle is shown in red and the target vehicles $j = 1, 2, 3, 4$ in green, violet, gray, blue. The future trajectories are indicated for some of the vehicles, with future positions shown in pale colors.

(a) Case 1: Merge into a faster lane. (b) Case 2: Merge into a slower lane. (c) Case 3: Lane change is rejected.

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Fig. 4: Velocity profiles corresponding to the safe trajectory of the controlled vehicle in red, and the target vehicles $j = 1, 2, 3, 4$ in green, violet, gray, blue. The dashed black line indicates the speed limit.


(22) California Department of Motor Vehicles, “California driver handbook,” California, United States, September 2014.


