

Basic Robot Kinematics

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April 17, 2008

Constants

a = length of link between shoulder and elbow

b = length of link between elbow and wrist

Variables

x, y, z = position of wrist from shoulder pivot of robot

r = distance between wrist and waist axis of robot

ρ = distance between wrist and shoulder pivot of robot

θ_1 = angle of waist

θ_2 = angle of link between shoulder and elbow

θ_3 = angle of link between elbow and wrist

β = angle between straight line from shoulder pivot to wrist and link between elbow and wrist

Forward Kinematics

$$x = (a \cos \theta_2 + b \cos \theta_3) \cos \theta_1 \quad (1)$$

$$y = (a \cos \theta_2 + b \cos \theta_3) \sin \theta_1 \quad (2)$$

$$z = a \sin \theta_2 + b \sin \theta_3 \quad (3)$$

Inverse Kinematics

$$\theta_1 = \tan^{-1} \frac{y}{x} \quad (4)$$

$$r = \sqrt{x^2 + y^2} \quad (5)$$

$$\rho = \sqrt{r^2 + z^2} \quad (6)$$

By the Law of Cosines,

$$b^2 = a^2 + \rho^2 - 2a\rho \cos \beta \quad (7)$$

$$\beta = \cos^{-1} \frac{a^2 + \rho^2 - b^2}{2a\rho} \quad (8)$$

Underhand

$$\theta_2 = \tan^{-1} \frac{z}{r} - \beta \quad (9)$$

$$\theta_3 = \tan^{-1} \frac{z - a \sin \theta_2}{r - a \cos \theta_2} \quad (10)$$

Overhand

$$\theta_2 = \tan^{-1} \frac{z}{r} + \beta \quad (11)$$

$$\theta_3 = \tan^{-1} \frac{z - a \sin \theta_2}{r - a \cos \theta_2} \quad (12)$$