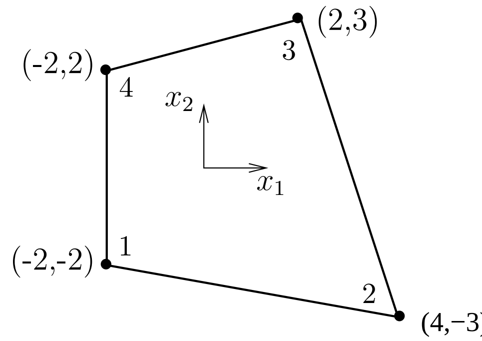
Assignment Nr. 9

due November 16th

Problem 1

Consider a 4-node isoparametric quadrilateral element in plane strain, with reference configuration as in the following figure. After a finite element analysis is conducted, the nodal displacements (u_1, u_2) of the element are found to be:

Node	u_1	u_2
1	0.005	-0.003
2	0.	0.002
3	0.004	0.
4	-0.005	0.001



Compute the normal strains $\epsilon_{11} = u_{1,1}$, $\epsilon_{22} = u_{2,2}$ and the engineering shear strain $\gamma_{12} = u_{1,2} + u_{2,1}$ at point P with coordinates $(x_1, x_2) = (1, 1)$. Also, compute all components of the stress tensor at point P, assuming that the material is isotropic linear elastic with $\lambda = 6 \times 10^5$ and $\mu = 4 \times 10^5$.

Problem 2

A 4-node isoparametric quadrilateral element Ω^e is used in the analysis of a linear elastic body in plane strain. Assuming that the stress field $\boldsymbol{\sigma}$ is constant over the element, determine the number of Gauss points required to *exactly* compute the integral

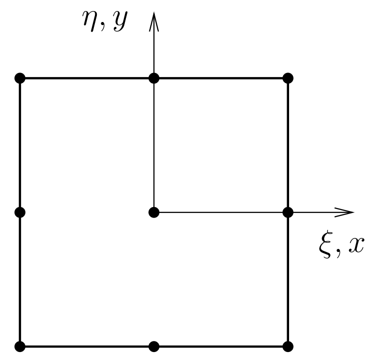
$$\boldsymbol{R}^e \; = \; \int_{\Omega^e} \boldsymbol{B}^{e\,T} \boldsymbol{\sigma} \, d\Omega$$

which emanates from the stress-divergence term

$$\boldsymbol{w}^{e\,T} \boldsymbol{R}^e = \int_{\Omega^e} \boldsymbol{\epsilon}^T(\boldsymbol{w}_h) \, \boldsymbol{\sigma} \, d\Omega \; .$$

Problem 3

Consider a 9-node isoparametric rectangular element, in which the coordinate systems of the natural and physical space coincide, as in the adjucent figure. Assuming that the element is used in modeling a linear elastic solid, whose material parameters remain constant within the element, determine the number of Gauss points per direction required to *exactly* integrate the element stiffness matrix. You do not have to actually compute the stiffness matrix.



Problem 4

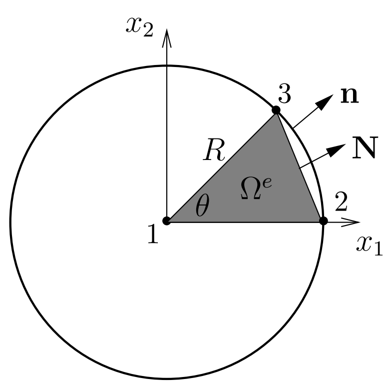
A long cylindrical body of radius R is subjected to boundary traction $\bar{\mathbf{t}} = p_0\,\mathbf{n}$, where p_0 is a constant and $\mathbf{n}$ is the outward unit normal to the boundary.

(a) Let the body be discretized using 3-node triangular elements. With reference to the following figure, compute the equivalent nodal forces on nodes 2 and 3 of a representative element Ω^e due to the prescribed traction, *assuming that p_0 is applied along the normal to the the finite element boundary* with constant outward unit normal $\mathbf{N}$.

(b) Determine the exact resultant traction vector $\mathbf{F}$, defined as

$$\mathbf{F} \;=\; \int \bar{\mathbf{t}}\, ds \;,$$

which applies on *the actual circular boundary* of length $s = R\theta$. How does $\mathbf{F}$ compare to the sum of the equivalent nodal forces computed in part (a)?



Problem 5

Perform a spectral analysis of the element stiffness $\mathbf{K}^e$ for a 4-noded rectangular element in plane strain with $\lambda = 20.0$ and $\mu = 10$ using the 2×2 Gaussian integration rule. You may assume that the element has dimensions 10×6 in the prescribed length unit. Subsequently, repeat your analysis using a 1×1 Gaussian integration rule. Do the eigenvalues change? Comment on the results.

In each case, plot the resulting eigenvectors versus the undeformed mesh and identify them according to the fundamental deformation mode that they represent. You may use MATLAB (or a similar programming language) for your calculations.