ME 104: Midterm II Review Solutions

Prof. Karl Hedrick: University of California, Berkeley

Chapter 12, Solution 106

For Earth, $R = 3690 \text{ mi} = 20.9088 \times 10^6 \text{ ft}, \quad g = 32.2 \text{ ft/s}^2$

$$GM_{\text{earth}} = gR^2 = (32.2)(20.9088 \times 10^6)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

For Venus, $GM = 0.82 \quad GM_{\text{earth}} = 11.543 \times 10^{15} \text{ ft}^3/\text{s}^2$

For a parabolic trajectory with $r_A = 9.3 \times 10^3 \text{ mi} = 49.104 \times 10^6 \text{ ft}$

$$(v_A)_1 = v_{\text{esc}} = \sqrt{\frac{2GM}{r_A}} = \sqrt{\frac{(2)(11.543 \times 10^{15})}{49.104 \times 10^6}} = 21.683 \times 10^3 \text{ ft/s}$$

First transfer orbit *AB*. $r_B = 190 \times 10^3 \text{ mi} = 1003.2 \times 10^6 \text{ ft}$

At Point *A*, where $\theta = 180°$

$$\frac{1}{r_A} = \frac{GM}{h_{AB}^2} + C \cos 180° = \frac{GM}{h_{AB}^2} - C \tag{1}$$

At Point *B*, where $\theta = 0°$

$$\frac{1}{r_B} = \frac{GM}{h_{AB}^2} + C \cos 0 = \frac{GM}{h_{AB}^2} + C \tag{2}$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_B + r_A}{r_A r_B} = \frac{2GM}{h_{AB}^2}$$

Solving for h_{AB},

$$h_{AB} = \sqrt{\frac{2GMr_A r_B}{r_B + r_A}} = \sqrt{\frac{(2)(11.543 \times 10^{15})(49.104 \times 10^6)(1003.2 \times 10^6)}{1052.3 \times 10^6}}$$

$$= 1.039575 \times 10^{12} \text{ ft}^2/\text{s}$$

$$(v_A)_2 = \frac{h_{AB}}{r_A} = \frac{1.039575 \times 10^{12}}{49.104 \times 10^6} = 21.174 \times 10^3 \text{ ft/s}$$

$$(v_B)_1 = \frac{h_{AB}}{r_B} = \frac{1.039575 \times 10^{12}}{1003.2 \times 10^6} = 1.03626 \times 10^3 \text{ ft/s}$$

Second transfer orbit *BC*. $r_C = 5600 \text{ mi} = 29.568 \times 10^6 \text{ ft}$

At Point *B*, where $\theta = 0$

$$\frac{1}{r_B} = \frac{GM}{h_{BC}^2} + C \cos 0 = \frac{GM}{h_{BC}^2} + C$$

At Point *C*, where $\theta = 180°$

$$\frac{1}{r_C} = \frac{GM}{h_{BC}^2} + C \cos 180° = \frac{GM}{h_{BC}^2} - C$$

PROBLEM 12.106 (Continued)

Adding,

$$\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_B + r_C}{r_B r_C} = \frac{2GM}{h_{BC}^2}$$

$$h_{BC} = \sqrt{\frac{2GMr_B r_C}{r_B + r_C}}$$

$$= \sqrt{\frac{(2)(11.543\times10^{15})(1003.2\times10^6)(29.568\times10^6)}{1032.768\times10^6}}$$

$$= 814.278\times10^9 \text{ ft}^2/\text{s}$$

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{814.278\times10^9}{1003.2\times10^6} = 811.69 \text{ ft/s}$$

$$(v_C)_1 = \frac{h_{BC}}{r_C} = \frac{814.278\times10^9}{29.568\times10^6} = 27.539\times10^3 \text{ ft/s}$$

Final circular orbit.

$$r_C = 29.568\times10^6 \text{ ft}$$

$$(v_C)_2 = \sqrt{\frac{GM}{r_C}}$$

$$= \sqrt{\frac{11.543\times10^{15}}{29.568\times10^6}}$$

$$= 19.758\times10^3 \text{ ft/s}$$

Speed reductions.

(*a*) At *A*:

$$(v_A)_1 - (v_A)_2 = 21.683\times10^3 - 21.174\times10^3$$

$$\Delta v_A = 509 \text{ ft/s} \blacktriangleleft$$

(*b*) At *B*:

$$(v_B)_1 - (v_B)_2 = 1.036\times10^3 - 811.69$$

$$\Delta v_B = 224 \text{ ft/s} \blacktriangleleft$$

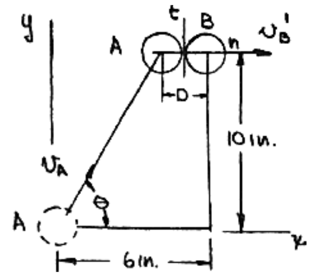
(*c*) At *C*:

$$(v_C)_1 - (v_C)_2 = 27.539\times10^3 - 19.758\times10^3$$

$$\Delta v_C = 7.78\times10^3 \text{ ft/s} \blacktriangleleft$$

Chapter 13, Solution 168

(*a*) Since v'_B is in the *x*-direction and assuming no friction, the common tangent between *A* and *B* at impact must be parallel to the *y*-axis.



Thus, $$\tan\theta = \frac{10}{6-D}$$

$$\theta = \tan^{-1}\frac{10}{6-2.37}$$

$$= 70.04°$$

$\theta = 70.0°$ ◀

(*b*) <u>Conservation of momentum in *x*(*n*) direction</u>.

$$mv_A\cos\theta + m(v_B)_n = m(v'_A)_n + mv'_B$$

$$(3)(\cos 70.04) + 0 = (v'_A)_n + v'_B$$

$$1.0241 = (v'_A)_n + (v'_B) \qquad (1)$$

<u>Relative velocities in the *n* direction</u>.

$e = 0.9$ $$(v_A\cos\theta - (v_B)_n)e = v'_B - (v'_A)_n$$

$$(1.0241 - 0)(0.9) = v'_B - (v'_A)_n \qquad (2)$$

$(1) + (2)$ $$2v'_B = 1.0241(1.9)$$

$\mathbf{v}'_B = 0.972\text{ ft/s} \rightarrow$ ◀

Chapter 13, Solution 188

Masses: $m_A = \dfrac{W_A}{g} = \dfrac{2 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.062112 \text{ lb} \cdot \text{s}^2/\text{ft}$

$$m_B = \frac{W_B}{g} = \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.124224 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Analysis of sphere A as it swings down.

Initial state: $\alpha = 30°, \quad h_0 = l(1-\cos\alpha) = (3)(1-\cos 30°) = 0.40192 \text{ ft}$

$$V_0 = W_A h_0 = (2)(0.40192) = 0.80385 \text{ ft} \cdot \text{lb}$$

$$T_0 = \frac{1}{2} m v_0^2 = \frac{1}{2}(0.062112)(2)^2 = 0.12422 \text{ ft} \cdot \text{lb}$$

Just before impact: $\alpha = 0, \quad h_1 = 0, \quad V_1 = 0$

$$T_1 = \frac{1}{2} m_A v_A^2 = \frac{1}{2}(0.062112) v_A^2 = 0.031056 v_A^2$$

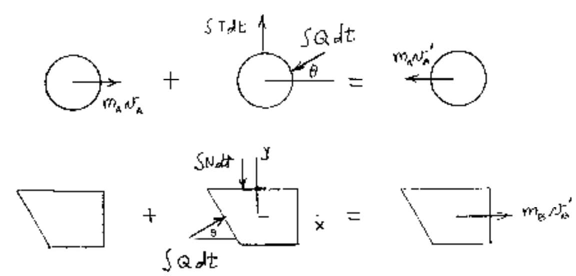
Conservation of energy: $T_0 + V_0 = T_1 + V_1$

$$0.12422 + 0.80385 = 0.031056 v_A^2 + 0$$

$$v_A^2 = 29.884 \text{ ft}^2/\text{s}^2$$

$$\mathbf{v}_A = 5.4666 \text{ ft/s} \longrightarrow \quad ◀$$

Analysis of the impact. Use conservation of momentum together with the coefficient of restitution $e = 0.8$.



Note that the ball rebounds horizontally and that an impulse $\int T dt$ is applied by the rope. Also, an impulse $\int N dt$ is applied to B through its supports.

<u>Both *A* and *B*.</u>

Momentum in *x*-direction:

$$m_A(v_A)_x + 0 = m_A(v_A)_x + m_B(v_B)_x$$

$$(0.062112)(5.4666) = (0.062112)(v'_A)_x + 0.124224(v'_B)_x \qquad (1)$$

Coefficient of restitution: $(v_A)_n = (v_A)_x \cos\theta$

$$(v_B)_n = 0, \quad (v'_A)_n = (v'_A)_x \cos\theta, \quad (v'_B)_x \cos 30°$$

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$

$$(v'_B)_x \cos\theta - (v'_A)_x \cos\theta = e[(v_A)_x \cos\theta]$$

Dividing by $\cos\theta$ and applying $e = 0.8$ gives

$$(v'_B)_x - (v'_A)_x = (0.8)(5.4666) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$(v'_A)_x = -1.093 \text{ ft/s}$$

$$(v'_B)_x = 3.28 \text{ ft/s}$$

$\mathbf{v}'_A = 1.093$ ft/s ← ◀

$\mathbf{v}'_B = 3.28$ ft/s → ◀