Multihull and Surface-Effect Ship Configuration Design: A Framework for Powering Minimization

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1 Configuration Design of Multihull Ships

It is desirable to have readily usable interactive tools that allow designers to make rational decisions on the overall performance of the hull design, particularly, the powering prediction of multihulls. Speed is an important design consideration since it can reduce excursion time. However, this has to be attained at the expense of increased power consumption and exhaust pollution. In an energy-conscious world, power consumption should always be minimized. Furthermore, higher power consumption is almost synonymous with a generation of larger wake wash, which is considered to be “unfriendly” to an estuary environment.

For a given amount of power, higher speeds can be achieved by raising the hulls above water using foils or air cushions, or simpler still, by reconfiguring the original hull into a formation of smaller hulls. Other hybrid solutions involving a combination of air cushion, multihulls, and foils have also been proposed. In the San Francisco Bay area, local and state authorities have authorized a ten-year plan for a “quadruple expansion” of ferry routes, using new, high-speed ferries with a design involving a combination of catamaran and air cushion. A fast tetrahull has also been successfully developed by Lockheed Martin in Sunnyvale, CA, which was found to have excellent motion characteristics in moderate seas.

2 Multihull Wave Resistance Theory

Wave resistance of ship hulls is often the inhibiting component of resistance at higher Froude numbers. In a revisit of the theory of Michell [1], which had provided the classical expression for the wave-making resistance of a monohull based on the assumption that the beam-to-length ratio is small, Yeung et al. [2] (henceforth referred to as “YPT”) were able to obtain a generalized expression useful for analyzing the effects of multihull interferences. The assumption is that the fluid is inviscid and the flow is irrotational. An extensive collection of references on ship resistance in various contexts can be found in review articles such as Kostyukov [3], Wehausen [4], and more recently, Gotman [5] and Tuck et al. [6]. Michell’s theory was considered inadequate in the 1970s as few practical monohulls would satisfy the stringent “thin-ship” assumption and most design conditions were aimed at speed below the first resistance hump. Yet, at the resistance hump, despite significant sinkage and trim [7], Michell’s theory has been found to yield predictions that are rather effective as a “first-cut” evaluation in ship design. Furthermore, for a given displacement, a multihull ship system will invariably consist of thinner hulls, thus making the individual hulls more consistent with the thin-ship assumption. Surface-effect ships (SESs), in particular, have, in fact, thin side hulls.

The presence of multihulls generates cross-flow effects. When this lifting contribution is neglected, based on either a slenderness assumption or on an appropriate camber reshaping of the member hulls, the total resistance of the hull system can be simply represented by the totality of the wave systems in the far field. Among many works on this subject, one should recall Eggers’ [8] excellent treatise on two hulls in staggered formation; this study also included the consideration of the effects from finite depth and channel walls. The cross-flow effects can be modeled by a dipole distribution [9], which requires the solution of an integral equation in a formulation similar to the Neumann–Kelvin (NK) problem that was discussed by Yeung [10]. Some excellent treatments on the NK resistance problem can be found in Ref. [11]; the computational efforts are quite demanding. Besides the hull boundary condition, nonlinear free-surface effects can also be important. A quasilinear correction of the free-surface effects was offered by Mizine et al. [12]. When multiple numbers of hulls are present, the quadratic form of the resistance expression, such as Eq. (6), yields terms that account for the interaction between each paired combination of the hulls. Recent analysis of YPT showed that this interference resistance can be expressed in a strikingly simple integral, parallelizing Michell’s original expression for a single hull. The new
expression contains the explicit effects of stagger and separation and requires only the knowledge of the Fourier signature of each of the interacting hulls. The expression together with Michell’s integral can be concurrently computed. On a desktop server, thousands of combinations of geometric configurations and speeds would take only a few minutes, thus enabling a rapid evaluation in the parameter space and a quick search for an optimum in the early stage of configuration design of multihulls. This framework has been put on a UC Berkeley server for web-based access by the professionals who need to make design decision. Basic validations of this theory and its limitations have been addressed by Yeung et al. [2].

After providing a brief exposition of the theoretical development, we present one case of design application to illustrate this interactive tool. The extension of this theory to model a pressure cushion and ship-hull interaction, as will be needed to treat a SES hull, is then described, with an example for illustration of the application.

2.1 Interference Resistance of a Pair of Hulls. The general linearized steady ship-resistance problem can be summarized as finding the solution of Laplace’s equation subjected to boundary conditions on the body, at the bottom of the fluid domain, and at infinity, but most importantly, the following combined free-surface condition for potential \( \phi(x,y,z) \):

\[
 k_0 \phi(x,y,0) + \phi_d(x,y,0) = \frac{P(x,y)}{\rho U}, \quad k_0 = g/U^2
\]

where \( U \) is the forward speed in direction \( x \). Here, \( P(x,y) \) is an imposed pressure on the water surface, which is trivially zero except in a region under a cushion. The related linearized fluid pressure \( p \) and longitudinal free-surface slope are given by

\[
 -\frac{p(x,y,z) - P(x,y)}{\rho} = -U \phi_x + g \zeta
\]

\[
 \zeta(x,y) = \frac{U}{\rho} \phi_{xx}(x,y,0) - \frac{P}{\rho g}
\]

The Green function \( G \) satisfying the homogeneous condition of Eq. (1), with \( P=0 \), is the Havelock source [4], which is given by

\[
 G(x-\xi;y=\eta;z,\xi) = \frac{1}{\pi} \frac{1}{r_1 + \frac{4ik_0}{\pi} \int_0^{\pi} d\theta \sec^2 \theta \int_0^\infty dk - k_0 \sec^2 \theta} \times \cos[k(x-\xi)\cos \theta] \cos[k(y-\eta)\sin \theta]
\]

\[
 + 4k_0 \int_0^{\pi} d\theta \sec^2 \theta e^{ik_0(\xi+y)\sec^2 \theta} \sin[k(x-\xi)\sec \theta]
\]

\[
 \times \cos[k_0(y-\eta)\sin \theta \sec^2 \theta] = G_L + G_w
\]

Here, \( 1/r \) and \( 1/r_1 \) are the usual Rankine source and its image about \( z=0 \). We also note, specifically, that \( x \)-derivatives of the first three terms, designated by \( G_L \), are odd in \( x-\xi \), while that of the last term, designated by \( G_w \), is even. Michell’s solution [1] for a single hull is given by a distribution of the Havelock source over the centerplane profile \( S_1 \) and of strength proportional to the longitudinal hull slope \( f_{x1} \):

\[
 \phi(x,y,z) = \frac{U}{2\pi} \int_{S_1} dx_1 \frac{d\xi}{f_{x1}}(\xi,\zeta) G(x-\xi;y-0;z,\xi)
\]

The general problem of a configuration of \( n \) hulls can be thought of as a collection of all possible pairs of combination of

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2 We note in that paper the value of \( \psi = 64.8 \) on page 160 and Fig. 23 should have been stated as 129.8.

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hulls. Thus, the \( n \)-hull problem can be easily understood by knowing how to treat the simpler problem of just one pair of hulls. With reference to Fig. 1 again, we assume two hulls to be moving at constant speed \( U \) in the \( x \)-direction, each defined by the hull function \( y_j = f_j(x, z_j) \) in its own body coordinates. If these hulls were individually alone, Michell’s well-known result mentioned above can be written as follows:

\[
 R_{w+j} = \pi p U^2 \int_1^0 \frac{d\lambda}{\lambda^2 - 1} |A_j(\lambda)|^2 \quad (j = 1, 2)
\]

where \( A_j(\lambda) \), the wavemaking-antenna (or Kochin) function of the \( j \)th hull, is given by

\[
 A_j(\lambda) = \frac{2i}{\pi} k_0^2 \int \int dxdz f_j(x,z) e^{ik_0^2 \xi^2 \zeta^2} = \frac{2i}{\pi} k_0^2 \int \int dxdz f_j(x,z) e^{ik_0^2 \xi^2 \zeta^2}
\]

Here, if \( \lambda \) is written as see \( \theta, \theta \) will denote the orientation of the local crest line of the Kelvin wave system relative to the -axis (see e.g., Ref. [13]). The classical results of (6) and (7) are usually given in terms of hull slope \( f_{x1} \). An integration by parts in \( x \) had been conducted to obtain our results after assuming the hulls to have closed ends. Hull shapes with transom sterns would require some modifications of the present treatment.

Equation (6) states that the wave resistance is proportional to the beam squared, with all other variables kept constant. Thus, if we start with a base line hull of beam \( B \) and split it into two identical hulls aligned with the initial one, the two separate hulls will each have beam \( B’=B/2 \), and \( R_{w+2} \) would only be one-half of the base-line hull resistance, yet having the same displacement. This argument is definitely true if the two hulls are infinitely far apart (not a practical situation). In reality, the resistance of the two hulls with finite separation and stagger is given by

\[
 R_{w+1} = R_{w1} + R_{w2} + R_{w12} = R_{w1} + R_{w2} + R_{w12} + R_{w12} \quad (8)
\]

The interference resistance, denoted by \( R_{w12} \), sums the effect of Hull 2 on Hull 1 \( (R_{w12}) \) and the effect of Hull 1 on Hull 2 \( (R_{w12}) \). Analytically, these effects can be expressed as

\[
 R_{w12} = R_{w21} + R_{w12} = \int dxdz f_2 \int dxdz f_1 \int dxdz f_2 \int dxdz f_1
\]

The source function itself, given by (4), consists of a double and single integral. The inner double integral represents the linearized dynamic pressure (Eq. (2), \( P=0 \)) on the \( i \)th hull resulting from the
velocity potential induced by the source distribution on the neighboring (ith) hull, while the outer double integral of Eq. (9) integrates this pressure weighted by the longitudinal slope of the hull in question. After substituting the expressions of the Green function in each of the two terms, making appropriate trigonometric expansions, and changing variables to relate the coordinate systems, we found that $G_{Lx}$ and $G_{Lx2}$, being odd in $(x_1, \theta)$, cancel out in the sum of $R_{w1-2}$ and $R_{w1-2}$ (see Appendix of YPT). In the end, we arrived at a rather simple expression involving the $A_j(\lambda)\lambda$ of the interacting hulls:

$$R_{w1-2} = 2\pi pU^2 \int_{\lambda} \frac{d\lambda}{\lambda^2} \cos[k_{sp} \sqrt{\lambda^2 - 1}]$$

$$\times (\text{Re}(A_j A_{j2} \cos[k_{sp} \lambda]) - \text{Im}(A_j A_{j2} \sin[k_{sp} \lambda]))$$

with $\text{Re}$ and $\text{Im}$ denoting real and imaginary parts, respectively. Equation (10) explicitly shows how the stagger $st = (s_{j2} - s_{j})$ and lateral separation $sp$ between the two hulls can influence the total wave resistance. The more negative the interference is, the smaller the total wave resistance the pair of hulls has. $R_{w1-2}$ is independent of the sign of $st$ if the two hulls are identical. If $A_j \neq A_{j2}$, $R_{w1-2}$ does depend on the sign of $st$. The sign of $sp$ does not affect $R_{w1-2}$.

The above analysis can be quickly generalized to a family of $n$ hulls. The total wave resistance exerted on the whole system is given by

$$R_w \equiv \sum_{i=1}^{n} R_{wi} + \sum_{i=1}^{n} \sum_{j>i}^{n} R_{wij} = \sum_{i=1}^{n} R_{wi} + R_{intf}$$

This involves the interference resistance $R_{wij}$ of any pair of hulls $i$ and $j$. The indices $(1,2)$ in (10) need only be replaced by $(i,j)$, with $st = (s_{j2} - s_{j})$ and $sp$ is the lateral separation between hulls $i$ and $j$. For example, for a tetrahull ($n=4$), there are six interference resistances to be accumulated.

$$R_{intf} = R_{w1-2} + R_{w1-3} + R_{w1-4} + R_{w2-3} + R_{w2-4} + R_{w3-4}$$

with no restrictions on the form of each hull. One may obtain the interference effects among the component hulls by first superposing the amplitude functions of each component to obtain $R_{w1}$, with the location of each of coordinate origins properly reconciled, and then subtracting the sum of resistance contributions from each hull alone (the first term on the right-hand side (RHS) of (11)). Such a procedure does not reveal immediately how the interference resistances depend on the configuration and speed parameters. It will also be unclear, at least a priori, how the integration of the interacting pressure fields associated with the $G_L$ terms can be cancelled.

To account for the frictional resistance of the individual hull in an elementary way, the ITTC [14] formula can be applied. Thus, the total powering of the multihull system $P_T$ is given by

$$P_T = \sum_{i=1}^{n} C_f(R_{wi}) \frac{1}{2} \rho S_i U^3 + R_{wT} U$$

where $C_f$ is the frictional resistance coefficient that is Reynolds number based on the length of the ith hull $R_{wi}$, and $S_i$ is its wetted surface area.

### 2.2 Tetrahull Versus Catamaran (Dihull)

Computations of the integrals given by Eqs. (6), (7), and (10) were made by first developing a spline surface of each of the hulls. Then, for a given $\lambda$, the Kochin functions (7) were computed using Gaussian quadrature and Filon quadrature. The square-root singularity at $\lambda = 1$ in (6) is locally integrable and can be removed altogether with the change of variable $\lambda = \sec \theta$ (see, e.g., Eq. (19) of YPT). The final integration in $\lambda$ or $\theta$ uses either regular or adaptive numerical integration as appropriate. The methodology has been put on the computational marine mechanics laboratory (CMMML) website of UC Berkeley for access by interested users. The fast yet accurate computations enable a full definition of the resistance function of the multihull system in the configuration-parameter space quickly.

Figure 2 shows the definition for the MULTRES code in place at the CMMML site. This is the most versatile version of a suite of codes that can compute two-hulls (CARES), three-hulls (TRIRES), $n$-hulls (n5) for various purposes. MULTRES computes the wave drag of a system of a maximum of five hulls with the main hull at the global coordinate origin. Hulls 2 and 3 can be identical to their centerplanes symmetrically located about the $x$-axis. Their stagger can be a variable. Hulls 4 and 5 can be individually positioned at any specified separation or stagger relative to the main hull. Each one of these five hulls can be described by customized offsets, standard hulls such as Series 60, or “nulled” to be flow transparent. The last option will imply that the Kochin function of that hull is set to zero and the hull is not present.

As an illustrative example, a design team wants to evaluate the merits of a tetrahull (four-hull) system, relative to a dihull system of the same displacement. A tetrahull was successfully developed previously by Lockheed Martin Inc. (LM) (see Ref. [15]), which had excellent resistance and motion characteristics, among other favorable performance features. In this example, this team focuses on the issue of powering requirements of the two design alternatives using a basic, hypothetical hull form. The hypothetical form has a normalized shape that can be stretched along the axes to obtain the desired overall principal ratios of $L/B$ and $B/T$. In terms of a normalized geometrical function $f_j(x, z)$, $x = x/L_j$, $z = z/T_j = [-1, 1]$, the dimensional hull form of the $j$th hull can be expressed as

$$f_j(x, z) = \frac{R_j}{z} f_j(\bar{x}, \bar{z})$$

For simplicity, the hypothetical normalized form $f$ is taken as the symmetrical demihull form of Lin and Day [16], which was used in the study of Yeung et al. [2]. The demihull was primarily a thin vertical strut faired into a spheroidal pontoon, constituting one-half of a small-water plane-area twin-hull (SWATH) system.

For either the tetrahull or dihull system, the main hull in MULTRES (Fig. 2) can be nullled, and the remaining hulls are set to the proper dimensions as below. Each of the member hulls of the tetrahull system is taken to be identical to the others and each is symmetrically located about the centerplane of the overall system. The overall length of the tetrahull system $L_t$ is 25 m, from the bow of the fore hull to the stern of the aft hull. Individual hull properties are shown below, noting that each has a physical length of 10 m, with a longitudinal gap of 5 m between the front pair and the aft pair. We call this tetrahull SS Lin–Day.

### Numerical Data

- $L_2 = 10 m$, $L_4 = 10 m$
- $B_2 = 2.2 m$, $B_4 = 2.2 m$
- $T_2 = 3.0 m$, $T_4 = 3.0 m$
- $s_{p2,3} = 10.0 m$, $s_{p4,5} = 12.5 m$
- $s_{L2} = 0.0 m$, $s_{L4} = -15.0 m$
- $V_f = 117.462 m^3$, $S_f = 302.01 m^2$

The dihull SWATH ship has two identical demihulls, with the overall length $L_d = 20 m$, being the sum of the structural lengths of a fore and aft pair of tetrahulls. The draft is the same as the SS Lin–Day, but the beam is adjusted so that it has the same displacement as SS Lin–Day. Figure 3 shows the underwater isometric views of the tetrahull and the catamaran being considered. The principal particulars of the dihull are:
The results from MULTURES for the total wave-resistance coefficients for both hull systems are shown in Fig. 4 for comparison. The wave-resistance coefficients $C_{wT}$ and $C_{w-intf}$ are defined by

$$ C_{wT} = R_{wT} / \left( \frac{1}{2} \rho S_s U^2 \right) $$

$$ C_{w-intf} = R_{w-intf} / \left( \frac{1}{2} \rho S_q U^2 \right) $$

where $R_{w-intf}$ is interference resistance given by the second term of Eq. (11) and $S_q$ is the total surface area. It is evident from the $C_w$ plot that the role played by interference resistance is of considerable importance for the tetrahull. It has a significant hump at $F_{nt}=0.35$, based on $L_t$ of the SS Lin–Day. The “first hump” of the tetrahull now occurs at a lower Froude number than that of the same-displacement dihull at $F_{nt}=0.45$. Essentially, the shorter-length hull members bring this hump to a lower speed. The interference effects within the tetrahull system also bring the resistance coefficient rapidly down to a lower value around $F_{nt}=0.52$.

Fig. 2 Entry web page for the MULTURES (MULT-RES) code

$L_2.3 = 20$ m, $L_d = 20$ m
$B_{2.3} = 2.199$ m, $T_{2.3} = 3.0$ m
$sp_{2.3} = 10.0$ m, $st_{2.3} = 0.0$ m
$\forall_d = 117.462$ m$^3$, $S_d = 297.54$ m$^2$

Fig. 3 Isometric views of a tetrahull, the SS Lin–Day (left), and a catamaran (dihull, right) of the same displacement for a comparative powering study. The elemental geometry is a normalized form of that used by Lin and Day [16].
A plot of total resistance and power shown as thicker lines is shown in the lower figure. It reveals that there is a range of speed of 8.5–13 m/s, in which the tetrahull outperforms its dihull counterpart, by having 100 kW in power saving. This is eventually offset by an increase in frictional drag at higher speeds.

It is of interest to note that the demanding first hump in the resistance coefficient of this tetrahull SS Lin–Day is a surmountable hurdle since it occurs at a relatively lower speed.
tetrahull are comparable to those in the report of Schmidt [15], even though the LM underwater forms are more “podlike” while our hypothetical form is more SWATH-like.

Figure 4 can be generated in a matter of tens of seconds, offering a great analysis tool for the professional to make design decisions. MULTIHULLS has also been used in follow-on computations to evaluate the effects of separation and stagger at the design speed of 12 m/s, but in the interest of brevity, the results are not included here.

As is well known, powering requirement is only one of the several critical issues of multihull design that need to be addressed. Aspects related to seakeeping performance of multihulls can be found in, say, Ref. [17] and represent continuing areas of research. Structural and controllability considerations are also important in the overall design process and we are not treating these issues here. It is also noteworthy that while the foregoing tool provides a sound direction for configuration design, a more refined prediction using viscous-fluid codes such as that in Ref. [18] is often needed to estimate the complete effects of form drag.

3 Air-Cushion Vehicles and Surface-Effect Ships

A generalized SES can be modeled as a combined structure of catamaran hulls with a pressure cushion in between. To accurately assess the wave resistance generated in SES, the interference resistance between the pressure cushion and hulls should be taken into account. The analysis problem will consist of the prediction of the resistance of the combined hull and pressure-distribution components; the design decision may involve evaluating the fractional support by buoyancy as opposed to the pressure cushion, with the fraction being unknown a priori. A number of classical reference views and related references can be found in Refs. [4], [20], [21], and more recently, Ref. [6]. Our aim is to focus on the interference problem between hulls and the cushion.

3.1 Resistance Generated by a Pressure Field. For a pressure patch \( P(x,y) \) in planform \( S_p \), the velocity potential \( \phi_p \), per Eq. (1), can be given in \( G \) as

\[
\phi_p = -\frac{U}{4\pi\eta g} \int_{S_p} P(\xi,\eta)G(x-\xi,y-\eta,0)d\xi d\eta
\]

(16)

after taking advantage of an integration by part in \( \xi \). The wave resistance is still expressible by Eq. (6), but the amplitude function will be given by \( A_p(\lambda) \):

\[
A_p(\lambda) = \frac{k_0\lambda^4}{\pi^2 U^2} \int_{S_p} P(\xi,\eta)e^{i\lambda(\xi^2+\eta^2)-1}d\xi d\eta
\]

(17)

A pressure-cushion profile that is infinitely differentiable in the horizontal plane was given by [19]

\[
P(\bar{x},\bar{y}) = \frac{P_m}{4} \left[ \tanh \alpha(\bar{x}+1) - \tanh \alpha(\bar{x}-1) \right] \\
\times \left[ \tanh \beta(\bar{y}+1) - \tanh \beta(\bar{y}-1) \right]
\]

(18)

where \( \alpha \) and \( \beta \) are the tapering parameters in the longitudinal and transverse directions (Fig. 5) and \( P_m \) is the nominal pressure at the center of the air cushion. Also, \( \bar{x} = 2x/L_p \) and \( \bar{y} = 2y/B_p \) are the normalized \( x \) and \( y \) variables. This special, analytical form yields

\[
A_p(\lambda) = \frac{k_0\pi L_p}{4\pi^2} L_p \sin(k_0 L_p/2) B_p \sin(k_0 \lambda(\lambda^2-1)/4) \\
\times \alpha \sinh \left( \frac{L_p}{4\alpha} \right) \beta \sinh \left( \frac{\lambda^2-1}{4\beta} \right)
\]

(19)

The wave resistance generated by the pressure cushion based on Eqs. (6) and (17) is shown in Fig. 6 as a function of \( \eta^2 \) and beam-to-length ratios \( (B_p/L_p) \) of the patch. The nondimensionalized resistance coefficient is given by

\[
\frac{R_{wp}}{2P_m^2 B_p/\rho g} = \frac{R_{wp}}{2\Delta(h/L_p)}
\]

where \( \Delta \) is the displacement (or “lift”) due to the cushion, and \( h/L \) is the (hydrostatic) head of \( P_m \) to \( L_p \) ratio. The plot indicates the interesting observation: The wave drag to displacement ratio is proportional to the head-to-length ratio times a function that depends on \( B_p/L_p \) and \( F_n \). A wide cushion always yields higher resistance. The highly oscillatory behavior is related to the interference of the bow and stern waves of the cushion.

A comparison is made between the above air-cushion vehicle (ACV) results and a monohull of the same length and the same displacement. For convenience, we take a Series-60 hull of \( C_B = 0.597 \), with \( L = 50 \) m, \( B = 6.558 \) m, and \( T = 2.622 \) m, which provides a matching displacement to \( P_m L_p B_p \). From Fig. 7, we can see, as \( F_n > 0.4 \) \((U > 8.85 \) m/s), that the cushion generates smaller resistance than the monohull. In addition, the wider cushion gives smaller \( P_m \), and therefore the lower resistance. We have not accounted for skin friction and skirt seal drag in this model.

3.2 Interference Resistance Between an Air Cushion and a Single Hull. We can extend the analysis of interference drag of Sec. 2.2 to the case of a pressure cushion \( P \) and a single hull \( \eta = f_1(x_1,z_1) \) as defined in Fig. 8. Note that the global origin is taken at the center of the pressure patch. The potential is now the sum of Eqs. (5) and (16). To obtain the resistance on the cushion, the
product of $P(x,y)\xi$, is needed, whereas to obtain the resistance on the hull, the combination of $p(x_1,0,z_1)f_1$ needs to be evaluated. The “self-induced resistances” of each of these hull/cushion components in the system yield $R_{\text{PH}}$, Eq. (6), requiring $A_1$ for the hull (Eq. (7)) or $R_p$, requiring $A_p$ (Eq. (17)) for the cushion alone. The interference term $R_{\text{PH}}$ is not as obvious and is treated as follows:

$$R_{\text{PH}} = R_p + R_p$$  \hspace{1cm} (21)

where $R_p$ is the resistance on the hull due to the pressure distribution and $R_p$ is the resistance on the pressure cushion generated by the presence of the hull. They can be expressed as

$$R_p = \frac{U^2}{2\pi g} \int_{s_p} dx_1 dx_2 f_1 \int_{s_p} d\xi_1 d\eta_1 P(x_1, \eta_1, x_2, \eta_2) G_{x_1, x_2}(x_1 - \xi_1; y_1 - \eta_1, 0)$$

$$R_p = \frac{U^2}{2\pi g} \int_{s_p} dx_1 dx_2 f_1 \int_{s_p} d\xi_1 d\eta_1 P(x_1, \eta_1, x_2, \eta_2) G_{x_1, x_2}(x_1 - \xi_1; y_1 - \eta_1, 0)$$  \hspace{1cm} (22)

Here, for convenience, we have split the pressure Kochin function $A_p$ defined in (17) as $A_p^\lambda$:

$$A_p^\lambda(\lambda) = \frac{k \lambda^2}{2\pi U^2} \int_{s_p} P(\xi, \eta) e^{i k \lambda x_1} \cos k_0 \xi_1 \cos k_0 \eta_1 dx_1 \eta_1$$

$$A_p^\lambda(\lambda) = \frac{k \lambda^2}{2\pi U^2} \int_{s_p} P(\xi, \eta) e^{i k \lambda x_1} \cos k_0 \xi_1 \cos k_0 \eta_1 dx_1 \eta_1$$

Once we have (24), the total resistance of any dihull system with a pressure distribution, as in Fig. 9, is given by

$$R_{nT} = R_{n1} + R_{n2} + R_{n1-2} + R_{np} + R_{n1-2P} + R_{n2-2P}$$  \hspace{1cm} (26)

3.3 Cushion Support of a Surface-Effect Ship. We will illustrate the application of our theory by considering the problem of assessing the total wave resistance of a SES with a varying fraction of the support by a cushion (Fig. 9). For convenience, we take the base line demihulls to be Wigley hulls with $L_{1,2}=50$ m, $T_{1,2}=3.125$ m, and $B_1=6.25$ m. The hull function, in normalized form, is given by (14), with $f_1=(1-x^2)(1-z^2)$.

Our SES design arrangement consists of applying a cushion over a planform of area $L_pB_p$, with $L_p=0.8L_1$, and $B_p=8B_1$. Note here that $sp$, in the present definition, is the separation between the two demihulls and is fixed to be 0.75$L_1$ in the current study. Constancy of the combined total displacement of the pressure and hull system $\Delta_T$ can be achieved by varying $B_1$, thus $B_p$.
cushion-support system of the SES can perform very well for most of the practical Froude-number range. There is a narrow high-resistance region as $\Lambda > 0.7$ and $F_n < 0.3$ because $C_{wT}$ is dominated by a cushion-generated resistance that is locally high at this $F_n$.

A cut of this total-resistance surface can be made at $\Lambda = 0.4$ to show the resistance components of the SES (Fig. 12). This $\Lambda$ corresponds to the lowest drag at hump speed. Compared to the base line catamaran, the SES is seen to experience an appreciably smaller wave drag in the $F_n$ region greater than 0.35. The resistance at the hump speed of 10.6 m/s ($F_n = 0.48$) is reduced by about 14%. At $F_n = 0.3$ (6.64 m/s), the second hump, the wave resistance generated by the SES is very close to that of the base line hull because of the high cushion-induced resistance. This is unlikely to be a problem since most SESs are meant to be operated at a much higher $F_n$.

In Fig. 13, the powering requirements for the base line catamaran and SES are shown. We can see that the power consumption in the SES is smaller than that of the base line catamaran, especially in the high-speed region. At $F_n = 0.8$, wave-drag related powering can be saved by 48% by applying a cushion, and the marginal increase in power per unit speed is reduced by a factor of 7.

4 Conclusions

In this paper, we examined the wave resistance and powering issue of high-speed marine vehicles, including catamarans, tetrahulls, and cushion supported SES, using the notion of interference resistances. A comparison between a dihull and a tetrahull was made and we found the tetrahull having lower total resistance for a short speed range at $F_n \sim 0.55$, with this result being geometry specific. The interference wave resistance between a pressure cushion and a hull was studied, and a new closed-form formula was given to predict this resistance involving the Kochin functions of the air cushion and the hull. Applications of this theory can be made in the early design of a cushion-hull supported SES. Compared to the same-displacement catamaran, a SES design with a cushion and demihulls can have significantly lower wave resistance and power consumption in most of the practical range of Froude number. In our case study, the resistance at hump speed was reduced by 14% and the powering saving from a reduced wave drag can be as much as 48% at $F_n$ around 0.8. The computational method is expedient and the analytical framework is easy to comprehend. The seal drag of the SES was, however, not considered.
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Appendix: Derivation of Interference Resistance \( R_{PH} \)

The interference resistance between the pressure cushion and the hull, \( R_{PH} \), is derived briefly in this appendix. \( R_{PH} \) is the sum of \( R_{P1} \) and \( R_{P2} \), where \( R_{P1} \) is the resistance experienced by the hull due to the pressure field generated by the pressure cushion. \( R_{P1} \) is the resistance experienced by the cushion due to the wave elevation generated by the hull. We first rewrite Eqs. (22) and (23) as

\[
R_{P1} = \frac{U^2}{2\pi} \int_{S_1} \int_{S_2} d\xi_1 d\xi_2 f_1 G_1(x_1 - \xi_1; \eta_1; \eta_2) d\xi_1 d\xi_2 f_1 G_1(x_2; \eta_1; \eta_2) (4.1)
\]

and splitting \( R_{P} \) as \( R_{P} = R_{P1} + R_{P2} \) as defined in Eq. (25), we can write \( R_{PH} \) as the real and imaginary parts of \( A_0 A_1 + A_1 A_0 \) in the final form given by Eq. (24).

\[
R_{PH} = \frac{4k_0^2}{\pi} \int_{S_1} \int_{S_2} f_1(x_1, x_2) dx_1 dx_2 \int_{S_1} P(\xi, \eta) d\xi d\eta \times \int_{0}^{\frac{\pi}{2}} \cos[k_0(\rho - \eta) \sin \theta \sec^2 \theta] d\theta
\]

References