Department of Mechanical Engineering
ME 185
J.Casey

SYLLABUS

0 INTRODUCTION

0.1 Continuum approach. Phenomenological theories.
Purely mechanical versus thermodynamical theories. Historical remarks. Scope of course.
0.2 The six primitive concepts of the purely mechanical theory: body, space, time, mass, force, torque. One, two, and three-dimensional bodies and combinations of them.
0.3 Coordinate systems. Reference frames in classical mechanics.
0.4 The three balance laws: mass, linear momentum, angular momentum. Remark on Newton's Laws versus Euler's Laws.
0.5 Material behavior and constitutive equations: examples from previous courses on elasticity, fluid mechanics, rigid body mechanics.
0.6 Scalars, vectors, tensors. Euclidean vector spaces. Index notation. Tensor product and direct notation. Multilinear mappings and determinants.

1 KINEMATICS

1.1 Body, configurations, motion. Convected Cartesian coordinates.
1.2 Particle velocity. The deformation gradient.
1.3 Material, referential, and spatial descriptions of fields.
1.4 The material time derivative.
1.5 Material transport of lines, surfaces and volumes.
1.5 Stretch of a line element. The Cauchy-Green deformation tensors and their properties. The stretch tensors. Maximal stretching. Method of Lagrange multipliers. Eulerian and Lagrangian strain tensors.
1.7 Deformation of area and volume elements.
1.8 The polar decomposition theorem.
1.9 Representation of the rotation tensor. Application to rigid body kinematics. Corotational frames, corotational derivatives.
1.10 The velocity gradient. The rate of deformation tensor, and the vorticity tensor and vector. Material derivative of deformation measures.
1.11 Streamlines and vortex-lines. Circulation. Vortex tubes.
1.12 Rigid motions superposed on a given motion of a deformable body. Objective fields. Objective rates of vectors and second-order tensors.
1.13 Linearization of kinematical measures. Fréchet derivative. Infinitesimal strain and rotation.

2 BALANCE LAWS

2.1 The divergence theorem.
2.2 The transport theorem.
2.3 Mass and mass density. Traction and body forces. Torques.
2.4 Conservation of mass: Material, referential, and spatial integral statements. Point

forms of conservation law.
2.5 Balance of linear momentum: Integral statements.
2.6 Balance of angular momentum: Integral statements.
2.7 Rigid body dynamics.
2.8 Applications of the balance laws in integral form. Control Volumes.
2.9 The Cauchy stress tensor. Existence theorem. Properties of the stress tensor.
2.10 Cauchy's first and second laws.
2.11 Application: the expansion of the universe under Newtonian gravitation.
2.12 Kelvin's kinematical theorem and Helmholtz's vorticity theorems.
2.13 Piola transforms. Piola-Kirchhoff stress tensors. Referential form of Euler's laws and Cauchy's laws.
2.14 Objectivity requirements.
2.15 The work-energy theorem.
2.16 Remarks on thermodynamical concepts, and on the first and second laws of thermodynamics.
2.17 On the derivation of Euler's Laws from the energy equation.

3 CONSTITUTIVE RELATIONS

3.1 Concept of a *material* in continuum mechanics. Examples of constitutive assumptions. Solids and fluids. Constrained and unconstrained materials.
3.2 Local action, materials with memory.
3.3 Restrictions due to objectivity requirements.
3.4 Material symmetry.
3.5 Ideal fluids. Incompressibility.
3.6 Viscous fluids. Linear viscous fluids. Navier-Stokes equations. Examples.
3.7 Elastic solids. Examples of simple deformations. Questions of uniqueness and stability of solutions.
3.8 Internally constrained materials.
3.9 Linearly elastic solids. Bending and torsion of bars. Saint Venant's principle.
3.10 Linear viscoelasticity.