ME 104: Homework 9 Solutions 2

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Chapter 16, Solution 136

Kinematics:

Velocity

$$\omega_{AB} = 0$$

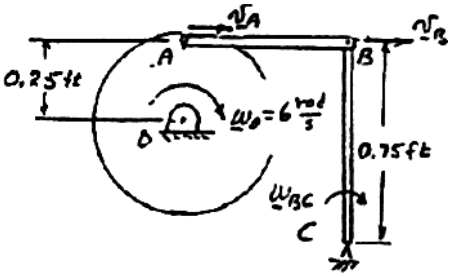
$$v_B = v_A$$

$$= (0.25 \text{ ft})(6 \text{ rad/s})$$

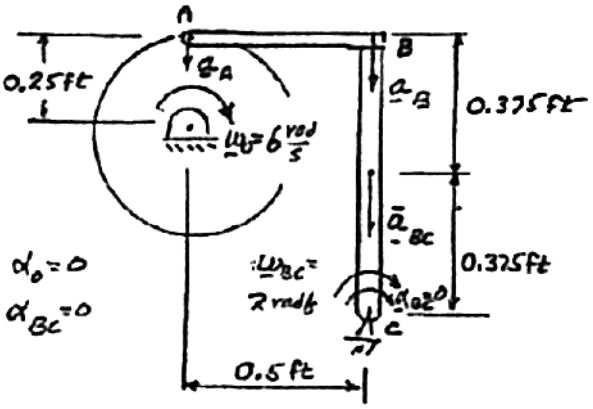
$$= 1.5 \text{ ft/s}^2$$

$$\omega_{BC} = \frac{v_B}{0.75 \text{ ft}} = \frac{1.5 \text{ m/s}^2}{0.75 \text{ ft}}$$

$$\boldsymbol{\omega}_{BC} = 2 \text{ rad/s} \circlearrowleft$$



Acceleration



$$a_A = (0.25 \text{ ft})(6 \text{ rad/s})^2$$

$$\mathbf{a}_A = 9 \text{ ft/s}^2 \downarrow$$

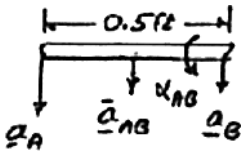
$$\mathbf{a}_B = (0.75)(3 \text{ rad/s}^2) = 3 \text{ ft/s}^2 \downarrow$$

$$\bar{a}_{BC} = (0.375 \text{ ft})(2 \text{ rad/s})^2$$

$$\bar{a}_{BC} = 1.5 \text{ ft/s}^2$$

$$\bar{a}_{AB} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(9 + 2)$$

$$\bar{\mathbf{a}}_{AB} = 6 \text{ ft/s}^2 \downarrow$$



$$a_A = a_B + (0.5 \text{ ft})\alpha_{AB}$$

$$9 \text{ ft/s}^2 = 3 \text{ ft/s}^2 + (0.5 \text{ ft})\alpha_{AB}$$

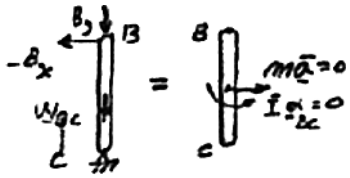
$$\boldsymbol{\alpha}_{AB} = 12 \text{ rad/s}^2 \circlearrowright$$

PROBLEM 16.136 (Continued)

Kinetics:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} (AB)^2 = \frac{1}{12} \frac{4 \text{ lb}}{32.2} (0.5 \text{ ft})^2$$

$$\bar{I}_{AB} = 2.588 \times 10^{3} \text{ lb} \cdot \text{ft/s}^2$$

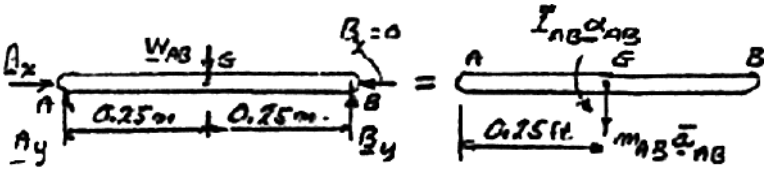


Rod *BC*:

Since $\alpha_{BC} = 0, \quad \bar{a} = 0$

$$\Sigma M_C = 0 \quad \text{yields} \quad B_x = 0$$

Rod *AB*:



$$\pm \rightarrow \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad A_x = 0$$

$$+ \circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad B_y (0.5 \text{ ft}) - W_{AB} (0.25 \text{ ft}) = \bar{I}_{AB} \alpha_{AB} - m_{AB} \bar{a}_{AB} (0.25 \text{ ft})$$

$$0.5 B_y - (4 \text{ lb})(0.25 \text{ ft}) = (2.588 \times 10^{-3} \text{ lb} \cdot \text{ft/s}^2)(12 \text{ rad/s}) - \frac{4 \text{ lb}}{32.2} (6 \text{ ft/s}^2)(11.25 \text{ ft})$$

$$0.5 B_y - 1 = 0.03106 - 0.1863$$

$$0.5 B_y = 0.8447$$

$$B_y = 1.689 \text{ lb} \qquad \mathbf{B} = 1.689 \text{ lb} \uparrow \blacktriangleleft$$

$$+ \uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A_y - W_{AB} + B_y = -m_{AB} \bar{a}_{AB}$$

$$A_y - 4 \text{ lb} + 1.689 \text{ lb} = -\frac{4 \text{ lb}}{32.2} (6 \text{ ft/s}^2)$$

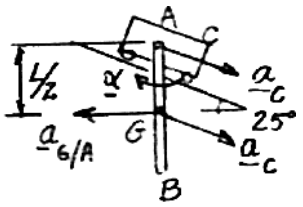
$$A_y = 1.565 \text{ lb} \qquad \mathbf{A}_y = 1.565 \text{ lb} \uparrow \blacktriangleleft$$

Chapter 16, Solution 144

Kinematics: We resolve the acceleration of G into the accel. of the cart and the accel. of G relative to A:

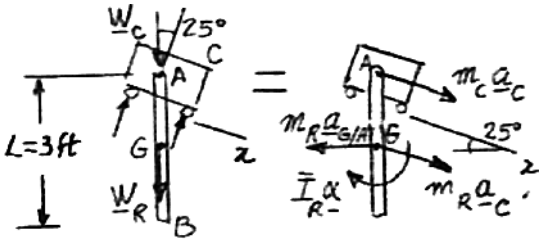
$$\bar{\mathbf{a}}_R = \mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$$

$$\bar{\mathbf{a}}_R = \mathbf{a}_C + \mathbf{a}_{G/A}$$



Where $$a_{G/A} = \frac{1}{2} L\alpha$$

Kinetics: Cart and rod



$$W_R = 30 \text{ lb}$$

$$W_C = 40 \text{ lb}$$

$$L = 3 \text{ ft}$$

$$\bar{I}_R = \frac{1}{12} m_R L^2$$

$$a_{G/A} = \frac{1}{2}(3)\alpha = 1.5\alpha$$

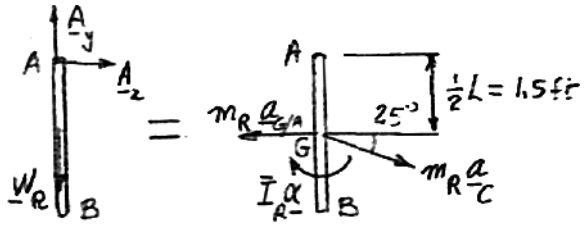
$$+\searrow \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad (W_C + W_R)\sin 25° = (m_C + m_R)\mathbf{a}_C - m_R a_{G/A}\cos 25°$$

$$(40+30)\sin 25° = \frac{1}{32.2}[(40+30)a_C - 30(1.5\alpha)\cos 25°]$$

$$70a_C = 70(32.2)\sin 25° + (4500\cos 25°)\alpha$$

$$a_C = (32.2)\sin 25° + \left(\frac{45}{70}\cos 25°\right)\alpha \qquad (1)$$

Rod



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m_R a_{G/R})\frac{L}{2} - (m_R a_C \cos 25°)\frac{L}{2}$$

$$\frac{1}{12}\frac{30}{g}(3)^2\alpha + \frac{30}{g}(1.5\alpha)(1.5) - \left(\frac{30}{g}a_C\cos 25°\right)(1.5) = 0$$

$$22.5\alpha + 67.5\alpha - (45\cos 25°)a_C = 0$$

$$\alpha = (0.5\cos 25°)a_C \qquad (2)$$

PROBLEM 16.144 (Continued)

(*a*) Acceleration of the cart.

Substitute for α from (2) into (1):

$$a_C = (32.2)\sin 25° + \left(\frac{45}{10}\cos 25°\right)(0.5\cos 25°)a_C$$

$$a_C = \frac{32.2\sin 25°}{1 - \frac{22.5}{10}\cos 25°}$$

$$= 18.490 \text{ ft/s}^2$$

$$\mathbf{a}_C = 18.49 \text{ ft/s}^2 \searrow 25° \blacktriangleleft$$

(*b*) Angular acceleration.

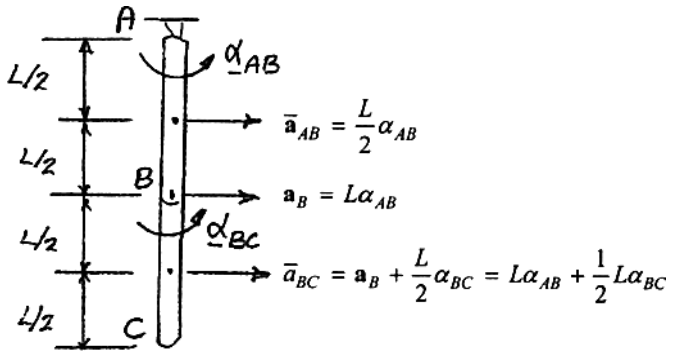
From (2): $\alpha = (0.5\cos 25°)(18.490)$

$$= 8.379 \text{ rad/s}^2$$

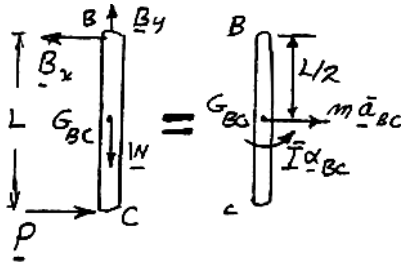
$$\boldsymbol{\alpha} = 8.38 \text{ rad/s}^2 \blacktriangleleft$$

Chapter 16, Solution 149*

Kinematics: Assume α_{AB} ↻ α_{BC} ↻ and $\omega_{AB} = \omega_{BC} = 0$



Kinetics: Bar *BC*



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad PL = \bar{I}\alpha_{BC} + (m\bar{a}_{BC})\frac{L}{2}$$

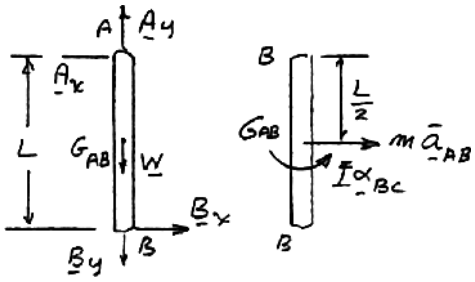
$$= \frac{m}{12}L^2\alpha_{BC} + m\left(L\alpha_{AB} + \frac{L}{2}\alpha_{BC}\right)\frac{L}{2}$$

$$P = \frac{1}{2}mL\alpha_{AB} + \frac{1}{3}mL\alpha_{BC} \qquad (1)$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P - B_x = m\bar{a}_{BC}$$

$$P - B_x = m\left(L\alpha_{AB} + \frac{1}{2}L\alpha_{BC}\right) \qquad (2)$$

Bar *AB*:



PROBLEM 16.149* (Continued)

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad B_x L = \bar{I}\alpha_{AB} + (m\bar{a}_{AB})\frac{L}{2}$$

$$= \frac{m}{12}L^2\alpha_{AB} + m\left(\frac{L}{2}\alpha_{AB}\right)\frac{L}{2}$$

$$B_x = \frac{1}{3}mL\alpha_{AB} \tag{3}$$

Add (2) and (3):

$$P = \frac{4}{3}mL\alpha_{AB} + \frac{1}{2}mL\alpha_{BC} \tag{4}$$

Subtract (1) from (4)

$$0 = \frac{5}{6}mL\alpha_{AB} + \frac{1}{6}mL\alpha_{BC}$$

$$\alpha_{BC} = -5\alpha_{AB} \tag{5}$$

Substitute for α_{BC} in (1):

$$P = \frac{1}{2}mL\alpha_{AB} + \frac{1}{3}mL(-5\alpha_{AB}) = -\frac{7}{6}mL\alpha_{AB}$$

$$\alpha_{AB} = -\frac{6}{7}\frac{P}{mL} \tag{6}$$

Eq. (5)

$$\alpha_{BC} = -5\left(-\frac{6}{7}\frac{P}{mL}\right) \quad \alpha_{BC} = \frac{30}{7}\frac{P}{mL} \tag{7}$$

Data:

$$L = 500 \text{ mm} = 0.5 \text{ m}, \quad m = 3 \text{ kg}, \quad P = 20 \text{ N}$$

$$\alpha_{AB} = -\frac{6}{7}\frac{P}{mL} = -\frac{6}{7}\frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})}$$

$$= -11.249 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{AB} = 11.43 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

$$\alpha_{BC} = \frac{30}{7}\frac{P}{mL} = \frac{30}{7}\frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})}$$

$$= 57.143 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{BC} = 57.1 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$