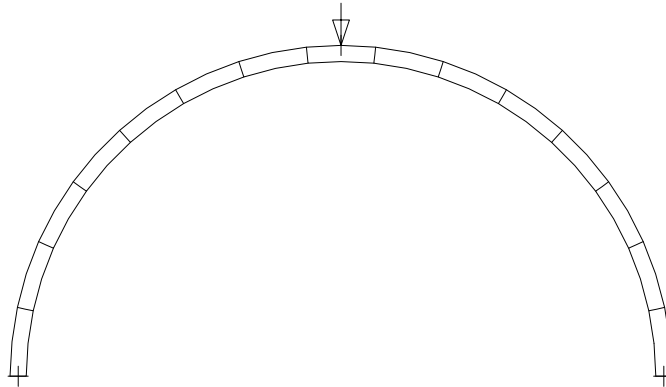


Assignment Nr. 6

due May 4th

Problem 1

Consider a semi-circular arch made of a non-linear elastic material, as in the figure below. The arch is simply supported at its two end points and is subject to a point load applied at its top. This problem exhibits a load-displacement curve with an initially ascending portion as the arch flattens, followed by a descending portion as the arch curves inversely, and, finally, an ascending portion, as the inverted arch is pulled downwards by the load. The FEAP input file `Iarch` contains a finite element mesh of 9-node quadrilaterals to be used for this snap-through simulation in plane strain under displacement control.



Conduct the FEAP analysis and plot the load-displacement curve. Modify the input file as necessary in order to accurately resolve the continuation past the two stability points.

Problem 2

FEAP can be used to estimate the buckling load of a cantilever beam due to uniform compressive traction load on its free end. The simplest way to estimate the buckling load is to determine the smallest applied tractions at which the global stiffness matrix of the beam becomes singular. This can be done, say, by a bisection method, where the analyst monitors the changes of sign in the diagonal of the factorized stiffness matrix, as reported in FEAP.

- (i) Estimate the buckling load for the cantilever beam of length $l = 10$ and height $h = 1$, whose FEAP model employing 9-node elements is contained in the input file `Icant` and whose reference configuration is plotted below. The material constants in this analysis are such that in the infinitesimal limit the beam has Young's modulus $E = 1000$ and Poisson's ratio $\nu = 0.3$. Also, the beam is assumed to be in plane strain. You should bracket the buckling load using the bisection method to within "relative" error of 1%.
- (ii) Perform an eigenvalue analysis to obtain the buckling mode. To this end, use the commands `iden` and `subs` to run a subspace iteration and find the mode associated with the (nearly) zero eigenvalue of the stiffness matrix. Plot the buckling mode using the `eigv` command (make sure to first appropriately scale the plot).
- (iii) Compare the buckling load obtained in (i) with the Euler buckling formula of elementary beam theory, according to which

$$P_{cr} = \frac{\pi^2 EI}{4(1 - \nu^2)l^2},$$

for a cantilever beam of depth b , where $I = \frac{bh^3}{12}$.

- (iv) Repeat the analysis in parts (i)-(iii) using a mesh of 4-node quadrilateral elements with the same total number of degrees of freedom as the mesh in `Icant`. What do you observe?



Problem 3

The objective of this problem is to numerically determine the planar curve of total length $2L$ that passes through points $(-a, 0)$ and $(a, 0)$, and whose centroid is as low as possible. Using calculus of variations, it can be established that the exact solution of this problem is given by

$$y = c \cosh(x/c) - \sqrt{c^2 + L^2},$$

where (x, y) are the coordinates of the plane and c a constant that satisfies $c \sinh a/c = L$. An approximate solution can be obtained by assuming that the curve is parametrized by N linear “finite element” subdomains, each of length $2L/N$. This leads to a non-linear algebraic minimization problem for the location of the centroid subject to the prescribed element size.

- (i) Write explicit expressions for the minimization function F and the constraint function \mathbf{c} in terms of the coordinates of the nodal points (be sure to exploit symmetry).
- (ii) Use the Lagrange multiplier formulation and Newton’s method to solve the constrained minimization problem of part (i) for $a = 1$, $L = 4$, and $N = 8$.
- (iii) Use the classical penalty formulation and Newton’s method to solve the constrained minimization problem of part (i) for $a = 1$, $L = 4$, and $N = 8$. Use penalty parameters $\epsilon_k = 10^k$, where $k = 1 - 5$.
- (iv) Use the augmented Lagrangian formulation with Uzawa’s algorithm and Newton’s method to solve the constrained minimization problem of part (i) for $a = 1$, $L = 4$, and $N = 8$. Also, choose a suitable (and fixed) penalty parameter ϵ of your choice and let the Uzawa parameter be $\zeta = \epsilon$.
- (v) Plot the exact curve and the curves corresponding to the solutions in parts (ii)-(iv). Comment on the quality of each approximate solution.