ME 104: Homework 8 Solutions

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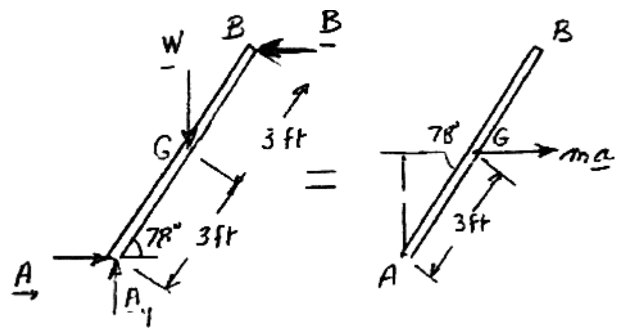
Chapter 16, Solution 3



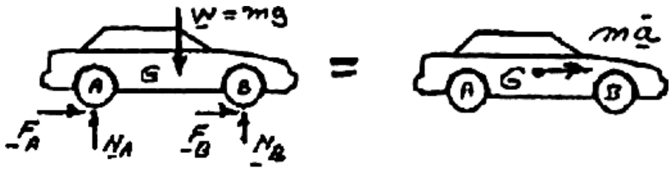
For the board to remain in the position shown, we need $B \geq 0$. When the board is just about to start rotating about A, we have $B = 0$.

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad W(3\text{ ft})\cos 78° = \frac{W}{g} a (3\text{ ft})\sin 78°$$

$$a = g\cot 78° = (32.2\text{ ft/s}^2)\cot 78° \qquad a = 6.84\text{ ft/s}^2 \blacktriangleleft$$

Chapter 16, Solution 5

(*a*) Four-wheel drive:



$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0 \qquad N_A + N_B = W = mg$$

Thus: $$F_A + F_B = \mu_k N_A + \mu_k N_B = \mu_k (N_A + N_B) = \mu_k W = 0.80mg$$

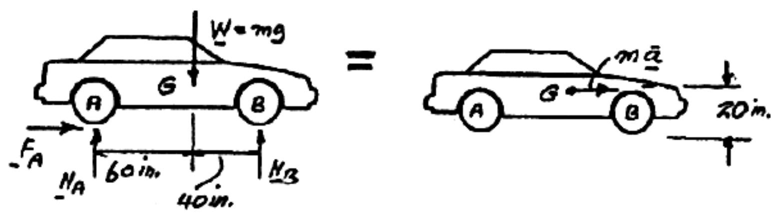
$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F_A + F_B = m\bar{a}$$

$$0.80mg = m\bar{a}$$

$$\bar{a} = 0.80g = 0.80(32.2 \text{ ft/s}^2)$$

$\bar{\mathbf{a}} = 25.8 \text{ ft/s}^2 \longrightarrow$ ◀

(*b*) Rear-wheel drive:



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad (40 \text{ in.})W - (100 \text{ in.})N_A = -(20 \text{ in.})m\bar{a}$$

$$N_A = 0.4W + 0.2m\bar{a}$$

Thus: $$F_A = \mu_k N_B = 0.80(0.4W + 0.2m\bar{a}) = 0.32mg + 0.16m\bar{a}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F_A = m\bar{a}$$

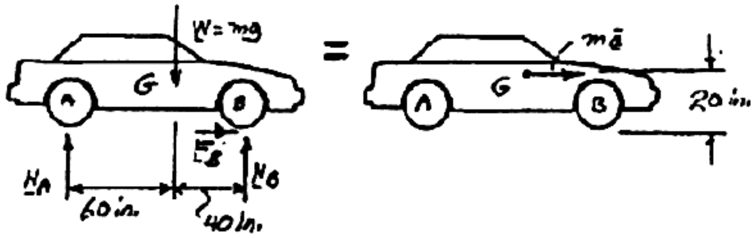
$$0.32mg + 0.16m\bar{a} = m\bar{a}$$

$$0.32g = 0.84\bar{a}$$

$$\bar{a} = \frac{0.32}{0.84}(32.2 \text{ ft/s}^2)$$

$\bar{\mathbf{a}} = 12.27 \text{ ft/s}^2 \longrightarrow$ ◀

(c)   Front-wheel drive:



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad (100\text{ in.})N_B - (60\text{ in.})W = -(20\text{ in.})m\bar{a}$$

$$N_B = 0.6W - 0.2m\bar{a}$$

Thus:

$$F_B = \mu_k N_B = 0.80(0.6W - 0.2m\bar{a}) = 0.48mg - 0.16m\bar{a}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F_B = m\bar{a}$$

$$0.48mg - 0.16m\bar{a} = m\bar{a}$$

$$0.48g = 1.16\bar{a}$$

$$\bar{a} = \frac{0.48}{1.16}(32.2\text{ ft/s}^2)$$

$$\bar{\mathbf{a}} = 13.32\text{ ft/s}^2 \longrightarrow \blacktriangleleft$$

Chapter 16, Solution 13

Kinematics: Assume that the barrel is sliding, but not tipping.

$$\mathbf{a} = 0 \quad \mathbf{a}_G = a \longrightarrow$$

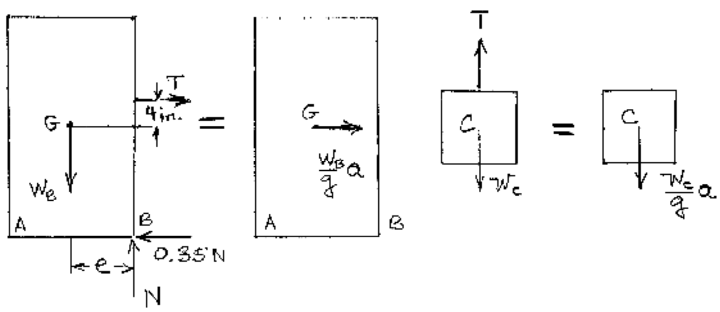
Since the cord is inextensible, $\mathbf{a}_C = a \downarrow$

Kinetics: Draw the free body diagrams of the barrel and the cylinder.

The barrel is sliding. $F_F = \mu_k N = 0.35N$

Assume that tipping is impending, so that the line of action of the reaction on the bottom of the barrel passes through Point B.

$$e = 10 \text{ in.}$$



For the barrel. $+\uparrow \Sigma F_y = 0: \quad N - W_B = 0 \quad N = W_B$

$$+\circlearrowright \Sigma M_G = 0: \quad Ne - 4T - (18)(0.35N) = 0$$

$$T = \frac{e - (18)(0.35)}{4} N = \frac{10 - 6.3}{4} W_B = 0.925 W_B$$

$$\xrightarrow{+} \Sigma F_x = \frac{W_B}{g} a: \quad T - 0.35N = \frac{W_B}{g} a$$

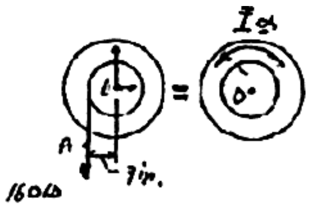
$$\frac{a}{g} = \frac{T}{W_B} - 0.35\frac{N}{W_B} = 0.925 - 0.35 = 0.575$$

For the cylinder: $+\downarrow \Sigma F = \frac{W_C}{g} a: \quad W_C - T = W_C \frac{a}{g}$

$$W_C = \frac{T}{1 - \frac{a}{g}} = \frac{0.925 W_B}{1 - 0.575} = (2.1765)(200 \text{ lb}) \qquad W_C = 435 \text{ lb} \blacktriangleleft$$

Chapter 16, Solution 34

Case 1:



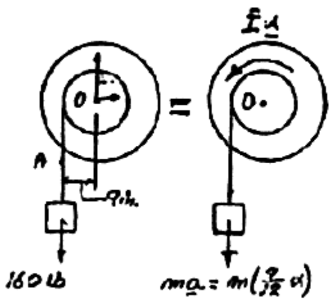
(*a*) $\circlearrowleft + \Sigma M_0 = \Sigma (M_0)_{\text{eff}}: \quad (160 \text{ lb})\left(\frac{9}{12}\text{ft}\right) = (15 \text{ lb} \cdot \text{ft} \cdot \text{s}^2)\alpha$

$$\boldsymbol{\alpha} = 8 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \frac{10 \text{ ft}}{\left(\frac{9}{12}\text{ ft}\right)} = 13.333 \text{ rad}$

$\omega^2 = 2\alpha\theta = 2(8 \text{ rad/s}^2)(13.33 \text{ rad}) \qquad \boldsymbol{\omega} = 14.61 \text{ rad/s} \circlearrowleft \blacktriangleleft$

Case 2:



(*a*) $\circlearrowleft + \Sigma M_0 = \Sigma (M_0)_{\text{eff}}: \quad (160)\left(\frac{9}{12}\right) = 15\alpha + ma\left(\frac{9}{12}\right)$

$$120 = 15\alpha + \frac{160}{32.2}\left(\frac{9}{12}\alpha\right)\left(\frac{9}{12}\right)$$

$$120 = (15 + 2.795)\alpha$$

$$\alpha = 6.7435 \text{ rad/s}^2$$

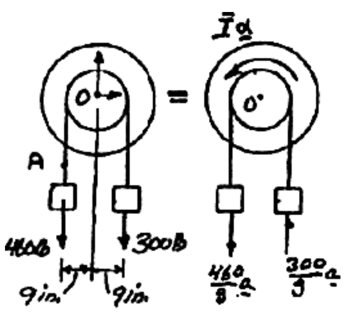
$$\boldsymbol{\alpha} = 6.74 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \frac{10 \text{ ft}}{\left(\frac{9}{12}\text{ ft}\right)} = 13.333 \text{ rad}$

$\omega^2 = 2\alpha\theta$

$= 2(6.7435 \text{ rad/s}^2)(13.333 \text{ rad}) \qquad \boldsymbol{\omega} = 13.41 \text{ rad/s} \circlearrowleft \blacktriangleleft$

Case 3:



(*a*) $\circlearrowleft + \Sigma M_0 = \Sigma (M_0)_{\text{eff}}:$

$$(460)\left(\frac{9}{12}\right) - (300)\left(\frac{9}{12}\right) = 15\alpha + \frac{460}{32.2}a\left(\frac{9}{12}\right) + \frac{300}{32.2}a\left(\frac{9}{12}\right)$$

$$120 = 15\alpha + \frac{460}{32.2}\left(\frac{9}{12}\right)^2\alpha + \frac{300}{32.2}\left(\frac{9}{12}\right)^2\alpha$$

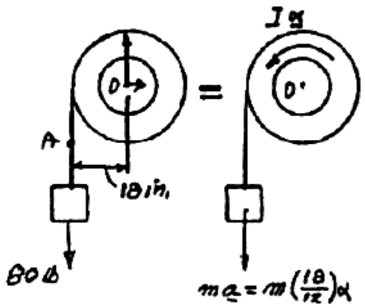
$$\alpha = 4.2437 \text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 4.24 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \frac{10 \text{ ft}}{\frac{9}{12}} = 13.333 \text{ rad}$

$\omega^2 = 2\alpha\theta = 2(4.2437)(13.333) \qquad \boldsymbol{\omega} = 10.64 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$

Case 4:



(*a*) $\overset{+}{\circlearrowleft} \Sigma M_0 = \Sigma (M_0)_{\text{eff}}: \quad (80)\left(\frac{18}{12}\right) = 15\alpha + \frac{80}{32.2} a \left(\frac{18}{12}\right)$

$$120 = 15\alpha + \frac{80}{32.2}\left(\frac{18}{12}\right)^2 \alpha$$

$$120 = (15 + 5.5901)\alpha$$

$$\alpha = 5.828 \text{ rad/s}^2$$

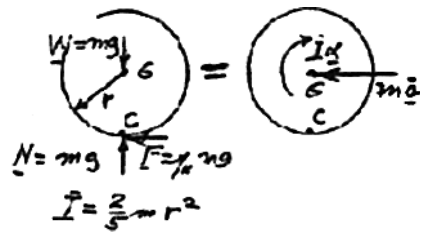
$\alpha = 5.83 \text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) $\theta = \dfrac{10 \text{ ft}}{\left(\frac{18}{12} \text{ ft}\right)} = 6.6667 \text{ rad}$

$\omega^2 = 2\alpha\theta = 2(5.828 \text{ rad/s}^2)(6.6667 \text{ rad})$ $\quad \omega = 8.82 \text{ rad/s} \circlearrowleft$ ◀

Chapter 16, Solution 69

Kinetics:



$$\overset{+}{\leftarrow}\ \Sigma F_x = \Sigma(F_x)_{\text{eff}}:\quad \mu_k mg = m\bar{a}$$

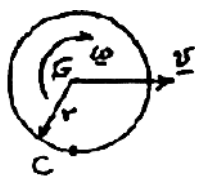
$$\bar{\mathbf{a}} = \mu_k g \longleftarrow$$

$$+\circlearrowright\ \Sigma M_G = \Sigma(M_G)_{\text{eff}}:\quad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = \frac{2}{5}mr^2$$

$$\boldsymbol{\alpha} = \frac{5}{2}\frac{\mu_k g}{r}\ \circlearrowright$$

Kinematics: When the ball rolls, the instant center of rotation is at C, and when



$$t = t_1 \quad v = r\omega \qquad (1)$$

$$\bar{v} = \bar{v}_0 - \bar{a}t = \bar{v}_0 - \mu_k g^t \qquad (2)$$

$$\omega = -\omega_0 + \alpha t = -\omega_0 + \frac{5}{2}\frac{\mu_k g}{r}t$$

When $t = t_1$:

Eq. (1): $v = r\omega$:

$$\bar{v}_0 - \mu_k g t_1 = \left(-\omega_0 + \frac{5}{2}\frac{\mu_k g}{r}t_1\right)r$$

$$\bar{v}_0 - \mu_k g t_1 = -\omega_0 r + \frac{5}{2}\mu_k g t_1$$

$$t_1 = \frac{2}{g}\frac{(\bar{v}_0 + r\omega_0)}{\mu_k g} \qquad (3)$$

$$\bar{v}_0 = 15\text{ ft/s},\quad \omega_0 = 9\text{ rad/s},\quad r = 4\text{ in.} = \frac{1}{3}\text{ft}$$

(*a*)

$$t_1 = \frac{2}{7}\frac{\left(15 + \frac{1}{3}(9)\right)}{0.1(32.2)} = 1.5972\text{ s}$$

$t_1 = 1.597\text{ s}$ ◀

PROBLEM 16.69 (Continued)

(*b*) Eq. (2):

$$\bar{v}_1 = v_0 - \mu_k g t_1$$
$$= 15 - 0.1(32.2)(1.5972)$$

$$\bar{v}_1 = 15 - 5.1429$$
$$= 9.857\text{ ft/s}$$

$\bar{v}_1 = 9.86\text{ ft/s}$ ◀

(*c*)

$$\overline{a} = \mu_k g = 0.1(32.2\text{ ft/s}^2) = 3.22\text{ ft/s}^2 \longleftarrow$$

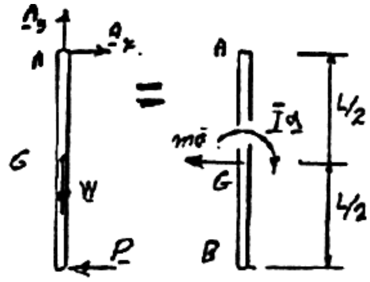
$$\xrightarrow{+}\; s_1 = \overline{v}_0 t_1 - \frac{1}{2}\overline{a}t_1^2$$

$$= (15\text{ ft/s})(1.597\text{ s}) - \frac{1}{2}(3.22\text{ ft/s}^2)(1.597\text{ s})^2$$

$$= 23.96 - 4.11 = 19.85\text{ ft}$$

$s_1 = 19.85\text{ ft}$ ◀

Chapter 16, Solution 76



$$\bar{a} = \frac{1}{2}\alpha \qquad \bar{I} = \frac{1}{12}mL^2$$

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$$

$$= \left(m\frac{L}{2}\alpha\right)\frac{L}{2} + \frac{1}{12}mL^2\alpha$$

$$PL = \frac{1}{3}mL^2\alpha$$

(*a*) Angular acceleration.

$$\alpha = \frac{3P}{mL}$$

$$= \frac{3(1.5\text{ lb})}{\left(\frac{4\text{ lb}}{32.2\text{ ft/s}^2}\right)(3\text{ ft})}$$

$$= 12.08\text{ rad/s}^2$$

$\boldsymbol{\alpha} = 12.08\text{ rad/s}^2 \circlearrowright$ ◀

(*b*) Components of the reaction at *A*.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A_y - W = 0$$

$$A_y = (N - 4\text{ lb})$$

$\mathbf{A}_y = 4.00\text{ lb}\uparrow$ ◀

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A_x - P = -m\bar{a}$$

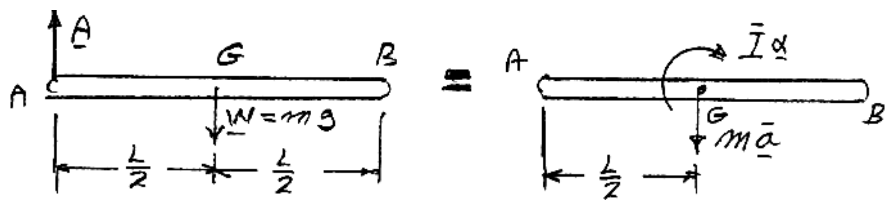
$$A_x = P - m\left(\frac{L}{2}\alpha\right) = P - m\frac{L}{2}\left(\frac{3P}{mL}\right) = -\frac{P}{2}$$

$$A_x = -\frac{P}{2} = -\frac{1.5\text{ lb}}{2} = -0.75\text{ lb}$$

$\mathbf{A}_x = 0.750\text{ lb} \longleftarrow$ ◀

Chapter 16, Solution 84

$$w = 0 \quad \bar{a} = \frac{L}{2}\alpha$$



$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad W\frac{L}{2} = \bar{I}\alpha + m\bar{a}\frac{L}{2}$$

$$mg\frac{L}{2} = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{2}\alpha\right)\frac{L}{2}$$

$$mg\frac{L}{2} = \frac{1}{3}mL^2\alpha \quad \alpha = \frac{3}{2}\frac{g}{L} \circlearrowright$$

(*b*) Reaction at *A*.

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A - mg = -m\bar{a} = -m\frac{L}{2}\alpha$$

$$A - mg = -m\left(\frac{L}{2}\right)\left(\frac{3}{2}\frac{g}{L}\right)$$

$$A - mg = -\frac{3}{4}mg$$

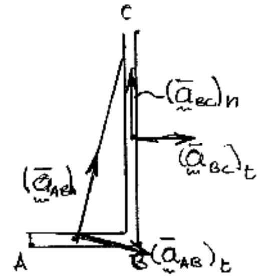
$$A = \frac{1}{4}mg \qquad \mathbf{A} = \frac{1}{4}mg \uparrow \blacktriangleleft$$

(*a*) Acceleration of *B*.

$$\mathbf{a}_B = \mathbf{a}_n + \mathbf{a}_{B/A} = 0 + L\alpha \downarrow$$

$$\mathbf{a}_B = L\left(\frac{3}{2}\frac{g}{L}\right) = \frac{3}{2}g \downarrow \qquad \mathbf{a}_B = \frac{3}{2}g \downarrow \blacktriangleleft$$

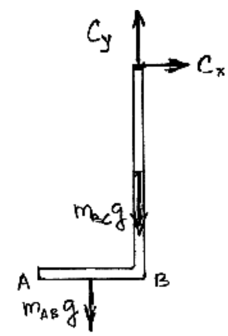
Chapter 16, Solution 87



Masses and lengths: $m_{AB} = 1\text{ kg}, \quad L_{AB} = 0.6\text{ m}, \quad m_{BC} = 2\text{ kg}, \quad L_{BC} = 0.3\text{ m}$

Moments of inertia: $\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2 = \frac{1}{12}(1)(0.3)^2 = 7.5\times 10^{-3}\text{ kg}\cdot\text{m}^2$

$$I_{BC} = \frac{1}{12} m_{BC} L_{BC}^2 = \frac{1}{12}(2)(0.6)^2 = 60\times 10^{-3}\text{ kg}\cdot\text{m}^2$$



Geometry: $r_{AB} = \sqrt{0.6^2 + 0.15^2} = 0.61847\text{ m}$

$$r_{BC} = \frac{1}{2} L_{BC} = 0.3\text{ m}$$

$$\tan\beta = \frac{0.15}{0.6}$$

$$\beta = 14.036°$$

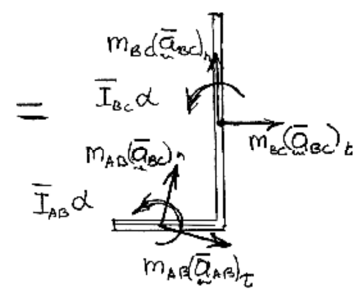
Kinematics: Let $\boldsymbol{\alpha} = \alpha$ ↻ be the angular acceleration of object ABC.

$$(\mathbf{a}_{AB})_t = r_{AB}\alpha \searrow \beta$$

$$(\bar{\mathbf{a}}_B)_n = r_{AB}\omega^2 \nearrow \beta$$

$$(\bar{\mathbf{a}}_{BC})t = r_{BC}\alpha \longrightarrow$$

$$(\bar{\mathbf{a}}_{BC})_n = r_{BC}\omega^2 \uparrow$$



Kinetics: $+↻\Sigma M_C = +↻\Sigma(M_C)_{\text{eff}}$: $\quad m_{AB}g\frac{L_{AB}}{2} = \bar{I}_{AB}\alpha + r_{AB}m_{AB}(\bar{a}_{AB})t$

$$+\bar{I}_{BC}\alpha + r_{BC}m_{BC}(\bar{a}_{BC})t$$

$$= \left(\bar{I}_{AB} + m_{AB}r_{AB}^2 + \bar{I}_{BC} + m_{BC}r_{BC}^2\right)\alpha$$

$$(1)(9.81)(0.15) = [(7.5\times 10^{-3}) + (1)(0.61847)^2$$

$$+(60\times 10^{-3}) + (2)(0.3)^2]\alpha \qquad \boldsymbol{\alpha} = 2.3357\text{ rad/s}^2 ↻$$

$\xrightarrow{+}\Sigma F_X = \xrightarrow{+}\Sigma(F_X)_{\text{eff}}$: $\quad C_x = m_{AB}(\bar{a}_{AB})_t\cos\beta + m_{AB}(\bar{a}_{AB})_n\sin\beta + m_{BC}(a_{BC})_t$

$$= m_{AB}r_{AB}(\alpha\cos\beta + \omega^2\sin\beta) + m_{BC}r_{BC}\alpha$$

$$= (1)(0.61847)(2.3357\cos 14.035° + 10^2\sin 14.035°) + (2)(0.3)(2.3357)$$

$$= 17.802\text{ N}$$

$+\uparrow\Sigma F_y = +\uparrow\Sigma(F_y)_{\text{eff}}$: $\quad C_y - m_{AB}g - m_{BC}g = -m_{AB}(\bar{a}_{AB})_t\sin\beta + m_{AB}(\bar{a}_{AB})_n\cos\beta + m_{BC}(\bar{a}_{BC})_n$

PROBLEM 16.87 (Continued)

$$C_y = (m_{AB} + m_{BC})g + m_{AB} r_{AB} (\omega^2 \cos\beta - \alpha \sin\beta) + m_{BC} r_{BC}\, \omega$$

$$C_y = (3)(9.81) + (1)(0.61847)(10^2 \cos 14.035° - 2.3357 \sin 14.035°)$$

$$+ (2)(0.3)(10)^2$$

$$C_y = 149.08 \text{ N}$$

Reaction at *C*.

$$C = \sqrt{17.802^2 + 149.08^2}$$

$$= 150.1 \text{ N}$$

$$\tan\beta = \frac{17.802}{149.08}$$

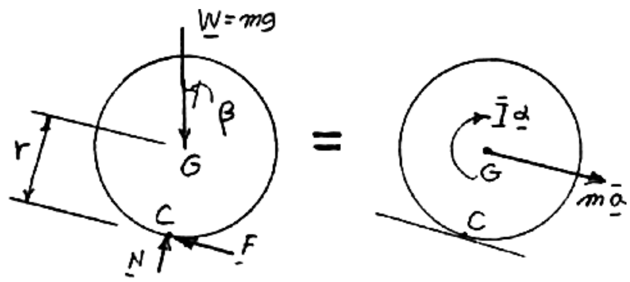
$$\beta = 83.2°$$

$$\mathbf{C} = 150.1 \text{ N} \measuredangle\, 83.2° \blacktriangleleft$$

Chapter 16, Solution 94

$$\bar{I}\alpha = m\bar{k}^2\alpha$$

$$m\bar{a} = mr\alpha$$



$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (W\sin\beta)r = (m\bar{a})r + \bar{I}\alpha$$

$$(mg\sin\beta)r = (mr\alpha)r + m\bar{k}^2\alpha$$

$$rg\sin\beta = (r^2 + \bar{k}^2)\alpha$$

$$\alpha = \frac{rg\sin\beta}{r^2 + \bar{k}^2}$$

$$\bar{a} = r\alpha = r\frac{rg\sin\beta}{r^2 + k^2}$$

$$\bar{a} = \frac{r^2}{r^2 + \bar{k}^2}g\sin\beta \blacktriangleleft$$

Chapter 16, Solution 141

Data: $l = 6$ m, $m = 50$ kg, $P = 500$ N, $\mu_k = 0.3$

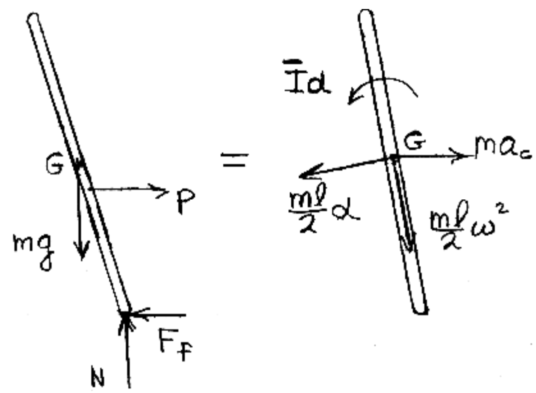
$$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}(50)(6)^2 = 150 \text{ kg}\cdot\text{m}^2$$

Kinematics: $\boldsymbol{\alpha} = \alpha \circlearrowleft$ $\mathbf{a}_C = a_C \longrightarrow$

$$\mathbf{a}_G = \mathbf{a}_C + (\mathbf{a}_{G/C})_t + (\mathbf{a}_{C/G})_n$$

$$= [a_C \longrightarrow] + \left[\frac{l}{2}\alpha \swarrow 10°\right] + \frac{l}{2}\omega^2 \searrow 80° \qquad (1)$$

Kinetics: Sliding to the right: $F_f = \mu_k N$



$+\circlearrowleft \Sigma M_G = +\circlearrowleft \Sigma(M_G)_{\text{eff}}$: $N\frac{l}{2}\sin 10° - F_f\frac{l}{2}\cos 10° + P\left(\frac{l}{2}\cos 10° - h\right) = \bar{I}\alpha$

$$\bar{I}\alpha - N\frac{l}{2}\cos 10° + \mu_k N\frac{l}{2}\sin 10° = P\left(\frac{l}{2}\cos 10° - h\right)$$

$$150\alpha - (3)(\sin 10° - 0.3\cos 10°)N = 500(3\cos 10° - 2) \qquad (2)$$

$+\uparrow \Sigma F_y = +\uparrow \Sigma(F_y)_{\text{eff}}$: $N - mg = -\left(\frac{ml}{2}\alpha\right)\sin 10° - \frac{ml}{2}\omega^2\cos 10°$

$$\left(\frac{ml}{2}\sin 10°\right)\alpha + N = mg - \frac{ml}{2}\omega^2\cos 10°$$

$$[(50)(3)\sin 10°]\alpha + N = (50)(9.81) - (50)(3)(1)^2\cos 10° \qquad (3)$$

Solving Eqs. (2) and (3) simultaneously,

$$\alpha = 2.5054 \text{ rad/s} \qquad N = 277.52 \text{ N}$$

$\xrightarrow{+} \Sigma F_x = \xrightarrow{+} \Sigma(F_x)_{\text{eff}}$: $P - F_f = ma_C - \frac{ml}{2}\alpha\cos 10° + \frac{ml}{2}\omega^2\sin 10°$

PROBLEM 16.141 (Continued)

$$ma_C = P - \mu_k N + \frac{ml}{2}\alpha\cos 10° - \frac{ml}{2}\omega^2 \sin 10°$$

$$50a_C = 500 - (0.3)(277.52) + (50)(3)(2.5054)\cos 10°$$
$$-(50)(3)(1)^2 \sin 10°$$

$$a_C = 15.2159 \text{ m/s}$$

Using Eq. (1),

$$\mathbf{a}_G = 15.2159 + (3)(2.5054) \swarrow 10° + (3)(1)^2 \searrow 80°$$

$$(a_G)_x = 15.2159 - 7.5162\cos 10° + 3\cos 80°$$
$$= 8.3348 \text{ m/s}^2$$

$$(a_G)_y = -7.5162\sin 10° - 3\sin 80°$$
$$= -4.2596 \text{ m/s}^2$$

$$a_G = \sqrt{(8.3348)^2 + (4.2596)^2}$$
$$= 9.36 \text{ m/s}^2$$

$$\tan\beta = \frac{4.2586}{8.3348}$$

$$\beta = 27.1° \qquad \mathbf{a}_G = 9.36 \text{ m/s}^2 \searrow 27.1° \blacktriangleleft$$

(*a*) Acceleration at Point *G*.

(*b*) Normal force. $\mathbf{N} = 278 \text{ N} \uparrow \blacktriangleleft$