

Assignment Nr. 9

due April 29th

Problem 1

Show that all geodesic curves on \mathcal{S}^2 are great circles, namely intersections of \mathcal{S}^2 with a plane that passes through the center of \mathcal{S}^2 .

Problem 2

Find the shortest distance between San Francisco International Airport (latitude $37^\circ 37'$ North, longitude $122^\circ 22'$ West) and Tokyo Narita International Airport (latitude $35^\circ 45'$ North, longitude $140^\circ 23'$ East). Assume that the earth is a sphere of radius $R = 6,371$ km.

Problem 3

Repeat Problem 3 using a more accurate geodesic model, which assumes that the earth is a Clarke-1866 ellipsoid with equatorial radius $R_e = 6,378$ km and polar radius $R_p = 6,357$ km. Note that you will need to solve the geodesic differential equation using a numerical method.

Problem 4

- Calculate the components of the metric tensor on a cylinder of unit radius parametrized using a cylindrical polar coordinate system (θ, z) , and evaluate the first fundamental form for a vector \mathbf{v} with coordinates (v^1, v^2) .
- Calculate the components of the metric tensor on a unit sphere parametrized using a spherical polar coordinate system (θ, ϕ) , and evaluate the first fundamental form for a vector \mathbf{v} with coordinates (v^1, v^2) .
- Comment on the difference between the two fundamental forms in parts (a) and (b) in relation to the surfaces to which they apply.

Problem 5

Calculate the curvature tensor \mathbf{R} , the Ricci tensor $\boldsymbol{\rho}$, and the scalar curvature κ for a sphere of radius r using spherical polar coordinates.

Problem 6

Let $\hat{g} : \mathbb{R} \mapsto \mathbb{R}^2$ be a smooth curve of the form $\hat{g}(s) = (s, y(s))$, such $y(s) > 0$ for all $s \in \mathbb{R}$. A 2-manifold \mathcal{M} embedded in \mathbb{R}^3 is obtained by rotating \hat{g} around the s -axis, so that its parametric equation takes the form

$$\phi(s, \theta) = (s, y(s) \cos \theta, y(s) \sin \theta) ,$$

where $0 \leq \theta < 2\pi$.

(a) Establish that the principal curvatures of \mathcal{M} are

$$\kappa_1(s) = -\frac{y''(s)}{(1 + (y'(s))^2)^{3/2}} \quad , \quad \kappa_2(s) = \frac{1}{y(s)(1 + (y'(s))^2)^{1/2}} .$$

(b) Find an algebraic expression for the special manifold \mathcal{M} belonging to the above defined class of 2-surfaces of revolution, in which the mean curvature is zero.