# Information and Control

## The Interconnection view

Sanjoy K. Mitter

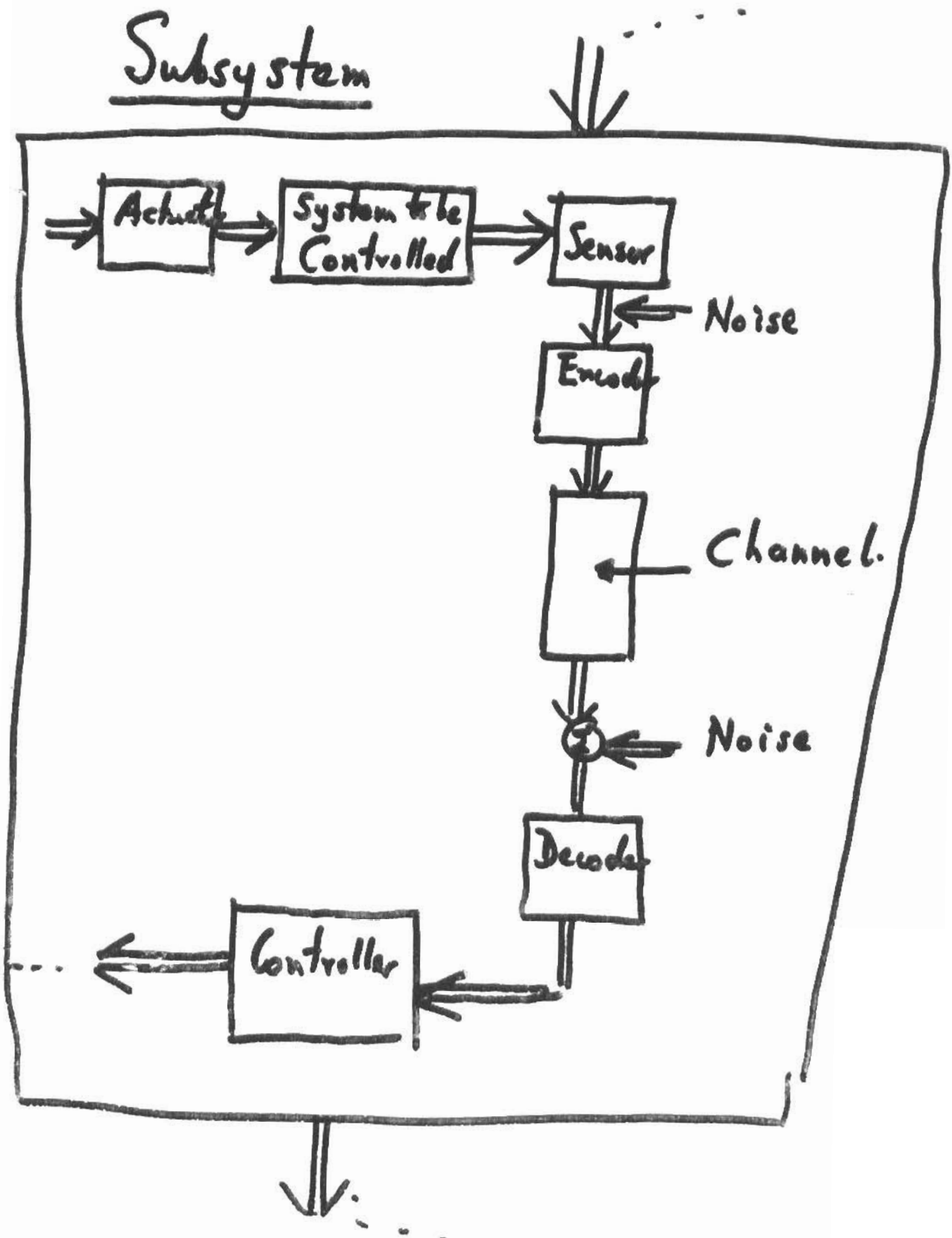
M.I.T.

Joint work with : V. Borkar, N. Elia, N. Newton, A. Sahai, S. Tatikonda.

# INTRODUCTION

- Systems, consisting of Subsystems linked through a Communication Links, where each subsystem is an interconnection of system to be controlled, sensor, controller, actuator, Themselves linked through a Communicatem Channel.

- Special Case :
  Dynamic Network of Sensors used to solve a Distributed Inference Problem


Subsystem
Actuator
System to be Controlled
Sensor
Noise
Encoder
Channel
Noise
Decoder
Controller

# Some Intellectual Unification between Information & Communication Theory Theories of Control Theories of Computation needed.

- Interconnections of Systems (Interaction) (Gluing)
- Obstructions
- Control through Information Transfer & Vice Versa

Anniversary

# Robin Milner's Turing Lecture (1997).

- Computing ..... Science of Interactive Systems
- Logic of Interaction

Software from being a prescription for how to do something – in Turing's terms a "list of <u>behaviours</u>" becomes much more akin to a <u>description</u> of behaviour, not only programmed on a computer, but occurring by luck or design <u>inside</u> or <u>outside</u>

| New Computing | vs. | Old Computing |
|---|---|---|
| Prescription | | Description |
| Hierarchical | | Heterarchical |
| Design | | Phenomena |
| Determinism | | Non-Determinism |
| End Result (Extension) | | Continuing Interaction (Intension) |

# Fundamental Entities of Interaction

Active Object (Agent)

Process

Parallel Composition

Message (as opposed to Operation on Datum) : Pass between Active peers.

# Logical Elements of Interaction

Synchronised Action

Channel or Vocative name

Algorithms = Game Theoretic Interactions

# Interactions : Statistical Physics

Given Hamiltonian (free)

$H(\omega)$. : Energy

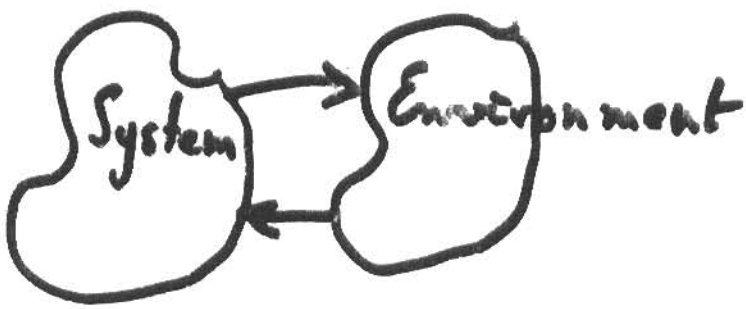
$$d\mu = \exp\left(-H(\omega)\right) d\mu_r$$

Perturbed Hamiltonian

$$- H(\omega) + \kappa V(\omega).$$

Couple Two Systems

# Systems Interacting with Environment (Reservoir, Heat Bath)

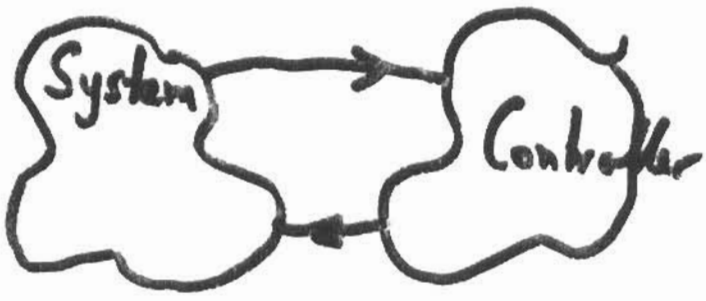



Energy Flow

Information Flow

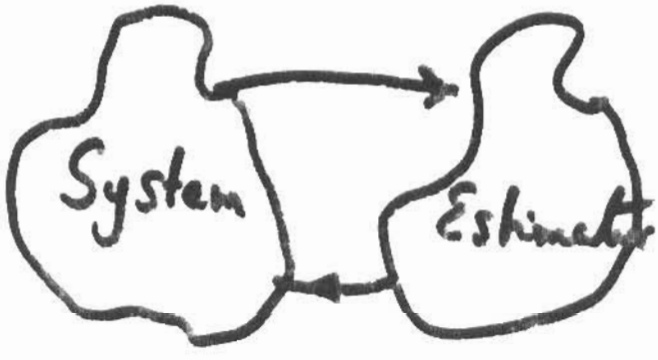
Non-equilibrium Statistical Mechanics

# Control as Interconnection




Optimal Energy Flow

Optimal <u>Information Flow</u>




Bayesian Inference : Information Theoretic View

# Control as Interconnection (Willems)

To be


$$\mathcal{P}_{full} = \left\{ (w,c) \in C^{\infty}(\mathbb{R}, \mathbb{R}^{w+c}) \;\middle|\; \begin{array}{l}(w,c) \text{ satisfies} \\ \text{Plant eqns.}\end{array} \right\}$$

$$\mathcal{P} = \left\{ w \in C^{\infty}(\mathbb{R}, \mathbb{R}^{w}) \;\middle|\; \exists\, c \text{ s.t } (w,c) \in \mathcal{P}_{full} \right\}$$

Elimination

$$\mathcal{C} = \left\{ c \in C^{\infty}(\mathbb{R}, \mathbb{R}^{c}) \;\middle|\; \begin{array}{l}c \text{ satisfies} \\ \text{Controller eqns.}\end{array} \right\}$$

## Interconnection

$$K_{full} = \left\{ (w,c) \in C^{\infty}(\mathbb{R}; \mathbb{R}^{w+c}) \;\middle|\; (w,c) \in \mathcal{P}_{full} \right\}$$
$$c \in C \}.$$

$$K = \{ w \mid \exists c \text{ s.t. } (w,c) \in K_{full} \}$$
(Manifest).

$K$ implements $C$

$K \subseteq \mathcal{P}$

## Regular Interconnection

Restriction on only the controllable part of the behaviour.

# Probabistic Interconnection

## Interconnection of Stochastic Kernels

$$x \longmapsto P(x; B)$$

to get desired Joint Distribution.

Willems

$$\Sigma = (T, W, \mathcal{B}) \qquad \mathcal{B} \subseteq W^T$$

set of trajectories

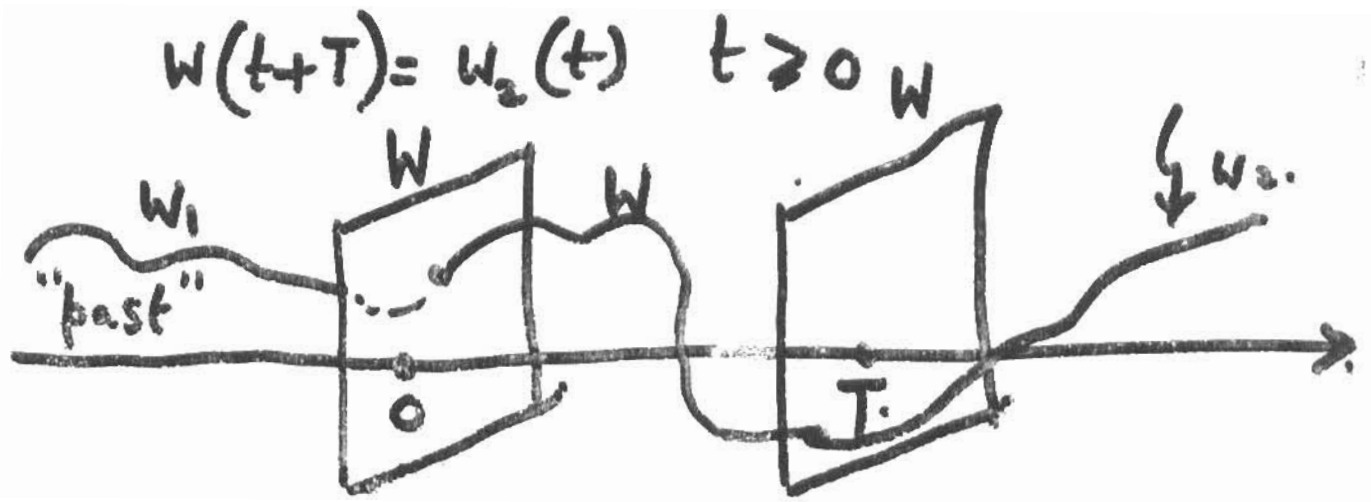
Probabistic: Family of measures on Paths.

Controllability of a Behaviour: Switch from one trajectory to another using a trajectory in the behaviour, provided we allow a delay.

$\Sigma = (\mathbb{R}, W, \mathcal{B})$ : shift invariant controllable if $\forall\ w_1, w_2 \in \mathcal{B}$, $\exists\ T \geq 0$ and $w \in \mathcal{B}$ s.t.

$$w(t) = w_1(t) \quad t < 0$$

$$w(t+T) = w_2(t) \quad t \geq 0$$



Information Flow
vs. Energy Flow
Relations to Statistical Mechanics
(Equilibrium & Non-Equilibrium)

Bayesian Inference & Filters

Information Theory: What needs to be changed?

Control in Information Terms
(Recent work of N. Elia)