ME 104: Homework 6 Solutions

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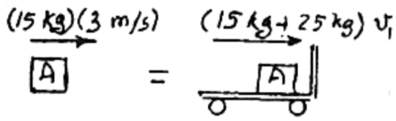
Chapter 14, Solution 1

There are no horizontal external forces acting during the impacts. The baggage carrier is to coast between impacts.

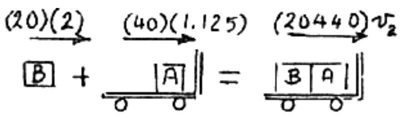
(*a*) 15-kg suitcase tossed on carrier first:

Let v_1 be the common velocity of suitcase *A* and the carrier after the first impact and v_2 be the common velocity of the two suitcases and the carrier of the second impact.



Conservation of momentum:

$$(15)(3) = (40)v_1 \qquad \mathbf{v}_1 = 1.125 \text{ m/s} \longrightarrow$$

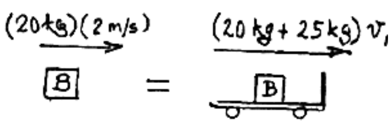


20-kg suitcase tossed next:

$$(20)(2) + (40)(1.125) = 60v_2 \qquad \mathbf{v}_2 = 1.417 \text{ m/s} \longrightarrow \blacktriangleleft$$

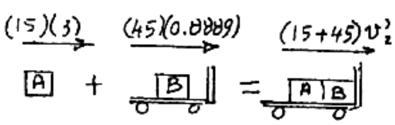
(*b*) 20-kg suitcase tossed on carrier first:

Let v_1' be the common velocity of suitcase *B* and the carrier after the first impact and v_2' be the common velocity of all after the second impact.



Conservation of momentum:

$$(20)(2) = (45)v_1' \qquad \mathbf{v}_1' = 0.8889 \text{ m/s} \longrightarrow$$



15-kg suitcase tossed next:

$$(15)(3) + (45)(0.8889) = 60v_2'$$

$$\mathbf{v}_2' = 1.417 \text{ m/s} \longrightarrow \blacktriangleleft$$

Chapter 14, Solution 5

The masses are m for the bullet and m_A and m_B for the blocks.

(a) The bullet passes through block A and embeds in block B. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) + m_B(0) = mv_0$

Final momentum: $mv_B + m_A v_A + m_B v_B$

Equating, $mv_0 = mv_B + m_A v_A + m_B v_B$

$$m = \frac{m_A v_A + m_B v_B}{v_0 - v_B} = \frac{(6)(5) + (4.95)(9)}{1500 - 9} = 0.0500 \text{ lb}$$

$m = 0.800$ oz ◀

(b) The bullet passes through block A. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) = mv_0$

Final momentum: $mv_1 + m_A v_A$

Equating, $mv_0 = mv_1 + m_A v_A$

$$v_1 = \frac{mv_0 - m_A v_A}{m} = \frac{(0.0500)(1500) - (6)(5)}{0.0500} = 900 \text{ ft/s}$$

$\mathbf{v}_1 = 900$ ft/s $\longrightarrow$ ◀

Chapter 14, Solution 019

Let t be the time elapsed since the first collision. No external forces in the xy plane act on the system consisting of cars A, B, and C during the impacts with one another. The mass center of the system moves at the velocity it had before the collision.

Initial mass center $\bar{\mathbf{r}}_0$:

$$(m_A + m_B + m_C)(\bar{x}_0\mathbf{i} + \bar{y}_0\mathbf{j}) = m_C(x_C\mathbf{i} + y_C\mathbf{j})$$

$$\bar{x}_0 = 0.3x_C = (0.3)(10) = 3 \text{ m}, \quad \bar{y}_0 = 0.3y_C = (0.3)(3) = 0.9 \text{ m}$$

Given velocities:

$$\mathbf{v}_A = v_A\mathbf{i}, \quad \mathbf{v}_B = (20 \text{ m/s})\mathbf{j}, \quad \mathbf{v}_C = -(25 \text{ m/s})\mathbf{i}$$

Velocity of mass center:

$$(m_A + m_B + m_C)\mathbf{v} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$\bar{\mathbf{v}} = 0.375\mathbf{v}_A + 0.325\mathbf{v}_B + 0.3\mathbf{v}_C$$

Since the collided cars hit the pole at

$$\mathbf{r}_P = x_P\mathbf{i} + y_P\mathbf{j}$$

$$\mathbf{x}_P\mathbf{i} + y_P\mathbf{j} = \bar{x}_0\mathbf{i} + \bar{y}_0\mathbf{j} + \bar{\mathbf{v}}t \qquad \text{Resolve into components.}$$

$$x: \quad x_P = \bar{x}_0 + 0.375v_At_P - 0.3v_Ct_P \qquad (1)$$

$$y: \quad y_P = \bar{y}_0 + 0.325v_Bt_P \qquad (2)$$

PROBLEM 14.19 (Continued)

<u>Data</u>: $x_P = 18 \text{ m}, \; y_P = 13.9 \text{ m}$

(a) From (2), $13.9 = 0.9 + (0.325)(20)t_P$ $\qquad t_P = 2.00 \text{ s}$ ◀

(b) From (1), $18 = 3 + (0.375)v_A(2.00) - (0.3)(25)(2.00)$

$v_A = 40 \text{ m/s}$ $\qquad v_A = 144.0 \text{ km/h}$ ◀

Chapter 14, Solution 39

Velocity vectors: $\mathbf{v}_0 = v_0(\cos 45°\,\mathbf{i} + \sin 45°\,\mathbf{j})$ $\quad v_0 = 15$ ft/s

$$\mathbf{v}_A = v_A\mathbf{j}$$
$$\mathbf{v}_B = v_B(\sin 30°\,\mathbf{i} - \cos 30°\,\mathbf{j})$$
$$\mathbf{v}_C = v_C(\cos 30°\,\mathbf{i} + \sin 30°\,\mathbf{j})$$

Conservation of momentum:

$$m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$

Divide by *m* and resolve into components.

$$\mathbf{i}:\quad v_0 \cos 45° = v_B \sin 30° + v_C \cos 30°$$

$$\mathbf{j}:\quad v_0 \sin 45° = v_A - v_B \cos 30° + v_C \sin 30°$$

Solving for v_B and v_C,

$$v_B = -0.25882 v_0 + 0.86603 v_A$$
$$v_C = 0.96593 v_0 - 0.5 v_A$$

Conservation of energy: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$

Divide by $\frac{1}{2}m$ and substitute for v_B and v_C.

$$v_0^2 = v_A^2 + (-0.25882 v_0 + 0.86603 v_A)^2$$
$$+ (0.96593 v_0 - 0.5 v_A)^2$$
$$= 2v_A^2 + v_0^2 - 1.41422 v_0 v_A$$

$v_A = 0.70711 v_0 = 10.61$ ft/s $\quad v_A = 10.61$ ft/s ◀

$v_B = 0.35355 v_0 = 5.30$ ft/s $\quad v_B = 5.30$ ft/s ◀

$v_C = 0.61237 v_0 = 9.19$ ft/s $\quad v_C = 9.19$ ft/s ◀

Chapter 14, Solution 46

Conservation of angular momentum about Q:

$$\overrightarrow{QA}_0 \times (m\mathbf{u}_0) + \overrightarrow{QB}_0 \times (4m\mathbf{v}_0) + \overrightarrow{QC}_0 \times (4m\mathbf{v}_0) = \overrightarrow{QA}_1 \times (m\mathbf{v}_A) + \overrightarrow{QB}_1 \times (4m\mathbf{v}_B) + \overrightarrow{QC}_1 \times (4m\mathbf{v}_C)$$

$$\overrightarrow{QA}_0 \times (m\mathbf{u}_0) + \overrightarrow{QB}_0 \times (4m\mathbf{v}_0) + 0 = 0 + \overrightarrow{QB}_1 \times (4m\mathbf{v}_B) + 0 \qquad (1)$$

where
$$\overrightarrow{QA}_0 = \mathbf{r}_{A_0} - \mathbf{r}_Q = (300\mathbf{i} + 300\mathbf{k}) - (240\mathbf{i} + 200\mathbf{j} + 100\mathbf{k})$$
$$= (60 \text{ mm})\mathbf{i} - (200 \text{ mm})\mathbf{j} + (200 \text{ mm})\mathbf{k}$$

$$\overrightarrow{QB}_0 = (\Delta x)\mathbf{i} + (\Delta y)\mathbf{j} + (\Delta z)\mathbf{k}$$

$$\overrightarrow{QB}_1 = \mathbf{r}_{B_1} - \mathbf{r}_Q = (147\mathbf{i} + 220\mathbf{j} + 130\mathbf{k}) - (240\mathbf{i} + 200\mathbf{j} + 100\mathbf{k})$$
$$- (93 \text{ mm})\mathbf{i} + (20 \text{ mm})\mathbf{j} + (30 \text{ mm})\mathbf{k}$$

$$\mathbf{u}_0 = -(600 \text{ m/s})\mathbf{i} + (750 \text{ m/s})\mathbf{j} - (800 \text{ m/s})\mathbf{k} \qquad \mathbf{v}_0 = (600 \text{ m/s})\mathbf{j}$$

and from the solution to Problem 14.27,

$$\mathbf{v}_B = v_B \boldsymbol{\lambda}_B = (716.98)(-0.42290\mathbf{i} + 0.89707\mathbf{j} - 0.12815\mathbf{k})$$
$$= -(303.21 \text{ m/s})\mathbf{i} + (643.18 \text{ m/s})\mathbf{j} - (91.88 \text{ m/s})\mathbf{k}$$

Calculating each term and dividing by m,

$$\overrightarrow{QA}_0 \times \mathbf{u}_0 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 60 & -200 & 200 \\ -600 & 750 & -800 \end{vmatrix} = 10{,}000\mathbf{i} - 72{,}000\mathbf{j} - 75{,}000\mathbf{k}$$

$$\overrightarrow{QB}_0 \times (4\mathbf{v}_0) = [(\Delta x)\mathbf{i} + (\Delta y)\mathbf{j} + (\Delta z)\mathbf{k}] \times (2400\mathbf{j})$$
$$= -2400(\Delta z)\mathbf{i} + 2400(\Delta x)\mathbf{k}$$

$$\overrightarrow{QB}_1 \times (4\mathbf{v}_B) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -93 & 20 & 30 \\ -1212.84 & 2572.72 & -367.52 \end{vmatrix} = -84{,}532\mathbf{i} - 70{,}565\mathbf{j} - 215{,}006\mathbf{k}$$

Collect terms and resolve into components.

$\mathbf{i}$: $\quad 10{,}000 - 2400(\Delta z) = -84{,}532 \qquad \Delta z = 39.388 \text{ mm}$

$\mathbf{k}$: $\quad -75{,}000 + 2400(\Delta x) = -215{,}006 \qquad \Delta x = -58.336 \text{ mm}$

Coordinates: $\quad x_{B_0} = x_Q + \Delta x = 240 - 58.336 \qquad x_{B_0} = 181.7 \text{ mm}$ ◀

$y_{B_0} = 0$ ◀

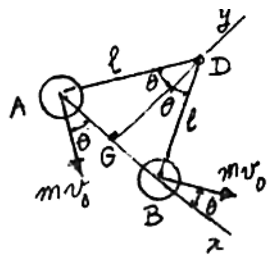
$z_{B_0} = z_Q + \Delta z = 100 + 39.388 \qquad z_{B_0} = 139.4 \text{ mm}$ ◀

Chapter 14, Solution 49

We consider the following two positions of the spheres A and B and the ring D.

Position 1: Immediately after cord CD breaks linear momentum:

$$\mathbf{L} = (2mv_0 \cos\theta)\mathbf{i} \quad (1)$$



Angular momentum about G:

$$H_G = 2(l\sin\theta)(mv_0 \sin\theta)\mathbf{k}$$

$$H_G = (2\,lmv_0 \sin^2\theta)\mathbf{k} \quad (2)$$

Position 2: After cords AD and BD become taut:

(a) Speed of mass center (now located at D).

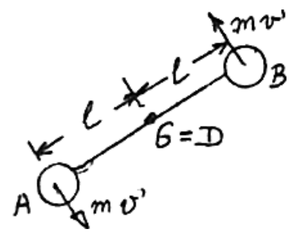
Recalling Eq. (1):

$$L = (2m)\overline{v} = (2\,mv_0 \cos\theta)\mathbf{i} \qquad \overline{v} = (v_0 \cos\theta)\mathbf{i}$$

$$v_D = \overline{v} = v_0 \cos\theta \quad (3)$$

(b) Relative speed v' at which A and B rotate about D.

Angular momentum about G:



$$H_G = (2\,mv')l$$

Recalling Eq. (2):

$$2\,mv'l = 2\,lmv_0 \sin^2\theta$$

$$v' = v_0 \sin^2\theta \quad (4)$$

PROBLEM 14.49 (Continued)

(c) Energy lost:

Considering system of 3 spheres:

Initially, $$v_C = \left(\frac{l'}{l}\right)v_A = (2\cos\theta)v_0$$

Therefore,

$$T_0 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2 = mv_0^2(1 + 2\cos^2\theta)$$

$$T_f = \frac{1}{2}(2m)v_D^2 + 2\left(\frac{1}{2}mv'^2\right) + \frac{1}{2}mv_C^2$$

$$= m\left[v_0^2\cos^2\theta + m(v_0\sin^2\theta)^2 + 2v_0^2\cos^2\theta\right]$$

$$= mv_0^2(3\cos^2\theta + \sin^4\theta)$$

$$\%\text{ loss} = 100\frac{T_0 - T_f}{T_0}$$

$$= 100\frac{1 + 2\cos^2\theta - 3\cos^2\theta - \sin^4\theta}{1 + 2\cos^2\theta}$$

$$= 100\frac{\sin^2\theta\cos^2\theta}{1 + 2\cos^2\theta} \quad (5)$$

Making $\theta = 30°$ in Eqs. (3), (4), and (5)

(a) $0.866v_0$, (b) $0.250\ v_0$, (c) 7.50% ◀

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Chapter 14, Solution 75

Let v be the air liner speed and u be the discharge relative velocity.

$$u = 2000 \text{ ft/s.}$$

Thrust formula for one engine: $F = \frac{dm}{dt}(u - v)$

Drag formula: $D = kv^2$

Three engines working. Cruising speed $= v_0 = 600 \text{ mi/h} = 880 \text{ ft/s}$

$$3F - D = 3\frac{dm}{dt}(u - v_0) - kv_0^2 = 0$$

$$\frac{dm}{dt} = \frac{kv_0^2}{3(u - v_0)} = \frac{k(880)^2}{3(2000 - 880)} = 230.48k$$

(a) *One engine fails*. Two engines working. Cruising speed $= v_1$

$$2F - D = 2\frac{dm}{dt}(u - v_1) - kv_1^2 = 0$$

$$(2)(230.48k)(2000 - v_1) - kv_1^2 = 0$$

$$v_1^2 + 460.96v_1 - 921.92 \times 10^3 = 0$$

$v_1 = 756.96 \text{ ft/s}$ $\qquad v_1 = 516 \text{ mi/h}$ ◀

(b) *Two engines fail*. One engine working. Cruising speed $= v_2$

$$F - D = \frac{dm}{dt}(u - v_2) - kv_2^2 = 0$$

$$(230.48k)(2000 - v_2) - kv_2^2 = 0$$

$$v_2^2 + 230.48v_2 - 460.96 \times 10^3 = 0$$

$v_2 = 573.41 \text{ ft/s}$ $\qquad v_2 = 391 \text{ mi/h}$ ◀