University of California, Berkeley
Department of Mechanical Engineering
ME185, Fall 2009

Homework 9 (due 17-Dec-2009)

Problem 1

Recall that the constitutive equation for a homogeneous isotropic elastic solid is

$$\mathbf{T} = \alpha_0 \mathbf{I} + \alpha_1 \mathbf{B} + \alpha_2 \mathbf{B}^2, \tag{1}$$

where $\mathbf{B}$ is the left Cauchy-Green tensor, and α_0, α_1, and α_2 are scalar-valued functions of the principal invariants, I_1, I_2, and I_3 of $\mathbf{B}$.

(a) If $\mathbf{e}$ is an eigenvector of $\mathbf{B}$, show that $\mathbf{e}$ is also an eigenvector of $\mathbf{T}$.

(b) Resolve (1) on an eigenbasis of $\mathbf{B}$.

(c) Let an elastic bar of square cross section undergo a motion of the form

$$x_1 = \lambda(t) X_1, x_2 = X_2, x_3 = X_3,$$

where the X_1 direction is axial, and $\lambda(0) = 1$, and $\ddot{\lambda}(t) = 0$. Calculate the stress field in the bar and the traction that must be supplied on the boundary of the bar. Also, calculate the body force field.

Problem 2

For a homogeneous, linearly elastic solid, the strain energy per unit volume is of the form

$$W = \frac{1}{2} C_{ijkl} e_{ij} e_{kl}, \tag{2}$$

where C_{ijkl} are the elastic constants.

(a) What does (2) reduce to in the case of an isotropic material?

(b) Obtain the stress-strain relation for the material in part (a), in terms of the Lamé constants λ and μ.

Problem 3

Consider a homogeneous, isotropic, linearly elastic material deforming in the absence of body forces, and let u_i be the Cartesian components of the displacement field. Show that the displacement is governed by Navier's equations

$$\mu u_{i,jj} + (\lambda + \mu)\, \mu u_{j,ji} = \rho_o \frac{\partial^2 u_i}{\partial t^2}.$$

Problem 4

Let a continuum $\mathscr{B}$ be subjected to a homogeneous deformation of the form

$$x_1 = \lambda_1 X_1, x_2 = \lambda_2 X_2, x_3 = \lambda_3 X_3,$$

and suppose that it is composed of a material which can support the given deformation by a uniform Cauchy stress field of the form

$$\begin{aligned} T_{11} &= \sigma_1, \\ T_{22} &= \sigma_2 \\ T_{33} &= \sigma_3, \\ T_{12} = T_{23} = T_{31} &= 0. \end{aligned}$$

(a) Calculate the components of the first Piola-Kirchoff stress tensor, $\mathbf{P}$.

(b) Consider a cuboid of material which in the reference configuration has edges of length L_1, L_2, and L_3, that are parallel to the basis vectors $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$, respectively. In the present configuration, this material will also occupy a cuboid, with edges l_1, l_2, and l_3, say. Calculate the traction vectors $\mathbf{t}_i$ and $\mathbf{p}_A$, and discuss the relationship between them in the present case.

Problem 5

Suppose that a material satisfies the constitutive equation

$$\mathbf{T} = \alpha \mathbf{B} + \beta \mathbf{D},$$

where α and β are the material constants, $\mathbf{B}$ is the left Cauchy-Green deformation tensor, and $\mathbf{D}$ is the rate of deformation tensor.

(a) For the homogeneous motion

$$x_1 = X_1 + \kappa(t)\, X_1, x_2 = X_2, x_3 = X_3,$$

calculate the Cauchy stress at time t.

(b) State the physical dimensions of α and β and explain what material properties these coefficients describe.

(c) Argue that the given constitutive equation is properly invariant under superposed rigid body motions of the continuum.

(d) Consider a constitutive equation

$$\mathbf{T} = \alpha\mathbf{B} + \beta\mathbf{D} + \gamma\mathbf{W} + \delta\mathbf{W}^2,$$

where $\mathbf{D}$ and $\mathbf{W}$ are the symmetric and skew parts of the velocity gradient, respectively, and α, β, γ and δ are constants. What can be said about the coefficients γ and δ? (Explain your answer.)