Scorbot Kinematics

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Constants

a = length of link between shoulder and elbow
b = length of link between elbow and wrist
c = distance between shoulder axis and base pivot of robot

Variables

x, y, z = position of wrist from base pivot of robot
r' = horizontal distance between wrist and shoulder axis of robot
ρ' = distance between wrist and shoulder axis of robot
θ_1 = angle of waist
θ_2 = angle of link between shoulder and elbow
θ_3 = angle of link between elbow and wrist
β = angle between straight line from shoulder pivot to wrist and link between elbow and wrist

Forward Kinematics

$$x = (a\cos\theta_2 + b\cos\theta_3 + c)\cos\theta_1 \tag{1}$$

$$y = (a\cos\theta_2 + b\cos\theta_3 + c)\sin\theta_1 \tag{2}$$

$$z = a\sin\theta_2 + b\sin\theta_3 \tag{3}$$

Inverse Kinematics

$$\theta_1 = \tan^{-1}\frac{y}{x} \tag{4}$$

$$r' = \sqrt{x^2 + y^2 - c^2} \tag{5}$$

$$\rho' = \sqrt{r'^2 + z^2} \tag{6}$$

By the Law of Cosines,

$$b^2 = a^2 + \rho'^2 - 2a\rho'\cos\beta \tag{7}$$

$$\beta = \cos^{-1} \frac{a^2 + \rho'^2 - b^2}{2a\rho'} \tag{8}$$

Underhand

$$\theta_2 = \tan^{-1} \frac{z}{r'} - \beta \tag{9}$$

$$\theta_3 = \tan^{-1} \frac{z - a\sin\theta_2}{r' - a\cos\theta_2} \tag{10}$$

Overhand

$$\theta_2 = \tan^{-1} \frac{z}{r'} + \beta \tag{11}$$

$$\theta_3 = \tan^{-1} \frac{z - a\sin\theta_2}{r' - a\cos\theta_2} \tag{12}$$