ME 104: Homework 7 Solutions

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Chapter 15, Solution 8

$$\omega_0 = 6900 \text{ rpm}\left(\frac{2\pi}{60}\right)$$

$$= 722.57 \text{ rad/s}$$

$$t = 4 \text{ min} = 240 \text{ s}$$

(*a*) $\omega = \omega_0 + \alpha t; \quad 0 = 722.57 + \alpha(240)$

$$\alpha = -3.0107 \text{ rad/s} \qquad \alpha = -3.01 \text{ rad/s}^2 \blacktriangleleft$$

(*b*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (722.57)(240) + \frac{1}{2}(-3.0107)(240)^2$

$$\theta = 173,416 - 86,708 = 86,708 \text{ rad}$$

$$\theta = 86,708 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) \qquad \theta = 13,800 \text{ rev} \blacktriangleleft$$

Chapter 15, Solution 10

$$\overrightarrow{AC} = (350 \text{ mm})\mathbf{i} - (200 \text{ mm})\mathbf{j} + (200 \text{ mm})\mathbf{k} \quad AC = 450 \text{ mm}$$

$$\boldsymbol{\lambda}_{AC} = \frac{\overrightarrow{AC}}{AC} = \frac{350\mathbf{i} - 200\mathbf{j} + 200\mathbf{k}}{450} = \frac{1}{9}(7\mathbf{i} - 4\mathbf{j} + 4\mathbf{k})$$

$$\boldsymbol{\omega} = \omega\boldsymbol{\lambda}_{AC} = (9 \text{ rad/s})\frac{1}{9}(7\mathbf{i} - 4\mathbf{j} + 4\mathbf{k})$$

$$\boldsymbol{\omega} = (7 \text{ rad/s})\mathbf{i} - (4 \text{ rad/s})\mathbf{j} + (4 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\alpha} = 0$$

Corner F:

$$\mathbf{r}_{F/B} = (-175 \text{ mm})\mathbf{i} + (100 \text{ mm})\mathbf{k}$$
$$= -(0.175 \text{ m})\mathbf{i} + (0.100 \text{ m})\mathbf{k}$$

$$\mathbf{v}_F = \boldsymbol{\omega} \times \mathbf{r}_{F/B}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & 4 \\ -0.175 & 0 & 0.100 \end{vmatrix}$$
$$= -0.4\mathbf{i} + (-0.7 - 0.7)\mathbf{j} - 0.7\mathbf{k}$$

$$\mathbf{v}_F = -(0.4 \text{ m/s})\mathbf{i} - (1.4 \text{ m/s})\mathbf{j} - (0.7 \text{ m/s})\mathbf{k} \blacktriangleleft$$

$$\alpha = 0$$

$$\mathbf{a}_F = \boldsymbol{\alpha} \times \mathbf{r}_{F/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{F/B})$$
$$= 0 + \boldsymbol{\omega} \times \mathbf{v}_F$$

$$\mathbf{a}_F = \boldsymbol{\omega} \times \mathbf{v}_F$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & 4 \\ -0.4 & -1.4 & -0.7 \end{vmatrix}$$
$$= (2.8 + 5.6)\mathbf{i} + (-1.6 + 4.9)\mathbf{j} + (-9.8 - 1.6)\mathbf{k}$$

$$\mathbf{a}_F = (8.4 \text{ m/s}^2)\mathbf{i} + (3.3 \text{ m/s}^2)\mathbf{j} - (11.4 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

Chapter 15, Solution 18

Uniformly accelerated motion.

$$\omega = \omega_0 + \alpha t = 0 + \alpha t \quad \omega = \alpha t$$

$$a_t = r\alpha \quad a_n = r\omega^2 = r\alpha^2 t^2$$

$$a_B^2 = a_t^2 + a_n^2 = r^2\alpha^2 + r^2\alpha^4 t^4 = r^2\alpha^2(1 + \alpha^2 t^4)$$

$$a_B = r\alpha(1 + \alpha^2 t^4)^{1/2}$$

$$r = 0.2 \text{ m}, \ \alpha = 0.3 \text{ rad/s}^2$$

$$a_B = (0.2)(0.3)(1 + (0.3)^2 t^4)^{1/2}$$

$$= 0.06(1 + 0.09t^4)^{1/2}$$

(*a*) $t = 0$: $a_B = 0.06(1 + 0)$ $a_B = 0.0600 \text{ m/s}^2$ ◀

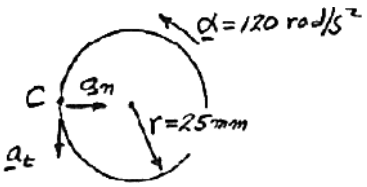
(*b*) $t = 2$ s: $a_B = 0.06(1 + 0.09 \times 2^4)^{1/2}$ $a_B = 0.0937 \text{ m/s}^2$ ◀

(*c*) $t = 4$ s: $a_B = 0.06(1 + 0.09 \times 4^4)^{1/2}$ $a_B = 0.294 \text{ m/s}^2$ ◀

Chapter 15, Solution 23

$$a_t = r\alpha = (0.025 \text{ m})(120 \text{ rad/s}^2)$$

$$\mathbf{a}_t = 3 \text{ m/s}^2 \downarrow$$



(*a*) $t = 0.5$ s:

$$\omega = \alpha t = (120 \text{ rad/s}^2)(0.5 \text{ s}) = 60 \text{ rad/s}$$

$$a_n = r\omega^2 = (0.025 \text{ m})(60 \text{ rad/s})^2$$

$$\mathbf{a}_n = 90 \text{ m/s}^2 \rightarrow$$

$$a_B^2 = a_t^2 + a_n^2 = 3^2 + 90^2 \qquad a_B = 90.05 \text{ m/s}^2 \blacktriangleleft$$

(*b*) $t = 2$ s:

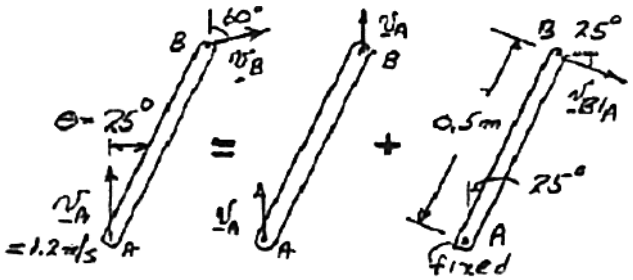
$$\omega = \alpha t = (120 \text{ rad/s}^2)(2 \text{ s}) = 240 \text{ rad/s}$$

$$a_n = r\omega^2 = (0.025 \text{ m})(240 \text{ rad/s})^2$$

$$a_n = 1440 \text{ m/s}^2$$

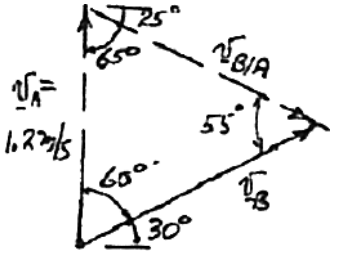
$$a_B^2 = a_t^2 + a_n^2 = 3^2 + 1440^2 \qquad a_B = 1440 \text{ m/s}^2 \blacktriangleleft$$

Chapter 15, Solution 41



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[v_B \angle 30°] = [1.2 \text{ m/s} \uparrow] + [v_{B/A} \searrow 25°]$$



Law of sines.

$$\frac{v_B}{\sin 65°} = \frac{v_{B/A}}{\sin 60°} = \frac{1.2 \text{ m/s}}{\sin 55°}$$

(*b*)

$$\mathbf{v}_B = 1.328 \text{ m/s} \angle 30° \blacktriangleleft$$

$$\mathbf{v}_{B/A} = 1.269 \text{ m/s} \searrow 65°$$

$$v_{B/A} = (AB)\omega_{AB}$$

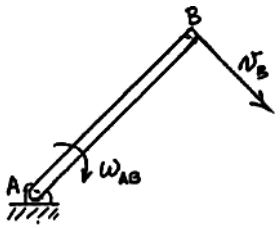
$$1.269 \text{ m/s} = (0.5 \text{ m})\omega_{AB}$$

$$\omega_{AB} = 2.538 \text{ rad/s}$$

(*a*)

$$\boldsymbol{\omega}_{AB} = 2.54 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

Chapter 15, Solution 50

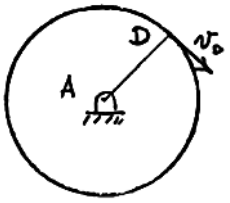


Arm AB: $\omega_{AB} = 90 \text{ rpm} = 3\pi \text{ rad/s}$ ↻

$\mathbf{v}_B = r_{AB}\omega_{AB} = (15)(3\pi) = 45\pi \text{ in./s}$ ↘

Gear A: $\omega_A = 120 \text{ rpm} = 4\pi \text{ rad/s}$

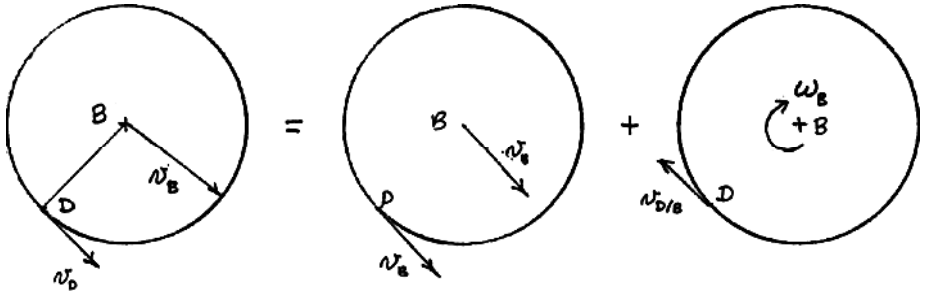
$v_D = r_A\omega_A = (6)(4\pi) = 24\pi \text{ in./s}$ ↘



Gear B: $\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$

24π ↘ $= 45\pi$ ↘ $+ \mathbf{v}_{D/B}$

$v_{D/B} = 21\pi \text{ in./s}$ ↖



$\omega_B = \dfrac{v_{D/B}}{r_B} = \dfrac{21\pi}{9} \text{ rad/s}$ ↻ $\omega_B = 70 \text{ rpm}$ ↻ ◀

Chapter 15, Solution 89

Locate the instantaneous center at intersection of lines draw perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

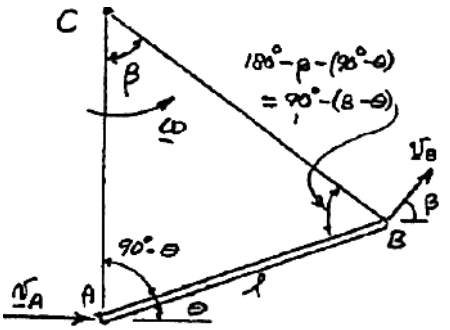
Law of sines.

$$\frac{AC}{\sin[90^\circ-(\beta-\theta)]}=\frac{BC}{\sin(90^\circ-\theta)}=\frac{l}{\sin\beta}$$

$$\frac{AC}{\cos(\beta-\theta)}=\frac{BC}{\cos\theta}=\frac{l}{\sin\beta}$$

$$AC=l\frac{\cos(\beta-\theta)}{\sin\beta}$$

$$BC=l\frac{\cos\theta}{\sin\beta}$$



Angular velocity:

$$v_A=(AC)\omega=l\frac{\cos(\beta-\theta)}{\sin\beta}\omega$$

$$\omega=\frac{v_A}{l}\cdot\frac{\sin\beta}{\cos(\beta-\theta)}$$

Velocity of B:

$$v_B=(BC)\omega=l\frac{\cos\theta}{\sin\beta}\cdot\left[\frac{v_A}{l}\cdot\frac{\sin\beta}{\cos(\beta-\theta)}\right]$$

$$v_B=v_A\frac{\cos\theta}{\cos(\beta-\theta)}$$

Data:

$$\theta=20^\circ,\quad \beta=50^\circ,\quad l=0.6\text{ m},\quad v_A=3\text{ m/s}$$

(a)

$$\omega=\frac{v_A}{l}\frac{\sin\beta}{\cos(\beta-\theta)}=\frac{3\text{ m/s}}{0.6\text{ m}}\cdot\frac{\sin 50^\circ}{\cos(50^\circ-20^\circ)}$$

$$\omega=4.423\text{ rad/s}$$

$\boldsymbol{\omega}=4.42$ rad/s ◀

(b)

$$v_B=v_A\frac{\cos\theta}{\cos(\beta-\theta)}$$

$$=(3\text{ m/s})\frac{\cos 20^\circ}{\cos(50^\circ-20^\circ)}$$

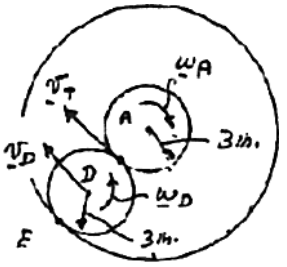
$$v_B=3.2552\text{ m/s}$$

$\mathbf{v}_B=3.26$ m/s ∠ 50° ◀

Chapter 15, Solution 119

Velocity. T = Tooth of gear D in contact with gear A

Gears: $$v_T = r\omega_A = (3 \text{ in.})\omega_A$$



Since $v_E = 0$, E is instantaneous center of gear D.

$$v_T = 2r\omega_D$$

$$(3 \text{ in.})\omega_A = 2(3 \text{ in.})\omega_D$$

$$\omega_D = \frac{1}{2}\omega_A$$

$$v_D = r\omega_D = (3 \text{ in.})\frac{1}{2}\omega_A = (1.5 \text{ in.})\omega_A$$

Spider:

$$v_D = (6 \text{ in.})\omega_S$$

$$(1.5 \text{ in.})\omega_A = (6 \text{ in.})\omega_S$$

$$\omega_S = \frac{1}{4}\omega_A$$

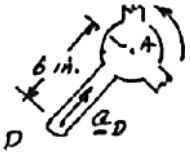
$$\boldsymbol{\omega}_A = 150 \text{ rpm} = 15.708 \text{ rad/s}$$

$$\boldsymbol{\omega}_D = \frac{1}{2}\omega_A = 7.854 \text{ rad/s}$$

$$\boldsymbol{\omega}_S = \frac{1}{4}\omega_A = 3.927 \text{ rad/s}$$

Acceleration.

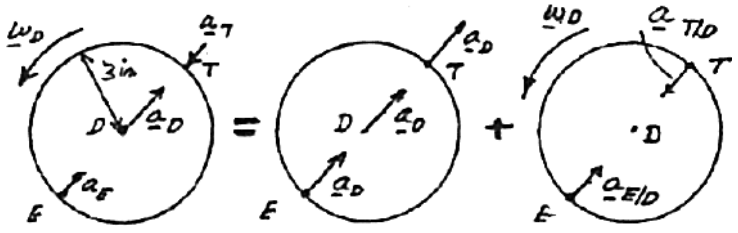
Spider: $$\omega_S = 3.927 \text{ rad/s}$$



$$a_D = (AD)\omega_S^2 = (6 \text{ in.})(3.927 \text{ rad/s})^2$$

$$\mathbf{a}_D = 92.53 \text{ in./s}^2$$

Gear D:



Plane motion = Trans. with D + Rotation about D

PROBLEM 15.119 (Continued)

(*a*) Tooth *T* in contact with gear *A*.

$$\mathbf{a}_T = \mathbf{a}_D + \mathbf{a}_{T/D} = a_D + (DT)\omega_D^2$$

$$= 92.53 \text{ in./s}^2 \nearrow + (3 \text{ in.})(7.854 \text{ rad/s})^2 \swarrow$$

$$= 92.53 \text{ in./s}^2 \nearrow + 185.06 \text{ in./s}^2 \swarrow$$

$$\mathbf{a}_T = 92.53 \text{ in./s}^2 \swarrow$$

$$a_T = 92.5 \text{ in./s}^2 \blacktriangleleft$$

(*b*) Tooth *E* in contact with gear *E*.

$$\mathbf{a}_E = \mathbf{a}_D + \mathbf{a}_{E/D} = \mathbf{a}_D + (ED)\omega_D^2$$

$$= 92.53 \text{ in./s}^2 \nearrow + (3 \text{ in.})(7.854 \text{ rad/s})^2 \nearrow$$

$$= 92.53 \text{ in./s}^2 \nearrow + 185.06 \text{ in./s}^2 \nearrow$$

$$\mathbf{a}_E = 277.6 \text{ in./s}^2 \nearrow$$

$$a_E = 278 \text{ in./s}^2 \blacktriangleleft$$