Chapter 1

(1.1) $a_1 = \frac{4\omega-2i}{2\omega}$, $a_2 = \frac{4\omega+2i}{2\omega}$

(1.3) $b_1 = 4$, $b_2 = \frac{2}{\omega}$

(1.5) $\frac{1}{4}\cos(3\omega t) + \frac{3}{4}\cos(\omega t)$

(1.7) $\frac{1}{8}(1-\cos(4\omega t))$

(1.9) $x(1.5) = 0.556$ m, $\dot{x}(1.5) = 6.50$ m/s,
$\ddot{x}(1.5) = -5.0$ m/s^2

(1.11) $\dot{x}(t)$ is maximal at $t = 0$.

(1.15) reduction by one half

(1.16) $k_{\text{eq}} = \frac{k}{n}$

(1.19) $x(t) = 5\sin(2t + \pi/4)$

(1.21) $\omega_n = \left[\frac{\frac{k_1 k_2}{k_1+k_2}+2k_3}{m}\right]^{\frac{1}{2}}$

(1.23) $k_{\text{eq}} = \frac{10}{11}k_1$

(1.25) $T = 180$ s, $\omega = 0.035$ rad/s, amplitude=20 m

(1.27) $k = 7.4 \times 10^6$ N/m

(1.29) (a) $\omega_n = 38.3$ rad/s, (b) $\omega_n = 31.8$ rad/s

(1.31) $\omega_n = 38.7$ rad/s - both cases

(1.33) $t^* = 0.479$ s

(1.35) $y(t) = -0.18\cos(8t)$ m

(1.37) $k = 10,000$ N/m

(1.40) $\omega_n = \sqrt{\frac{k_1 k_2 k_3}{m(k_1 k_2 + k_2 k_3 + k_3 k_1)}}$

(1.43) 50 percent

(1.45) $m' = \frac{m}{8}$

(1.47) $\frac{48EI}{l^3}$

(1.49) 31.3 rad/s

(1.51) $\omega_n = \sqrt{\frac{k}{m}}$

(1.53) $\omega_n = \sqrt{\frac{48EI}{l^3}}$

(1.55) $\omega_n = 2$ rad/s

(1.57) 3.59 rad/s

(1.59) 2.56 rad/s

(1.61) 3.05 rad/s

(1.63) $\omega_n = \sqrt{\frac{8g}{(9\pi-16)r}}$

(1.65) $\omega_n = \sqrt{\frac{g}{9r_1}}$, $T = 6\pi\sqrt{\frac{r_1}{g}}$

(1.67) $\omega_n = \sqrt{\frac{k_\theta - mgl\cos(\theta_{\text{eq}})}{ml^2}}$

(1.69) $\omega_n = \sqrt{\frac{3g}{h(1+6\frac{m_1}{m})}}$

(1.71) $\omega_n = \sqrt{\frac{g}{r}}$

(1.73) $\omega_n = 5.72$ rad/s

(1.77) $\omega_n = 14.3$ rad/s

(1.79) $\omega_n = 9.68$ rad/s

(1.81) $\omega_n = 8.38$ rad/s

(1.83) $k_\theta = 6.25$ N·m/rad

(1.85) $\omega_n = \sqrt{\frac{2g}{l}}$

(1.87) $\omega_n = \sqrt{\frac{4Tr^2}{I_0 l}}$

(1.91) Equally effective

(1.93) k affects ω_d and not $\zeta\omega_n$

(1.95) 2, 115

(1.97) $e^{-at} \to e^{-\frac{at}{2}}$

(1.99) $m\ddot{x} + (k_1 + k_2)x = 0$

(1.101) $m\ddot{x} + c\dot{x} + kx = 0$

(1.103) $(m_1 + m_2)\ddot{x} + l\ddot{\theta}m_2\cos(\theta)$
$-m_2 l\dot{\theta}^2\sin(\theta) + kx = 0$
$l\ddot{\theta} + \ddot{x}\cos(\theta) + g\sin(\theta) = 0$

(1.105) $m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 -$
$c_1\dot{x}_2 - k_1 x_2 = 0$
$m_2\ddot{x}_2 + (c_1 + c_2)\dot{x}_2 + (k_1 + k_2)x_2 -$
$c_1\dot{x}_1 - k_1 x_1 = 0$

(1.107) $\ddot{\theta} + (\frac{g}{l} + \frac{\ddot{y}}{l})\theta = 0$

(1.109) $\ddot{x} = 0$

(1.111) $m_1\ddot{x}_1 + k_1 x_1 - kx_2 = 0$
$m_2\ddot{x}_2 + kx_2 - kx_1 = 0$

(1.113) $m\ddot{y} + 2(k_1 + k_2)y + l(k_2 - \frac{k_1}{2})\theta = 0$
$\frac{ml^2}{12}\ddot{\theta} + \frac{l^2}{2}(k_2 + \frac{k_1}{4})\theta + l(k_2 - \frac{k_1}{2}) = 0$

Chapter 2

(2.1) c=50 N$\cdot$ s/m

(2.3) settling time not affected

(2.5) $v = 3.94$ m/s

(2.7) mass moves 2.66 percent of base motion

(2.9) $\Delta x = 0.05$ mm

(2.11) $\omega_n = 44.72$ rad/s

(2.13) springs 1 and 2 provide the best isolation

(2.16) $\omega_n = 1.814$ rad/s

(2.19) $|\ddot{y}| = 0.32g$

(2.21) $0 \leq \omega < 100.6$ rad/s

(2.23) if $a > h$ we have separation at all ω

(2.25) $\omega_n = 2$ rad/s

(2.27) $x(t) = a_1 \cos(\omega_n t) + a_2 \sin(\omega_n t) + \frac{\overline{f}}{2\omega_n} t \sin(\omega_n t)$

(2.29) $\omega < 61.6$ rad/s and $\omega > 64.8$ rad/s

(2.31) $f = \frac{l_3 k_1}{l_1} y$

(2.33) $\omega = \sqrt{\frac{a+x_0}{a}} \omega_n$

(2.35) $\omega_n^2 = \frac{k l_2^2}{m_1 l_1^2 + m_2 l_3^2}$

(2.37) $g(\omega) = \frac{-\omega^2}{k - m\omega^2}$

(2.38) $g(\omega) = -\omega^2$

(2.41) $\frac{\overline{f}_w}{\overline{y}} = \frac{k_1 k_2}{k_1 + k_2 - m\omega^2}$

(2.43) $\zeta = 0.06$, $c = 0.48$ N$\cdot$s/m

(2.45) $c = 636.6$ N$\cdot$s/m

(2.47) $x(t) = \sqrt{20}\mathrm{Re}[e^{i(3t+0.4636)}]$

(2.49) $x(t) = (\frac{1}{2} - \frac{1}{2i})e^{it} + (\frac{1}{2} + \frac{1}{2i})e^{-it}$

(2.51) $\frac{\overline{x}}{i\omega\overline{y}} = \frac{c}{i\omega c - \omega^2 m}$

(2.53) $g(\Omega) = \frac{\Omega^2}{1 - \Omega^2 + 2i\zeta\Omega}$

(2.57) $\dot{x}(t) = -36\sin(100t) + 3.9\cos(100t)$ mm/s

(2.59) $|g(\omega)| = \sqrt{\frac{1600 + 2\omega^2}{(50 - \omega^2)^2 + 2\omega^2}}$

(2.61) $y_2(t) = \frac{-5k_1 \sin(10t) - 50c\cos(10t)}{k_2}$

(2.63) $\phi = -1.56$ rad

(2.65) response magnitude = 0.216 m

(2.69) $\overline{x} = \frac{(l_2 m_2 - l_1 m_1)\omega^2}{-\omega^2(m_1 + m_2) + k_1 + k_2}$

(2.71) $(m_1 + m_2 + 10.8)\ddot{x} + kx = 0.8r\omega^2\cos(\omega t)$

(2.73) transmitted force magnitude = 99.1 N

(2.75) $|x_{\mathrm{peak}}| \approx 5$ cm

(2.79) $|g(0)| \approx 4$ mm

(2.81) error of about 6 percent

(2.83) error of about 13.7 percent

(2.85) neither is applicable

(2.87) $\zeta = 0.034$

(2.89) $\zeta = 0.52$

(2.91) $a = \frac{\sqrt{f^2 - (\frac{4}{\pi}\mu m g)^2}}{k - m\omega^2}$

(2.93) $a = 5.21$ mm

(2.95) $a = 6.83$ mm

(2.98) $\mu = 0.33$

(2.101) $\omega = 5860$ Hz

(2.103) $\omega = 87.6$ rad/s

Chapter 3

(3.2) 2 terms are required

(3.6) $a_i' = -a_i$ and $b_i' = -b_i$

(3.9) $x(t) = \sum_{n=1}^{\infty} \frac{20A}{(n\pi)^2}[2\sin(\frac{n\pi}{2}) - \sin(\frac{n\pi}{4}) - \sin(\frac{3n\pi}{4})] \times \frac{\sin(\frac{n\pi t}{4})}{2.56 - (\frac{n\pi}{4})^2}$

(3.13) $x_n = \frac{\frac{8a}{(n\pi)^2}\sin(\frac{n\pi}{2})}{4 - (\frac{2n\pi}{T})^2} \sin(\frac{2n\pi t}{T})$

(3.15) $x(2) = 4.43 \times 10^{-5}$ m

$\dot{x}(2) = -9.31 \times 10^{-4}$ m/s

(3.17) $h(t) = -y_0[\cos(\omega_n t) - 1]$

(3.19) $|\ddot{x}|_{\max} = 140.4$ m/s^2

(3.21) new acceleration = $9.8g$

(3.23) cutting k in half doubles safe drop height

(3.25) reducing mass increases $|\ddot{x}|_{\max}$

(3.27) ω_n must be reduced by 8.7 percent

Chapter 4

(4.1) $\omega_1 = 2.57 \times 10^3$ rad/s, $\omega_2 = 5.21 \times 10^2$ rad/s

(4.3) $x_1(t) = -0.01\cos(7.67t) + 4.905t^2$

$x_2(t) = 0.1666\cos(7.67t) + 4.905t^2$

(4.6) $\omega_1 = 1.9714$ rad/s, $\omega_2 = 5.2390$ rad/s

(4.8) $\omega_1 = 22.3$ rad/s, $\omega_2 = 44.9$ rad/s

(4.11) $\omega_1 = 0.6824$ rad/s, $\omega_2 = 2.2948$ rad/s

(4.13) $\omega_1 = 1.155$ rad/s, $\omega_2 = 1.414$ rad/s

(4.15) $\omega_1 = 0.9307$ rad/s, $\omega_2 = 1.2959$ rad/s

(4.19) $\omega_1 = \sqrt{2}$ and X_1 is an eigenvector

(4.21) $\omega_1 = 1.1756$ rad/s, $\omega_2 = 1.9021$ rad/s

(4.28) $\omega_1 = 169.3$ rad/s, $\omega_2 = 477.5$ rad/s

(4.31) $\omega_1 = 110.3$ rad/s, $\omega_2 = 405.6$ rad/s

(4.33) $\omega_1^2 = 4$, $\omega_2^2 = 9$, $k = \frac{22.5}{32}$

(4.35) $\omega_1 = 2.2913$ rad/s, $\omega_2 = 2.3979$ rad/s

(4.39) $\omega_1 = 0.3912$ rad/s, $\omega_2 = 2.0000$ rad/s,

$\omega_3 = 2.0608$ rad/s

(4.39) $\omega_1 = 6.82$ rad/s, $\omega_2 = 13.2$ rad/s

(4.45) 1.2 percent error

(4.47) $v = 14$ mph

(4.49) $\lambda_n = \frac{4}{n}$ m, $n = 1, 2, 3 \cdots$

(4.55) $x - y = 0.0692\sin(18.08t) - 0.0060\cos(18.08t) -$

$0.0282\sin(44.47t) - 0.0010\cos(44.47t)$

(4.57) (e)

(4.59) $k_3 = 236,000$ N/m

(4.61) $m_2 = 1.95$ kg, $k_2 = 70.2$ N/m

(4.63) for both cases, vibration amplitude = 2.5 mm

(4.65) total excursion = 8.53 cm

(4.67) $\omega = \sqrt{\frac{k_1+k_2}{m_1}}$

(4.69) no frequencies allow $x_2 = x_4 = 0$

(4.71) $\frac{k_4+k_5}{m_3} = 900$

(4.75) $3.6403\ddot{\eta}_1 + 77.52\eta_1 = 0$

$1.0807\ddot{\eta}_2 + 355.24\eta_2 = 2\cos(10t)$

(4.77) $\tilde{X}_1^T = \frac{1}{\sqrt{33}}[1\ 2\ 1\ 2]$

$\tilde{X}_2^T = \frac{1}{\sqrt{107}}[2\ 1\ -1\ 3]$

(4.79) $\tilde{X}_3^T = [1\ -\sqrt{2}\ 1]$

(4.83) $\ddot{\eta}_1 + 22.9\eta_1 = -0.6613$

$\ddot{\eta}_2 + 168.6\eta_2 = 0.6085$

$\ddot{\eta}_3 + 2881\eta_3 = 5.3798$

(4.85) $\ddot{\eta}_1 + 1.2164\eta_1 = 0$

$\ddot{\eta}_2 + 2.0208\eta_2 = 0$

$\ddot{\eta}_3 + 3.0853\eta_3 = 0$

(4.87) magnitude of modal response: $\frac{5}{3\sqrt{2}}$

(4.89) $-0.0823 \pm 1.13i$, $-0.118 \pm 1.64i$,

$-0.163 \pm 2.12i$

(4.92) $X_1^T = [-0.3723\ 0.1434]$,

$X_2^T = [-0.0247\ 0.0428]$

$X = X_1\sin(5t) + X_2\cos(5t)$

(4.97) $\omega_1 = 0$, $\omega_2 = 0.8165$ rad/s

(4.99) X_i^T isn't allowable

(4.103) $x_1 = 4.3$ cm, $x_2 = 2.7$ cm, $x_3 = 1.9$ cm

(4.106) $f_1 = 102$ N, $f_2 = -56$ N, $f_3 = 2$ N

(4.108) $f_1 = -80$ N, $f_2 = 115$ N, $f_3 = -11$ N

Chapter 5

(5.1) $k\overline{y}(l) + T\overline{y}_x(l) = 0$

(5.6) ω_n goes from $\beta_n\sqrt{\frac{T}{\rho_0}}$ to $\frac{\beta_n}{\sqrt{2}}\sqrt{\frac{T}{\rho_0}}$

(5.8) ω_n goes from $\beta_n\sqrt{\frac{T}{\rho_0}}$ to $1.049\beta_n\sqrt{\frac{T}{\rho_0}}$

(5.10) elastic modes: $\omega_n = \frac{n\pi}{l}\sqrt{\frac{T}{\rho}}$, $\overline{w}(x) = \cos(\frac{n\pi x}{l})$

(5.12) $\tan(\beta l) = -\frac{T\beta l}{kl}$

(5.15) $\tan(\beta l) = \frac{m}{m_1\beta}$

(5.18) $\beta_n = \frac{n\pi}{l}$

(5.21) $\xi(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{(2n-1)\pi x}{2l})\cos(\sqrt{\frac{EA}{m}}\frac{(2n-1)\pi t}{2l})$

$a_1 = 0.8606$, $a_2 = -0.0244$, $a_3 = 0.0825$

(5.25) $\xi(x,t) = \frac{bl}{3\pi}\sqrt{\frac{m}{EA}}\sin\left(\frac{3\pi x}{l}\right)\sin\left(\sqrt{\frac{EA}{m}}\frac{3\pi t}{l}\right)$

(5.27) $T = 6970$ N

(5.31) $\omega_1 = 3183$ rad/s, $\omega_2 = 9549$ rad/s,

$\omega_3 = 15,915$ rad/s

(5.33) $\omega_1 = 4.84$ rad/s, $\omega_2 = 30,921$ rad/s

(5.35) The first natural frequency in torsion is lower than that for longitudinal vibrations.

(5.38) No difference.

(5.41) $k = 69.5$ N/m

(5.43) No

(5.45) $\beta_1 = \frac{\pi}{2}$, $\beta_2 = 1.602$

(5.49) doubling of frequency

(5.51) $\cos(\beta l)\cosh(\beta l) = 1$

(5.53) $\omega_n = \sqrt{\frac{EI\beta_n^4}{\rho} + \frac{k}{\rho}}$

(5.56) $X_{xxxx} - \frac{T}{EI}X_{xx} - \frac{\omega^2\rho}{EI}X = 0$

(5.58) $X(x) = 0$

(5.61) $y(x,t) = \sum_{m=1}^{\infty}\overline{a}_m\cos(\omega t)$

$\overline{a}_m = \frac{2\overline{f}}{\rho l}\frac{\sin(m\pi x_0)}{\left(\frac{T(m\pi)^2}{\rho} - \omega^2\right)}$

(5.63) $w(x,t) = \frac{3\sin(\pi x)\sin(5t)}{\left(\frac{EI\pi^4}{\rho} - 25\right)} - \frac{2\sin(2\pi x)\sin(6t)}{\left(\frac{16EI\pi^4}{\rho} - 36\right)}$

(5.65) $w = \frac{\overline{f}}{2\rho}t^2$

Chapter 6

(6.5) 15 % contamination induces 5 % frequency shift

(6.7) $m_3 = 8$ kg

(6.9) 3 % error

(6.11) 3 errors

(6.13) problem is ill-posed

(6.15) no

(6.17) yes

(6.19) Estimates are too high - analysis is flawed.

(6.23) $\omega_1 = 2.806$ rad/s, $\omega_2 = 5.672$ rad/s

(6.27) $\omega_1 = 88.69$ rad/s, $\omega_2 = 179.2$ rad/s

(6.29) $\omega_1 = 2.000$ rad/s

Chapter 7

(7.3) phase lag is $90°$

(7.5) velocity response - direct force excitation

(7.7) $\omega_1 = 0.187$ rad/s, $\omega_2 = 3.49$ rad/s

(7.9) $\omega_1 = 4.47$ rad/s, $\omega_2 = 3.16$ rad/s

(7.13) fixed approximation is reasonable

(7.17) $\epsilon = 0.14$

(7.19) exact: $\omega_1 = 30.03$ rad/s, $\omega_2 = 70.34$ rad/s

Chapter 8

(8.1) $\frac{\sum x_i}{n} = 0$

(8.3) $\frac{\sum x_i}{n} = 0.0385$

(8.6) 14.5

(8.8) $FT(x) = \frac{1}{\pi\omega^2}\left[2e^{-2i\omega} - e^{-4i\omega} - 1\right]$

(8.10) $FT(y) = e^{-i\omega\tau}FT(x)$

(8.12) time delay t_0 causes phase shift in Fourier Transform of ωt_0

(8.15) $x(t) = 1.502e^{-0.1t}\sin(1.9975t)$

(8.17) $R_{xx}(\tau) = \frac{2S_0}{\tau}(\sin(\omega_2\tau) - \sin(\omega_1 t))$

(8.19) $E[x^2] = 0.0179$

(8.21) $R_{xx}(0) = 0.0179$

(8.26) $\overline{x^2} = 0.785S_0$