Assignment Nr. 12

due December 7th

Problem 1

Write a program to solve the one-dimensional convection-diffusion equation

$$u_{,t} + 10u_{,x} = 0.1u_{,xx}$$

in the domain $(0, 1)$, with periodic boundary conditions $u(0, t) = u(1, t)$ and $u'(1, t) = 0$, and initial condition

$$u(x, 0) = \begin{cases} 0 & \text{for } x \in (0, 0.45) \\ 20(x - 0.45) & \text{for } x \in [0.45, 0.5] \\ 20(0.55 - x) & \text{for } x \in (0.5, 0.55] \\ 0 & \text{for } x \in (0.55, 1) \end{cases} .$$

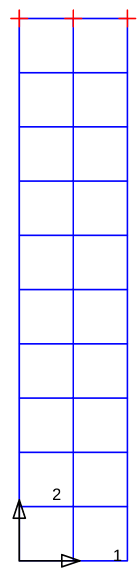
For the spatial discretization, use linear finite elements with 2-point integration and upwinding with optimal grid diffusion. For the temporal discretization, use the forward Euler method with constant time step-size $\Delta t = h^2/6$, where h is the length of a typical element (the choice of the step-size is critical for stability). Run the program for uniform meshes of 20 and 200 elements and plot the distribution of the dependent variable u at times $t = 0.01$, 0.05, and 0.1.

Hint: You may enforce the periodic boundary condition on the left end either by time-lagging, namely setting $u(0, t_{n+1}) = u(1, t_n)$, or by direct coupling between the unknowns $u(0, t)$ and $u(1, t)$.

Problem 2

Use FEAP to solve the equations of linear elastodynamics in a rectangular domain of size $a = 2$ and $b = 10$ under plane strain conditions. The elastic constants are taken to be $E = 100$ and $\nu = 0.3$, while the mass density is $\rho = 1$. The body is fixed on the top edge and is subjected to quasi-static deformation through a non-homogeneous Dirichlet boundary condition of unit upwards vertical displacement on the bottom edge. Also, an initial condition of zero velocity is imposed throughout the domain. At time $t = 1$, the Dirichlet boundary condition on the bottom edge is released and the body undergoes a free vibration.

For the finite element solution, use the grid of 4-node elements shown in the figure. Employ the implicit Newmark integrator ($\beta = 0.25$, $\gamma = 0.5$) and the explicit Newmark integrator ($\beta = 0.0$, $\gamma = 0.5$) with time steps $\Delta t = 0.05$ and 0.01 and carry out the analyses up to time $t = 10$.



Submit plots of the σ_{22} stress at time $t = 10$ for all solutions. Also, show in a single plot the history of the vertical displacement u_2 of the center node on the bottom edge for all solutions and comment on the relation between the different solutions.