ME 104: Homework 5 Solutions

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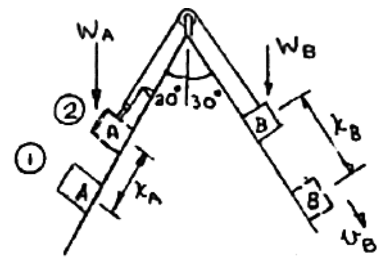
Chapter 13, Solution 19

(*a*) Kinematics:

$$x_B = 2x_A$$
$$v_B = 2v_A$$



A and B. Assume *B* moves down.

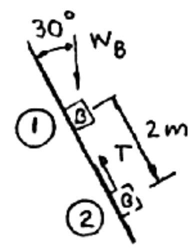
$$v_1 = 0$$
$$T_1 = 0$$
$$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$
$$= \frac{1}{2}(2\text{ kg})\left(\frac{v_B^2}{4} + v_B^2\right)$$
$$T_2 = \frac{5}{4}v_B^2$$
$$U_{1-2} = -m_A g(\cos 30°)(x_A) + m_B g(\cos 30°)x_B$$
$$x_B = 2\text{ m}$$
$$x_A = 1\text{ m}$$
$$U_{1-2} = (2)(9.81)\left(\frac{\sqrt{3}}{2}\right)[-1+2]$$
$$U_{1-2} = 16.99\text{ J}$$

Since work is positive, block *B* does move down.

$$T_1 + U_{1-2} = T_2$$
$$0 + 16.99 = \frac{5}{4}v_B^2$$
$$v_B^2 = 13.59$$

$$\mathbf{v}_B = 3.69\text{ m/s} \searrow 60° \blacktriangleleft$$

(*b*) B alone.



$$v_1 = 0$$
$$T_1 = 0$$
$$v_2 = 3.69\text{ m/s, (from } (a))$$
$$T_2 = \frac{1}{2}m_B v_2^2 = \frac{1}{2}(2)(3.69)^2 = 13.59\text{ J}$$
$$U_{1-2} = (m_B g)(\cos 30°)(x_B) - (T)(x_B)$$
$$U_{1-2} = \left[(2\text{ kg})(9.81\text{ m/s}^2)\left(\frac{\sqrt{3}}{2}\right) - (T)\right](2\text{ m})$$
$$U_{1-2} = 33.98 - 2T$$

PROBLEM 13.19 (Continued)

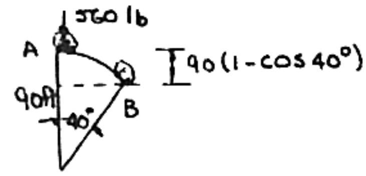
$$T_1 + U_{1-2} = T_2 \quad 0 + 33.98 - 2T = 13.59$$

$$2T = 33.98 - 13.59 = 20.39$$

$T = 10.19\text{ N}$ ◀

Chapter 13, Solution 41

$$v_A = 0 \quad T_A = 0 \quad T_B = \frac{1}{2} m v_B^2 = \frac{1}{2}\left(\frac{560}{g}\right) v_B^2 = \frac{280}{g} v_B^2$$



$$U_{AB} = W(90)(1 - \cos 40°)$$

$$U_{AB} = (560 \text{ lb})(90 \text{ ft})(0.234)$$

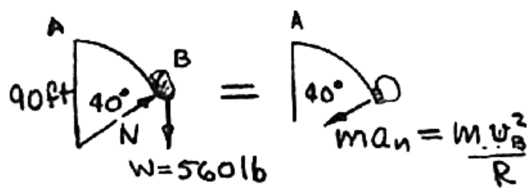
$$U_{AB} = 11,791 \text{ ft} \cdot \text{lb}$$

$$T_A + U_{A-B} = T_B \qquad 0 + 11,791 = \frac{280}{g} v_B^2$$

$$v_B^2 = \frac{(11,791 \text{ ft} \cdot \text{lb})(32.2 \text{ ft/s}^2)}{(280 \text{ lb})}$$

$$v_B^2 = 1356 \text{ ft}^2/\text{s}^2$$

Newton's law at B



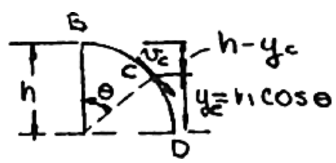
$$+\nearrow N - W\cos 40° = -\frac{m v_B^2}{R}; \quad v_B^2 = 1356 \text{ ft}^2/\text{s}^2$$

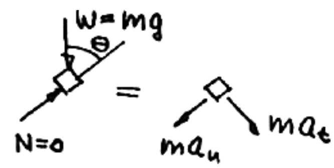
$$N = (560 \text{ lb})(\cos 40°) - \frac{(560 \text{ lb})(1356 \text{ ft}^2/\text{s}^2)}{(32.2 \text{ ft/s}^2)(90 \text{ ft})}$$

$$N = 429 - 262 = 167.0 \text{ lb}$$

$N = 167.0$ lb ◀

Chapter 13, Solution 44





Block leaves surface at C when the normal force $N = 0$.

$$+\swarrow mg\cos\theta = ma_n$$

$$g\cos\theta = \frac{v_C^2}{n}$$

$$v_C^2 = gh\cos\theta = gy \qquad (1)$$

<u>Work-energy principle</u>.

(*a*)

$$T_B = \frac{1}{2}mv^2 = \frac{1}{2}m(8)^2 = 32m$$

$$T_C = \frac{1}{2}mv_C^2 \qquad U_{B-C} = W(h-y) = mg(h-y_C)$$

$$T_B + U_{B-C} = T_C$$

$$32m + mg(h-y) = \frac{1}{2}mv_C^2$$

Using Eq. (1)

$$32 + g(h-y_C) = \frac{1}{2}gy_C \qquad (2)$$

$$32 + gh = \frac{3}{2}gy_C$$

$$y_C = \frac{(32+gh)}{\left(\frac{3}{2}g\right)}$$

$$y_C = \frac{(32+(32.2)(3))}{\frac{3}{2}(32.2)}$$

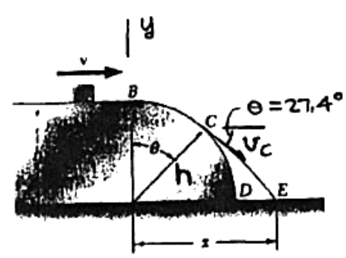
$$y_C = 2.6625\text{ ft} \qquad (3)$$

$$y_C = h\cos\theta \quad \cos\theta = \frac{y_C}{h} = \frac{2.6625}{3} = 0.8875$$

$\theta = 27.4°$ ◀

PROBLEM 13.44 (Continued)

(*b*) From (1) and (3)



$$v_C = \sqrt{gy}$$

$$v_C = \sqrt{(32.2)(2.6625)}$$

$$v_C = 9.259\text{ ft/s}$$

<u>At C</u>: $(v_C)_x = v_C\cos\theta = (9.259)(\cos 27.4°) = 8.220\text{ ft/s}$

$$(v_C)_y = -v_C \sin\theta = -(9.259)(\sin 27.4°) = 4.261 \text{ ft/s}$$

$$y = y_C + (v_C)_y t - \frac{1}{2}gt^2 = 2.6625 - 4.261t - 16.1t^2$$

<u>At *E*:</u> $y_E = 0 \quad t^2 + 0.2647t - 0.1654 = 0$

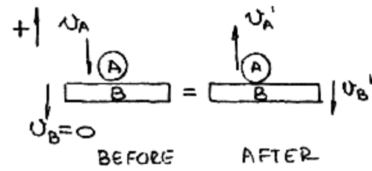
$$t = 0.2953 \text{ s}$$

<u>At *E*:</u> $x = h(\sin\theta) + (v_C)_x t = (3)(\sin 27.4°) + (8.220)(0.2953)$

$$x = 1.381 + 2.427 = 3.808 \text{ ft}$$

$x = 3.81$ ft ◀

Chapter 13, Solution 178



Velocity of *A* and *B* after impact

$$m_A = \frac{W_A}{g} = \frac{1.3}{32.2} = 0.04037 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_B = \frac{W_B}{g} = \frac{2.6}{32.2} = 0.08075 \text{ lb} \cdot \text{sec}^2/\text{ft}$$

Sphere *A* falls. Use conservation of energy to find v_A, the speed just before impact. Use the plate surface as the datum.

$$T_1 = 0, \quad V_1 = m_A g h_0, \quad T_2 = \frac{1}{2} m_A v_A^2, \quad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \qquad 0 + m_A g h_0 = \frac{1}{2} m_A v_A^2 + 0$$

With $h_0 = 1.8$ ft,

$$v_A = \sqrt{2 g h_0} = \sqrt{(2)(32.2)(1.8)}$$

$$\mathbf{v}_A = 10.767 \text{ ft/s} \downarrow$$

Analysis of the impact. Conservation of momentum.

$$m_A \mathbf{v}_A + m_B \mathbf{v}_B = m_A \mathbf{v}_A' + m_B \mathbf{v}_B' \quad \text{with} \quad \mathbf{v}_B = 0$$

Dividing by m_A and using *y*-components $+\uparrow$ with $(m_B/m_A = 2)$

$$-10.767 + 0 = (v_A')_y + 2(v_B')_y \qquad (1)$$

Coefficient of restitution. $(v_B')_y - (v_A')_y = e[(v_A)_y - (v_B)_y]$

$$(v_B')_y - (v_A')_y = e(v_A)_y = -10.767e \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously with $e = 0.8$ gives

$$(v_A')_y = 2.153 \text{ ft/s}$$
$$(v_B')_y = -6.460 \text{ ft/s}$$
$$\mathbf{v}_A' = 2.153 \text{ ft/s} \uparrow,$$
$$\mathbf{v}_B' = 6.460 \text{ ft/s} \downarrow$$

(*a*) Sphere *A* rises. Use conservation of energy to find *h*.

$$T_1 = \frac{1}{2} m_A (v_A')^2, \quad V_1 = 0, \quad T_2 = 0, \quad V_2 = m_A g h$$

$$T_1 + V_1 = T_2 + V_2: \qquad \frac{1}{2} m_A (v_A')^2 + 0 = 0 + m_A g h$$

$$h = \frac{(v_A')^2}{2g} = \frac{(2.153)^2}{(2)(32.2)} \qquad h = 0.0720 \text{ ft} \blacktriangleleft$$

PROBLEM 13.178 (Continued)

(*b*) Plate *B* falls and compresses the spring. Use conservation of energy.

Let δ_0 be the initial compression of the spring and Δ be the additional compression of the spring after impact. In the initial equilibrium state,

$$+\uparrow \Sigma F_y = 0: \quad k\delta_0 - W_B = 0 \quad \text{or} \quad k\delta_0 = W_B \tag{3}$$

Just after impact: $T_1 = \frac{1}{2} m_B (v'_B)^2, \quad V_1 = \frac{1}{2} k\delta_0^2$

At maximum deflection of the plate, $T_2 = 0$

$$V_2 = (V_2)_g + (V_2)_e = -W_B\Delta + \frac{1}{2} k(\delta_0 + \Delta)^2$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m_B (v'_B)^2 + \frac{1}{2} k\delta_0^2 = 0 - W_B\Delta + \frac{1}{2} k\delta_0^2 + k\delta_0\Delta + \frac{1}{2} k\Delta^2$$

Invoking the result of Eq. (3) gives

$$\frac{1}{2} m_B (v'_B)^2 = \frac{1}{2} k\Delta^2 \tag{4}$$

Data: $m_B = 0.08075 \text{ lb} \cdot \text{s}^2/\text{ft}, \quad v'_B = 6.460 \text{ ft/s}$

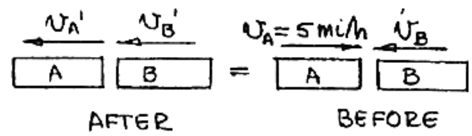
$$\Delta = 3h = (3)(0.072) = 0.216 \text{ ft}$$

$$k = \frac{m_B (v'_B)^2}{\Delta^2} = \frac{(0.08075)(6.460)^2}{(0.216)^2}$$

$k = 72.2$ lb/ft ◀

Chapter 13, Solution 180

(*a*) At *C*:



Conservation of total momentum.

$$m_A = m_B = m$$

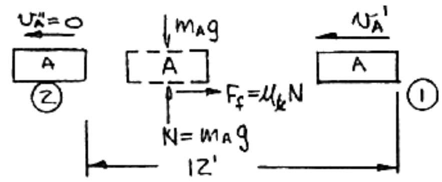
$$5 \text{ mi/h} = 7.333 \text{ ft/s}$$

$$\overset{+}{\leftarrow} \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \qquad (1)$$

$$-7.33 + v_B = v_A' + v_B'$$

Work and energy.

Car *A* (after impact):



$$T_1 = \frac{1}{2} m_A (v_A')^2$$

$$T_2 = 0$$

$$U_{1-2} = F_f (12)$$

$$U_{1-2} = \mu_k m_A g (12 \text{ ft})$$

$$T_1 + U_{1-2} = T_2$$

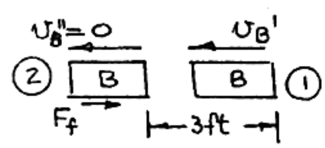
$$\frac{1}{2} m_A (v_A')^2 - \mu_k m_A g (12) = 0$$

$$(v_A')^2 = (2)(12 \text{ ft})(0.3)(32.2 \text{ ft/s}^2)$$

$$= 231.84 \text{ ft/s}^2$$

$$v_A' = 15.226 \text{ ft/s}$$

Car *B* (after impact):



$$T_1 = \frac{1}{2} m_B (v_B')^2$$

$$T_2 = 0$$

$$U_{1-2} = \mu_k \, m_B \, g(3)$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2} m_B (v_B')^2 - \mu_k m_B g(3)$$

$$v_B'^2 = (2)(3 \text{ ft})(0.3)(32.2 \text{ ft/s}^2)$$

$$(v_B')^2 = 57.96 \text{ ft}^2/\text{s}^2$$

$$v_B' = 7.613 \text{ ft/s}$$

PROBLEM 13.180 (Continued)

From (1)

$$v_B = 7.333 + v_A'^2 + v_A'$$
$$= 7.333 + 15.226 + 7.613$$

$v_B = 30.2 \text{ ft/s} = 20.6 \text{ mi/h}$ ◀

(*b*) Relative velocities.

$$\overset{+}{\longleftarrow} \; (-v_A - v_B)e = v_B' - v_A'$$

$$(-7.333 - 30.2)e = 7.613 - 15.226$$

$$e = \frac{-(7.613)}{-(37.53)} = 0.2028$$

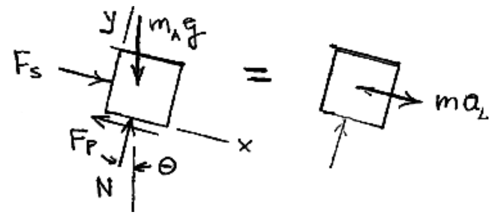
$e = 0.203$ ◀

Chapter 13, Solution 201

Data: $m_A = 2$ kg, $m_B = 1$ kg, $k = 800$ N/m, $x = 0.1$ m, $d = 1.5$ m

$\mu_k = 0.2$, $e = 0.8$, $\theta = 20°$, $\alpha = 40°$, $l = 1.0$ m

Block slides down the incline.



$$+\nearrow \Sigma F_y = 0:$$

$$N - m_A g \cos\theta = 0$$

$$N = m_A g \cos\theta = (2)(9.81)\cos 20° = 18.4368 \text{ N}$$

$$F_f = \mu_k N = (0.2)(18.4368) = 3.6874 \text{ N}$$

Use work and energy. Datum for V_g is the impact point near B.

$$T_1 = 0, \quad (V_1)_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(800)(0.1)^2 = 4.00 \text{ J}$$

$$(V_1)_g = m_A g h_1 = m_A g(x+d)\sin\theta = (2)(9.81)(1.6)\sin 20° = 10.7367 \text{ J}$$

$$U_{1\to 2} = -F_f(x+d) = -(3.6874)(1.6) = -5.8998 \text{ J}$$

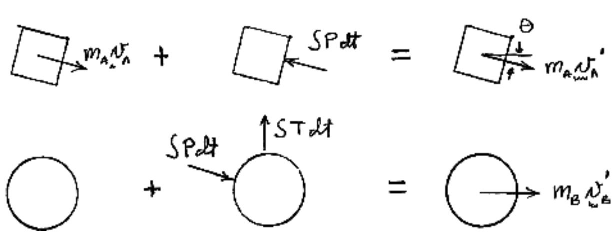
$$T_2 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(1)(v_A^2) = 1.000\, v_A^2 \quad V_2 = 0$$

$$T_1 + V_1 + U_{1\to 2} = T_2 + V_2: \quad 0 + 4.00 + 10.7367 - 5.8998 = 1.000 v_A^2 + 0$$

$$v_A^2 = 8.8369 \text{ m}^2/\text{s}^2$$

$$\mathbf{v}_A = 2.9727 \text{ m/s} \searrow 20°$$

Impact: Conservation of momentum.



Both A and B, horizontal components $\xrightarrow{+}$:

$$m_A v_A \cos\theta + 0 = m_A v_A' \cos\theta + m_B v_B$$

$$(2)(2.9727)\cos 20° = 2v_A' \cos 20° + (1.00)v_B \qquad (1)$$

PROBLEM 13.201 (CONTINUED)

Relative velocities: $(v'_B)_n - (v'_A)_n = e[(v_B)_n - (v_A)_n]$

$$v'_B \cos\theta - v'_A = e[v_A - 0]$$

$$v'_B \cos 20° - v'_A = (0.8)(2.9727) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$v'_A = 1.0382 \text{ m/s}$$

$$v'_B = 3.6356 \text{ m/s}$$

Sphere *B* rises: Use conservation of energy.

$$T_1 = \frac{1}{2} m_B (v'_B)^2 \quad V_1 = 0$$

$$T_2 = \frac{1}{2} m_B v_2^2 \qquad V_2 = m_B g h_2 = m_B g l (1 - \cos\alpha)$$

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2} m_B (v'_B)^2 + 0 = \frac{1}{2} m_B v_2^2 + m_B g (1 - \cos\alpha)$$

$$v_2^2 = (v'_B)^2 - 2gl(1 - \cos\alpha)$$

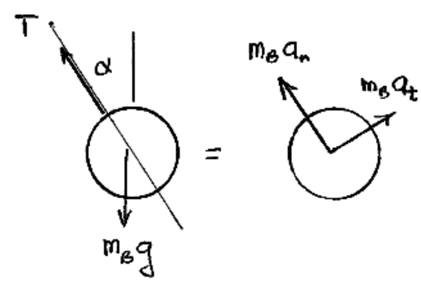
$$= (3.6356)^2 - (2)(9.81)(1 - \cos 40°)$$

$$= 8.6274 \text{ m}^2/\text{s}^2$$

(*a*) Speed of *B*. $v_2 = 2.94$ m/s ◀

(*b*) Tension in the rope. $\rho = 1.00$ m

$$a_n = \frac{v_2^2}{\rho} = \frac{8.6274}{1.00} = 8.6274 \text{ m/s}^2$$



$$+\nwarrow \sum F_n = m_B a_n:$$

$$T - m_B g \cos\alpha = m_B a_n$$

$$T = m_B (a_n + g\cos\alpha)$$

$$= (1.0)(8.6274 + 9.81\cos 40°)$$

$T = 16.14$ N ◀